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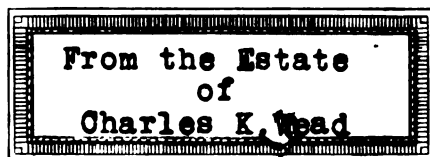
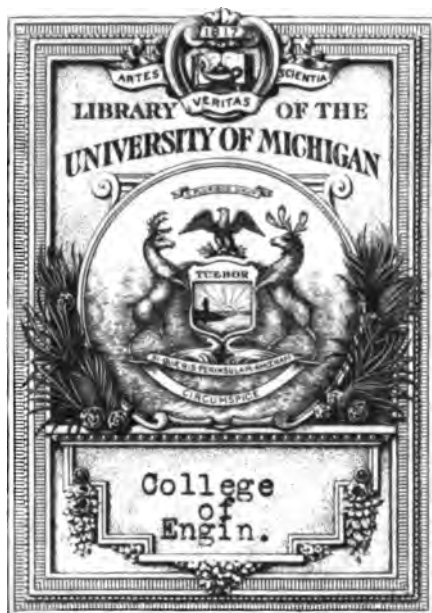
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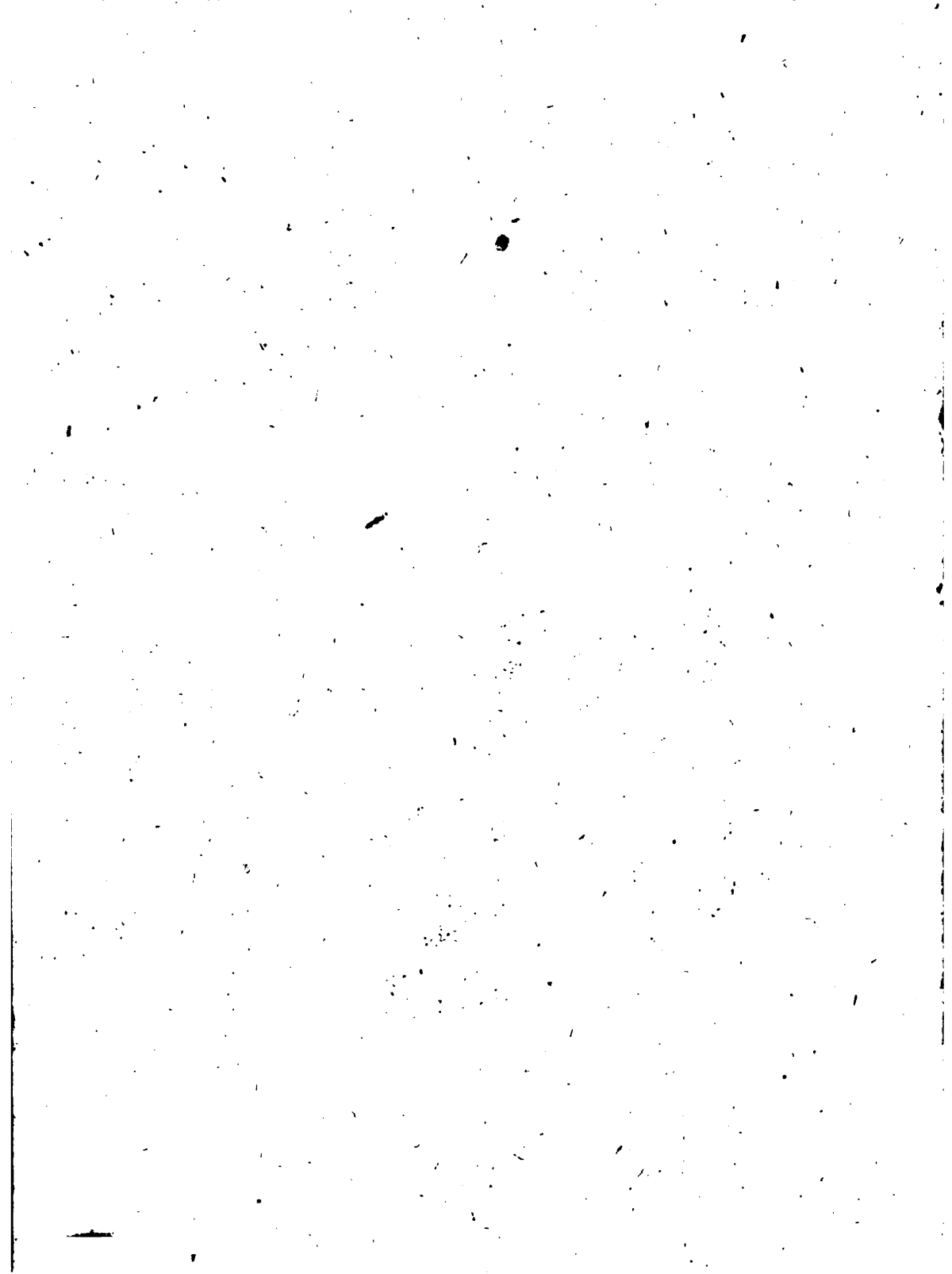


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**Integraltafeln,**

oder

**Sammlung von Integralformeln.**

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*E. K. Meier 1810*

# Integraltafeln,

oder

Sammlung von Integralformeln.

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Von

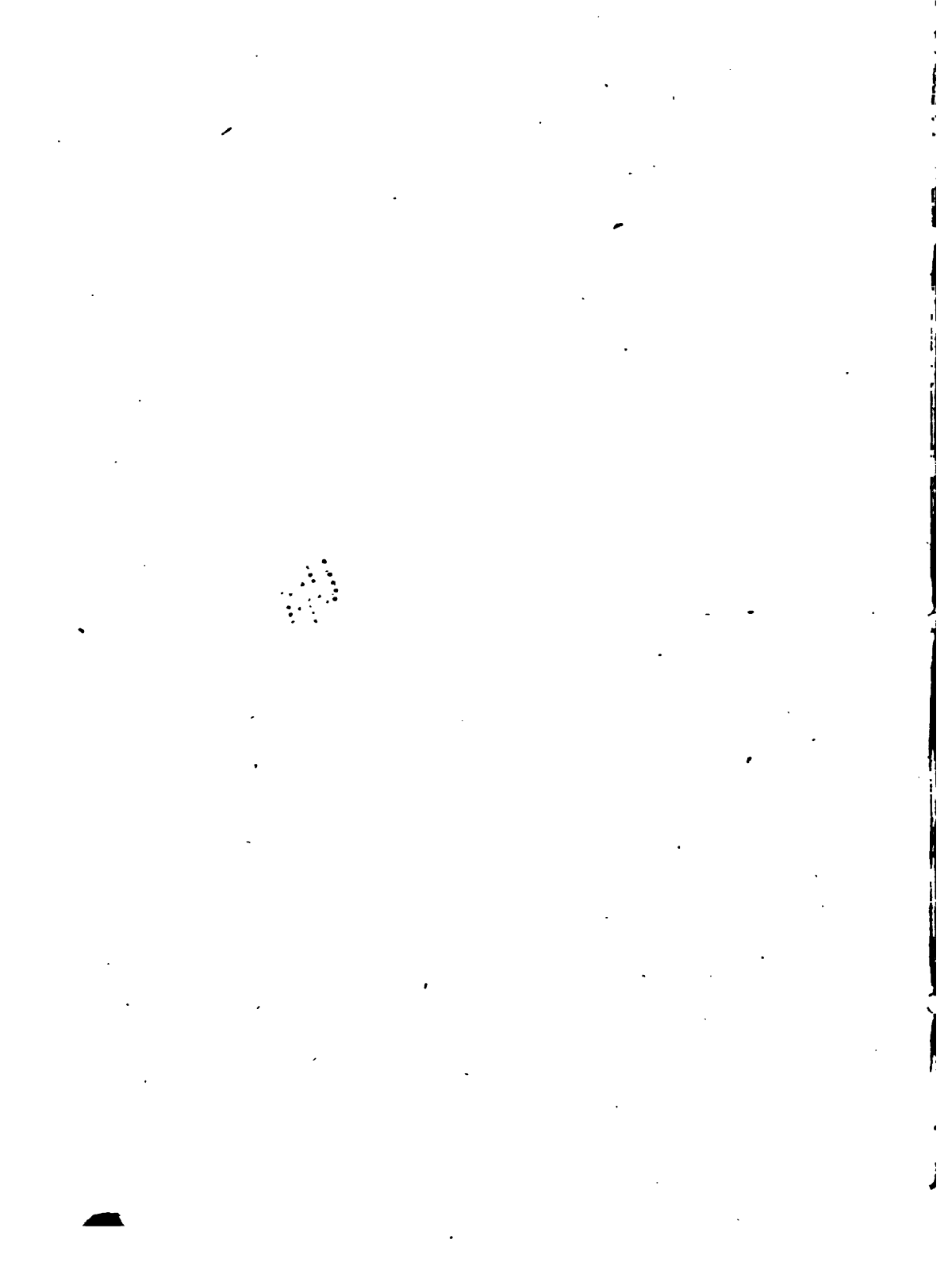
Meier Hirsch.

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Berlin,

bei Duncker und Humblot.

1810.



From the Estate  
of Charles K. Meach  
7-8-29

V o r r e d e.

Tafeln für häufig wiederkehrende Fälle sind zu allen Zeiten als ein wirksames Mittel angesehen worden, theils die Uebersicht zu erleichtern und dem Gedächtnisse zu Hülfe zu kommen, theils die Mühe des öftern Aufsuchens zu ersparen. Das Bedürfniß solcher Tafeln wird desto dringender, je öfterer die nämlichen Fälle wiederkehren, und je größer die Mühe ist, welche das Aufsuchen verursacht. Am fühlbarsten aber wird ihr Mangel in der Analysis, wo die Menge der Formeln von Tag zu Tag anwächst, und die Uebersicht so sehr erschwert, daß es selbst dem geübtesten Analysten nicht mehr möglich ist, sie alle inuner bey der Hand zu haben. Hat man sich auch durch ein vieljähriges Studium mit den allgemeineren vertraut gemacht, so müssen doch die speciellen, welohe man zu irgend einem Zwecke brauchen will, immer erst in einem oder dem andern Werke aufgesucht, oder, wenn sie sich nicht finden sollten, berechnet werden. Das letztere ist aber nicht die Sache eines jeden, und überdies muß bey dem ausübenden Theile der Mathematiker noch der damit verbundene Zeitaufwand in Anschlag gebracht werden. Wer weiß, ob nicht vielleicht manche Untersuchung, die für die Wissenschaft selbst, oder für ihre Anwendung hätte Gewinn werden können, bloß dieserhalb unterblieben ist? — Was schon Leibnitz, der selbst sich vorsetzte analytische Tafeln zu verfertigen, von ihrem Nutzen sagt, ist bekannt genug, und es würde nicht schwer

sey, noch mehrere Autoritäten anzuführen, wenn der Gegenstand ihrer bedürfte.

Was hier von dem Nutzen analytischer Tafeln im Allgemeinen gesagt worden, gilt von Integraltafeln insbesondere in einem hohen Grade. In unsern Lehrbüchern, und selbst in Eulers und Lacroix's ausführlichen Werken, findet man, und zwar mit Recht, nur diejenigen speciellen Formeln angeführt, welche zur Erläuterung der vorgetragenen Sätze dienen können. Stößt man daher in der Ausübung, wie es gar nicht selten zu geschehen pflegt, auf ein Integral, welches sich weder in diesen, noch in anderen Werken findet, so ist man genöthigt es selbst zu suchen. Integrale sind aber nicht so leicht gefunden, wie etwa die einzelnen Potenzen eines Binoms aus der Binomialformel; es wird dazu schon weit mehr Gewandtheit und Fertigkeit in der Behandlung analytischer Formeln erfordert, als man bey den meisten voraussetzen darf. Und doch behauptet die Integralrechnung den bedeutendsten Platz in der reinen Analysis, sie dringt so mächtig in das Gebiet der Anwendung ein, daß selbst der ausübende Mathematiker, der sich etwas über das Gewöhnliche erheben will, ihre Hülfe nicht mehr entbehren kann. Der Verfasser glaubt daher in der Ausarbeitung des vorliegenden Werkes, wo man alle, sowohl allgemeine als speciell Integralformeln, die muthmaßlich bey analytischen Untersuchungen oder in der Ausübung gebraucht werden möchten, nicht bloß berechnet, sondern auch so geordnet findet, wie es ihm zum Nachschlagen am bequemsten schien, etwas Verdienstliches unternommen zu haben. Er wurde in dieser Meinung durch die Aufmunterung mehrerer sehr schätzbarer Männer bestärkt, und er fürchtet nichts so sehr, als daß die Ausfüh-

rung ihren Erwartungen nicht entsprechen möchte. Besonders hält er es für seine Pflicht, dem geheimen Oberbaurathe Herrn Eytelwein, für so manche Winke in Hinsicht auf die bessere Einrichtung dieser Tafeln, seinen verbindlichsten Dank zu sagen.

Für Richtigkeit der Rechnung und des Druckes ist die möglichste Sorge getragen worden; sollten sich jedoch hier und da noch Fehler finden, welche dem Verf. entgangen sind, so würde ihm durch die Anzeige derselben eine dankeswerthe Gefälligkeit erwiesen werden.

Am Schlusse dieser Vorrede muß ich noch bemerken, daß es ein Irrthum war, wenn ich in meiner Samml. von Aufgaben aus der Theorie der Gleichungen die allgemeine Auflösung derselben nicht nur für möglich hielt, sondern sie sogar gefunden zu haben glaubte. Man wird daher das achte Capitel, wie auch das, was in der Vorrede von diesem Gegenstande gesagt worden, mit Mißtrauen lesen müssen. Zwar habe ich die Auflösung einer Menge sehr merkwürdiger unzerlegbarer Gleichungen gefunden, aber keinesweges die allgemeine Auflösung derselben, in dem Sinne der Euler, Lagrange und anderer großen Analysten, von deren Unmöglichkeit ich gegenwärtig überzeugt bin. Der Fehler entsprang aus Uebereilung, und ist so leicht zu entdecken, daß jeder, der bis dahin gekommen ist, ihn von selbst finden wird.

Berlin, im May 1810.

Meier Hirsch.



## E i n l e i t u n g.

Die Einrichtung und Anordnung der Tafeln kann man am besten aus einer flüchtigen Durchsicht kennen lernen. Es soll daher hier bloß das Nöthige über den Gebrauch und die richtige Anwendung der Formeln selbst angezeigt werden.

1) Die Zeichen, welche in diesem Werke gebraucht worden, sind die überall üblichen, das Differentialzeichen  $\partial$  etwa ausgenommen. Der Kürze wegen wurden auch die, nun schon in Deutschland hinlänglich bekannten Hindenburgschen Zeichen,  $\mathfrak{A}$ ,  $\mathfrak{B}$ ,  $\mathfrak{C}$ , etc.,  $\frac{1}{i}\mathfrak{A}$ ,  $\frac{1}{i}\mathfrak{B}$ ,  $\frac{1}{i}\mathfrak{C}$ , etc.,  $-\frac{1}{i}\mathfrak{A}$ ,  $-\frac{1}{i}\mathfrak{B}$ ,  $-\frac{1}{i}\mathfrak{C}$ , etc., für die Binomialcoefficienten der Potenzen  $m$ ,  $\frac{p}{q}$ ,  $-\frac{p}{q}$  gebraucht.

Bey den bestimmten Integralen wurde dem Integralzeichen  $\int$  oben ein Strich angehängt, um diese Integrale von den andern zu unterscheiden. Die Logarithmen, welche hier vorkommen, sind durchgängig die natürlichen oder hyperbolischen, und die Kreisbogen, sämmtlich auf den Halbmesser  $= 1$  bezogen. V. Z. heißt Verkürzungszeichen

2) Man weiß, daß bey jeder Integration zu dem gefundenen Integrale noch eine willkürliche Constante gefügt werden muß. Diese Constante ist, um den Platz nicht zu verengen, in den Tafeln weggelassen worden, weil sie sonst durchgängig hätte gesetzt werden müssen. Bey denjenigen Formeln, welche das Integralzeichen noch enthalten, ist dies nicht nöthig, jedoch muß es immer nach der vollständigen Entwicklung geschehen. Wie die Constante bestimmt wird, findet man in allen Lehrbüchern angezeigt.

3) Die Coefficienten in den Integralen wurden fast durchgängig unbestimmt gelassen. Wie überall in der Analysis, muß man auch hier, wenn die Formeln auf einzelne Fälle angewandt werden sollen, für die unbestimmten Größen die bestimmten setzen,

welche ihnen nach dem jedesmaligen Fall zukommen. So findet man

S. 68,  $\int \frac{\partial x}{x^4 X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \text{etc.}$  (für  $X = a + bx + cx^2$ ).

Wollte man nun z. B.  $\int \frac{\partial x}{x^4(-3+2x-x^2)}$  haben, so darf man nur  $a = -3$ ,  $b = 2$ ,  $c = -1$  setzen; hierdurch erhält man

$$\int \frac{\partial x}{x^4 X} = \frac{1}{9x^3} + \frac{1}{9x^2} + \frac{1}{27x} + \frac{2}{81} \log \frac{x^2}{X} - \frac{7}{81} \int \frac{\partial x}{X}$$

worin  $X = -3 + 2x - x^2$ . Das Integral von  $\frac{\partial x}{X}$  findet sich

S. 61, und da hier  $k = 4ac - b^2 = 8$ , also positiv, so hat man

$$\int \frac{\partial x}{X} = \frac{2}{\sqrt{8}} \text{Arc Tang} \frac{2-2x}{\sqrt{8}} = \frac{1}{\sqrt{2}} \text{Arc Tang} \frac{1-x}{\sqrt{2}}.$$

Wird dieser Werth substituirt, so erhält man

$$\begin{aligned} \int \frac{\partial x}{x^4(-3+2x-x^2)} &= \frac{1}{9x^3} + \frac{1}{9x^2} + \frac{1}{27x} + \frac{2}{81} \log \frac{x^2}{-3+2x-x^2} \\ &\quad - \frac{7}{81\sqrt{2}} \text{Arc Tang} \frac{1-x}{\sqrt{2}} + \text{Const.} \end{aligned}$$

Gesetzt man wollte  $\int \frac{x^2 \partial x}{(2-3x+4x^2)^{\frac{3}{2}}}$  haben, so muß man

die Tafel für  $\int \frac{x^2 \partial x}{(a+bx+cx^2)^{\frac{3}{2}}}$  aufschlagen (S. 190). Hier findet man,

wenn  $a = 2$ ,  $b = -3$ ,  $c = 4$  gesetzt wird,

$$\int \frac{x^2 \partial x}{X^{\frac{3}{2}}} = \left(-\frac{x}{8} - \frac{1}{64}\right) \frac{1}{X\sqrt{X}} + \frac{41}{128} \int \frac{\partial x}{X^{\frac{3}{2}}};$$

ferner auf derselben Seite, da hier  $k = 4ac - b^2 = 23$ ,

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = \left(\frac{1}{69X} + \frac{32}{1587}\right) \frac{16x-6}{\sqrt{X}}.$$

Wird dieser Werth substituirt, so erhält man,

$$\int \frac{x^2 \partial x}{X^{\frac{3}{2}}} = -\frac{8x+1}{64X\sqrt{X}} + \frac{41}{128} \left(\frac{1}{69X} + \frac{32}{1587}\right) \frac{16x-6}{\sqrt{X}} + \text{Const.}$$

4) Bisweilen ereignet es sich, daß für gewisse Werthe der Coefficienten ein Nenner in der Integralformel = 0, mithin die Formel unbrauchbar wird. In einem solchen Falle kann man im-

mer gewiß seyn, daß das gegebene Differential einer Umformung fähig ist, weshalb es nicht mehr zu derjenigen Gattung von Differentialen gehört, zu welcher man es zählte. So z. B. findet man,

wenn  $\int \frac{\partial x}{x^3(3-6x+3x^2)^{\frac{1}{2}}}$  S. 189 aufgesucht wird, daß  $\int \frac{\partial x}{x^3 X^{\frac{1}{2}}}$

die Formel  $\int \frac{\partial x}{X^{\frac{1}{2}}}$  enthalte, und S. 188, daß diese letztere Formel

die GröÙe  $k = 4ac - b^2 = 0$  im Nenner habe. Es läßt sich

nämlich dem Integral  $\int \frac{\partial x}{x^3(3-6x+3x^2)^{\frac{1}{2}}}$  die Form  $\int \frac{\partial x}{3^{\frac{1}{2}} x^3 (1-x)^3}$

$= \frac{1}{3\sqrt{3}} \int \frac{\partial x}{x^3 (1-x)^3}$  geben, und in dieser Form gehört es zu der

Gattung  $\int \frac{\partial x}{x^m(a+bx)^3}$ . Man findet S. 42, wenn  $a = 1$ ,  $b = -1$  gesetzt wird,

$$\frac{1}{3\sqrt{3}} \int \frac{\partial x}{x^3(1-x)^3} = \frac{1}{3\sqrt{3}} \left( -\frac{1}{2x^2} - \frac{2}{x} + 9 - 6x \right) \frac{1}{(1-x)^2} - \frac{2}{\sqrt{3}} \log \frac{1-x}{x} + \text{Const.}$$

5) Bey denjenigen Integralformeln, welche keine andere involviren, und unmittelbar durch Logarithmen und Kreisbogen ausgedruckt werden müssen, (man pflegt sie, obgleich etwas un- eigentlich, Elementarintegrale zu nennen) erscheinen hier größtentheils in mehreren Formen, unter welchen man nach Be- lieben wählen kann. Der Verfasser wurde hierzu durch die nicht unwichtigen Gründe bewogen, daß erstens in gewissen Fällen die eine Form vor der andern einen wirklichen Vorzug hat, und daß es zweitens die Vergleichung mit andern Werken erleichtert, wo bisweilen ausschließend die eine oder die andere Form gebraucht wird. Außer den angeführten giebt es aber noch unendlich viele andere Formen, wohin auch diejenigen zu rechnen sind, welche aus der Veränderung der Constante entspringen. Die vorzüglich- sten darunter können am besten hier ihren Platz finden. Wenn nämlich  $X$  irgend eine Function von  $x$  bezeichnet, so kann man

anstatt $\text{Arc Sin } X$	$+ \text{Const.}$	setzen	$- \text{Arc Cos } X$	$+ \text{Const.}$
..... $\text{Arc Cos } X$	$+ \text{Const.}$	.....	$- \text{Arc Sin } X$	$+ \text{Const.}$
..... $\text{Arc Tang } X$	$+ \text{Const.}$	.....	$- \text{Arc Cot } X$	$+ \text{Const.}$
..... $\text{Arc Cot } X$	$+ \text{Const.}$	.....	$- \text{Arc Tang } X$	$+ \text{Const.}$
..... $\text{Arc Sec } X$	$+ \text{Const.}$	.....	$- \text{Arc Cosec } X$	$+ \text{Const.}$
..... $\text{Arc Cosec } X$	$+ \text{Const.}$	.....	$- \text{Arc Sec } X$	$+ \text{Const.}$
..... $\log -X$	$+ \text{Const.}$	.....	$\log X$	$+ \text{Const.}$
..... $\log \sqrt{-X}$	$+ \text{Const.}$	.....	$\log \sqrt{X}$	$+ \text{Const.}$

6) Mit Hülfe dieser Tafeln lassen sich auch unendlich viele Integrale durch bloßes Zusammensetzen finden, wie sich am leichtesten an einigen Beispielen zeigen läßt.

Es werde das Integral von

$$\partial Z = \frac{(3x^{12} - 2x^9 - 5x^6 + 2x^4 + 9)\partial x}{x^6(3 - 2x^2)^{\frac{7}{2}}}$$

gesucht. Setzt man der Kürze wegen  $3 - 2x^2 = X$ , so ist

$$Z = 3 \int \frac{x^6 \partial x}{X^{\frac{7}{2}}} - 2 \int \frac{x^3 \partial x}{X^{\frac{7}{2}}} - 5 \int \frac{\partial x}{X^{\frac{7}{2}}} + 2 \int \frac{\partial x}{x^2 X^{\frac{7}{2}}} + 9 \int \frac{\partial x}{x^6 X^{\frac{7}{2}}}.$$

Werden diese Integrale in den Tafeln für

$$\int \frac{x^m \partial x}{(a + bx^2)^{\frac{7}{2}}}, \quad \int \frac{\partial x}{x^m (a + bx^2)^{\frac{7}{2}}},$$

(S. 150. 151) aufgesucht, so findet man für  $a = 3$ ,  $b = -2$ ,

$$\begin{aligned} 3 \int \frac{x^6 \partial x}{X^{\frac{7}{2}}} &= \left( \frac{23x^5}{10} - \frac{21x^3}{4} + \frac{27x}{8} \right) \frac{1}{X^2 \sqrt{X}} - \frac{3}{8} \int \frac{\partial x}{\sqrt{X}} \\ -2 \int \frac{x^3 \partial x}{X^{\frac{7}{2}}} &= \left( -\frac{x^3}{3} + \frac{1}{5} \right) \frac{1}{X^2 \sqrt{X}} \\ -5 \int \frac{\partial x}{X^{\frac{7}{2}}} &= -5 \int \frac{\partial x}{X^{\frac{7}{2}}} \\ +2 \int \frac{\partial x}{x^2 X^{\frac{7}{2}}} &= -\frac{2}{3x} \cdot \frac{1}{X^2 \sqrt{X}} + 8 \int \frac{\partial x}{X^{\frac{7}{2}}} \\ +9 \int \frac{\partial x}{x^6 X^{\frac{7}{2}}} &= \left( -\frac{3}{5x^5} - \frac{4}{3x^3} - \frac{64}{9x} \right) \frac{1}{X^2 \sqrt{X}} + \frac{256}{3} \int \frac{\partial x}{X^{\frac{7}{2}}} \end{aligned}$$

mithin

$$Z = \left( \frac{23x^5}{10} - \frac{21x^4}{4} - \frac{x^3}{3} + \frac{27x}{8} + \frac{1}{5} - \frac{70}{9x} - \frac{4}{3x^3} - \frac{3}{5x^5} \right) \frac{1}{X^2 \sqrt{X}} - \frac{5}{8} \int \frac{dx}{\sqrt{X}} + \frac{265}{9} \int \frac{dx}{X^{\frac{7}{2}}}$$

Es ist aber

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left( \frac{32x^5}{405} - \frac{8x^3}{27} + \frac{x}{3} \right) \frac{1}{X^2 \sqrt{X}}, \quad \int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{2}} \text{Arc Sin } x\sqrt{\frac{2}{3}};$$

werden daher diese Werthe substituirt, so erhält man nach der gehörigen Reduction,

$$Z = \left\{ \frac{33727x^5}{7290} - \frac{13583x^3}{972} - \frac{x^2}{3} + \frac{2849x}{216} + \frac{1}{5} - \frac{70}{9x} - \frac{4}{3x^3} - \frac{3}{5x^5} \right\} \frac{1}{X^2 \sqrt{X}} - \frac{3}{8\sqrt{2}} \text{Arc Sin } x\sqrt{\frac{2}{3}} + \text{Const.}$$

7) Sollte man auch ein Integral nicht unmittelbar in diesen Tafeln finden, so wird es sich doch immer, wenn überhaupt die vollständige Integration möglich ist; sehr leicht auf eines oder das andere der vorhandenen reduciren lassen. So z. B. findet sich das

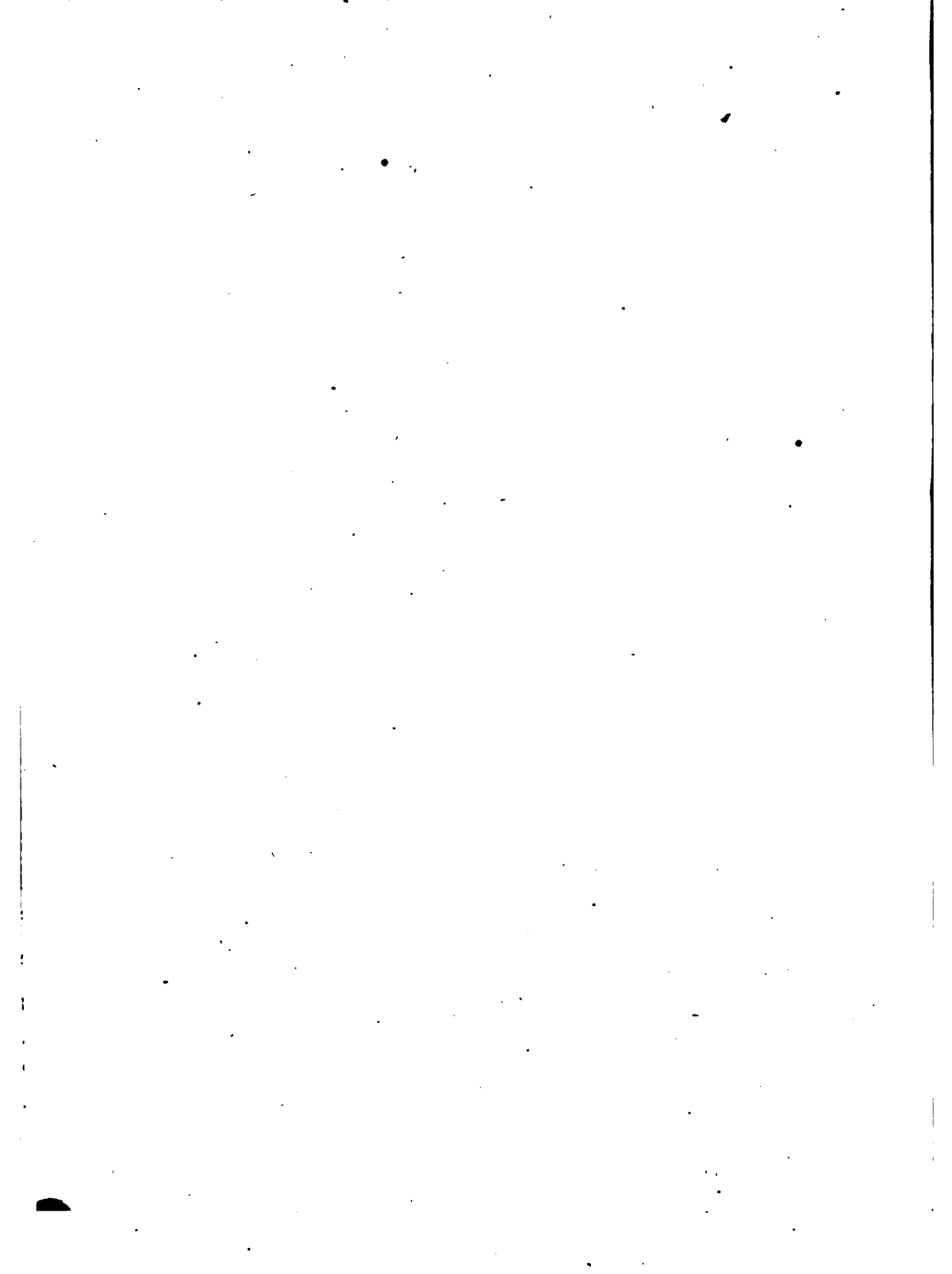
Integral  $\int \frac{x^{\frac{2n+1}{2}} dx}{(a+bx)^{\frac{3}{2}}}$  in dieser Form nicht hier, wohl aber in der

Form  $\int \frac{x^{n+2} dx}{(ax+bx^2)^{\frac{3}{2}}}$ , welche aus jener erhalten wird, wenn man

den Zähler und Nenner des Bruches mit  $x^{\frac{1}{2}}$  multiplicirt.

**Kürze**  
**Darstellung der Methoden**  
**zur**  
**Zerlegung der gebrochenen rationalen Functionen**  
**in Partialbrüche,**  
**mit den**  
**nöthigen Erläuterungen und Beispielen.**

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Es sey  $\frac{U}{V}$  der zu zerlegendе Bruch;  $U, V$ , bezeichnen zwey ganze rationale Functionen von nachstehender Form:

$$U = Ax^m + Bx^{m-1} + Cx^{m-2} + Dx^{m-3} + \text{etc.}$$

$$V = x^\mu + ax^{\mu-1} + bx^{\mu-2} + cx^{\mu-3} + \text{etc.}$$

und  $m < \mu$ ; die Coefficienten positiv oder negativ, oder auch zum Theil = 0. Es wird angenommen, daß man den Nenner  $V$  in lauter reelle Factoren von den folgenden vier Formen zerlegen könne:

I.  $x + a,$

II.  $(x + a)^2$

III.  $x^2 + ax + b,$

IV.  $(x^2 + ax + b)^2$

Es wird gefordert, den Bruch  $\frac{U}{V}$  in solche Brüche zu zerlegen, deren Nenner diese Formen haben.

Da jeder dieser Factoren eine eigene Behandlungsart erfordert, so entspringen daraus vier verschiedene Fälle, für welche nun die Methoden mit den nöthigen Erläuterungen und Beispielen gegeben werden sollen.

### Erster Fall

Der Nenner  $V$  habe den Factor  $x + a$ , und enthalte denselben nur Einmal. Es sey

$$V = (x + a) Q$$

so ist  $Q$  eine bekannte ganze und rationale Function; denn es ist  $\frac{U}{Q} = \frac{V}{x + a}$ . Man setze

$$\frac{U}{V} = \frac{A}{x + a} + \frac{P}{Q}.$$



Das noch unbekannte  $A$  bezeichne eine constante Grösse, das ebenfalls unbekannte  $P$  eine ganze rationale Function von  $x$ . Es wird alsdann  $A$  und  $P$  nach den folgenden Methoden bestimmt.

### *Erste Methode.*

Man setze  $x + a = 0$ , also  $x = -a$ . Man substituirt diesen Werth des  $x$  in den beiden bekannten Functionen  $U$ ,  $Q$ , und bezeichne die constanten Grössen, worin sie sich hierdurch verwandeln, mit  $U'$ ,  $Q'$ . Es ist alsdann immer

$$A = \frac{U'}{Q'}.$$

Hat man auf diese Weise  $A$  bestimmt, so erhält man  $P$  aus der Formel

$$P = \frac{U - AQ}{x + a},$$

wenn die angezeigte Division wirklich verrichtet wird; der Zähler  $U - AQ$  wird immer durch den Nenner  $x + a$  ohne Rest theilbar seyn.

### *Zweite Methode.*

Es sey  $\partial V = Z \partial x$ , also  $Z$  eine bekannte Function von  $x$ ; ferner  $U'$ ,  $Z'$ , das, worin sich die Functionen  $U$ ,  $Z$ , verwandeln, wenn man darin  $-a$  für  $x$  setzt; so ist immer

$$A = \frac{U'}{Z'}.$$

Die Function  $P$  wird wie bey der ersten Methode gefunden.

### *Bemerkungen.*

1) Ist  $x$  selbst ein Factor des Nenners  $V$ , so muß man, um aus den Functionen  $U$ ,  $Q$ ,  $Z$ , die constanten Grössen  $U'$ ,  $Q'$ ,  $Z'$ , zu erhalten, 0 für  $x$  setzen.

2) Enthält der Nenner  $V$ , ausser dem Factor  $x + a$ , noch andere dergleichen Factoren,  $x + a'$ ,  $x + a''$ ,  $x + a'''$ , etc., so läßt sich aus jedem derselben ein eigener Partialbruch bilden, und die Zähler dieser Brüche  $A'$ ,  $A''$ ,  $A'''$ , etc., lassen sich auf die nämliche Art finden, wie der Zähler  $A$  für den Factor  $x + a$ .

3) Ist der Nenner  $V$  aus lauter solchen Factoren  $x + a$ ,  $x + a'$ ,  $x + a''$ ,  $x + a'''$ , etc., zusammengesetzt, so läßt sich der Bruch  $\frac{U}{V}$  in lauter Partialbrüche von der Form  $\frac{A}{x + a}$  zerlegen, und die Summe derselben wird alsdann dem Bruche  $\frac{U}{V}$  gleich seyn.

4) Diese Methoden sind jedoch nur alsdann anwendbar, wenn die Factoren sämmtlich von einander verschieden sind; denn im entgegengesetzten Falle wird für denjenigen Factor, welcher mehrere Mal vorkommt, sowohl  $Q' = 0$ , als  $Z' = 0$ , und daher nach beiden Methoden  $A = \frac{U'}{0} = \infty$ .

### Beispiel.

Es sey  $\frac{U}{V} = \frac{2x + 3}{x^3 + x^2 - 2x} = \frac{2x + 3}{(x - 1)(x + 2)x}$  der zu zerlegende Bruch, also

$$U = 2x + 3; V = x^3 + x^2 - 2x = (x - 1)(x + 2)x.$$

Für den ersten Factor  $x - 1$  ist nun  $Q = (x + 2)x$ , und  $x - 1 = 0$ , giebt  $x = 1$ . Man findet also, wenn dieser Werth substituirt wird,  $U' = 5$ ,  $Q' = 3$ , und daher nach der ersten Methode  $A = \frac{U'}{Q'} = \frac{5}{3}$ . Auch ist  $\partial V = (3x^2 + 2x - 2)\partial x$ , und daher  $Z = 3x^2 + 2x - 2$ ; mithin  $Z' = 6x + 2$ , und daher nach der zweiten Methode  $A = \frac{U'}{Z'} = \frac{5}{6x + 2}$ , wie vorher. Der Partialbruch für den Factor  $x - 1$  ist demnach  $\frac{\frac{5}{3}}{x - 1} = \frac{5}{3(x - 1)}$ .

Für den Factor  $x + 2$  ist  $Q = (x - 1)x$ , und  $x + 2 = 0$  giebt  $x = -2$ . Demnach ist, wenn  $-2$  für  $x$  substituirt wird,  $U' = -1$ ,  $Q' = 6$ , daher nach der ersten Methode  $A = \frac{U'}{Q'} = -\frac{1}{6}$ . Da ferner  $Z = 3x^2 + 2x - 2$ , so ist, wenn  $-2$  für  $x$  gesetzt wird,  $Z' = 6$ , und folglich  $A = \frac{U'}{Z'} = -\frac{1}{6}$ , wie vorher. Der Partialbruch ist demnach  $-\frac{1}{6(x + 2)}$ .

Für den Factor  $x$  ist  $Q = (x - 1)(x + 2)$ , und also, wenn

$x=0$  gesetzt wird,  $U'=3$ ,  $Q'=-2$ ; daher nach der ersten Methode,  $A=\frac{U'}{Q'}=-\frac{3}{2}$ . Ferner ist  $Z=3x^2+2x-2$ , und für  $x=0$ ,  $Z'=-2$ ; daher nach der zweiten Methode  $A=\frac{U'}{Z'}=-\frac{3}{2}$ , wie vorher. Der Partialbruch ist also  $-\frac{3}{2x}$ .

Aus allem diesen ergibt sich, daß

$$\frac{U}{Z} = \frac{6}{3(x-1)} - \frac{1}{6(x+2)} - \frac{3}{2x}.$$

### Zweiter Fall.

Der Nenner  $V$  des Bruches  $\frac{U}{V}$  enthalte den Factor  $x+a$  mehrere Mal, und es sey  $V=(x+a)^n Q$ ; mithin  $Q$  eine bekannte ganze rationale Function von  $x$ . Man setze

$$\frac{U}{V} = \frac{A}{(x+a)^n} + \frac{A'}{(x+a)^{n-1}} + \frac{A''}{(x+a)^{n-2}} + \dots + \frac{A^{(n-2)'}}{(x+a)^2} + \frac{A^{(n-1)'}}{x+a} + \frac{P}{Q}.$$

Es wird gefordert, die constanten Zähler  $A, A', A'', \text{etc.}$ , zu bestimmen.

#### Erste Methode.

Wenn  $Q', U', U_1, U_2, U_3, \text{etc.}$ , das bezeichnen, worin sich die mit  $Q, U, U_1, U_2, U_3, \text{etc.}$ , bezeichneten Functionen verwandeln, wenn man  $x+a=0$ , also  $x=-a$  setzt: so hat man zur Bestimmung von  $A, A', A'', A''', \text{etc.}$ , nachstehende Formeln:

(1) $A = \frac{U'}{Q'}$	(2) $\frac{U - A Q}{x+a} = U_1$
(3) $A' = \frac{U_1'}{Q'}$	(4) $\frac{U_1 - A' Q}{x+a} = U_2$
(5) $A'' = \frac{U_2'}{Q'}$	(6) $\frac{U_2 - A'' Q}{x+a} = U_3$
(7) $A''' = \frac{U_3'}{Q'}$	(8) $\frac{U_3 - A''' Q}{x+a} = U_4$
etc.	etc.

Es wird nämlich zuerst, sowohl in  $U$  als in  $Q$ , für  $x$  sein Werth  $-a$  gesetzt, und dadurch der Werth der constanten Größen  $U$ ,  $Q'$ , gefunden. Die Formel (1) giebt hierauf den Werth von  $A$ ; und wird dieser Werth in der Formel (2) substituirt, und die angezeigte Division durch  $x + a$  wirklich verrichtet, so giebt dieses für  $U_1$  eine ganze Function. Wird in derselben  $-a$  für  $x$  gesetzt, so erhält man  $U'_1$ , und hieraus, vermittelst der Formel (3), den Werth von  $A'$ . Wird dieser Werth in der Formel (4) substituirt, und durch  $x + a$  wirklich dividirt, so erhält man für  $U_2$  eine ganze Function. Aus derselben erhält man ferner durch die Substitution von  $-a$  für  $x$  den Werth der constanten GröÙe  $U'_2$ , und die Formel (5) giebt alsdann den Werth von  $A''$ . Durch die Substitution dieses Werthes in der Formel (6) erhält man  $U_3$ , und somit auch den Werth der constanten GröÙe  $U'_3$ , und die Formel (7) giebt den Werth von  $A'''$ . Mit dieser Operation wird so lange fortgefahren, bis die sämtlichen Zähler  $A$ ,  $A'$ ,  $A''$ ,  $A'''$  ....  $A^{n-1}$  bestimmt sind.

Ist  $A^{n-1}$  gefunden, so läßt sich auch der Zähler  $P$  des ergänzenden Bruches  $\frac{P}{Q}$  finden. Denn aus  $A^{n-1}$  erhält man auf dem angegebenen Wege  $U_n$ ; und es ist alsdann immer  $P = U_n$ .

### Zweite Methode.

Es ist immer

$$A = \frac{U}{Q}$$

$$A' = \frac{1}{1 \cdot \partial x} \partial \cdot \frac{U}{Q}$$

$$A'' = \frac{1}{1 \cdot 2 \cdot \partial x^2} \partial^2 \cdot \frac{U}{Q}$$

$$A''' = \frac{1}{1 \cdot 2 \cdot 3 \cdot \partial x^3} \partial^3 \cdot \frac{U}{Q}$$

$$A^{iv} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot \partial x^4} \partial^4 \cdot \frac{U}{Q}$$

und im Allgemeinen

$$A^n = \frac{1}{1 \cdot 2 \cdot 3 \cdot \dots \cdot m \cdot \partial x^m} \partial^m \cdot \frac{U}{Q}$$

wenn in den erhaltenen Resultaten  $-a$  für  $x$  gesetzt wird.

Um daher die Größen,  $A', A'', \dots, A^{(n-1)}$  zu bestimmen, muß man die Function  $\frac{U}{Q}$   $n-1$  Mal nach einander differentiiren, die gefundenen Differentiale, in der Ordnung, wie sie einander folgen, durch  $1.\partial x$ ,  $1.2.\partial x^2$ ,  $1.2.3.\partial x^3$ ,  $\dots$ ,  $1.2.3\dots n-1.\partial x^{n-1}$  dividiren, und in den  $n-1$  erhaltenen Resultaten  $-a$  für  $x$  setzen.

### Beispiel.

Es sey der zu zerlegende Bruch

$$\frac{U}{V} = \frac{3x^2 + x - 2}{(x-1)^3 (x^2 + 1)}$$

und es werde gesetzt:

$$\frac{U}{V} = \frac{A}{(x-1)^3} + \frac{A'}{(x-1)^2} + \frac{A''}{x-1} + \frac{P}{Q};$$

man soll die constanten Größen  $A, A', A''$ , wie auch die Function  $P$  finden.

### Rechnung nach der ersten Methode.

Es ist hier  $U = 3x^2 + x - 2$ ,  $Q = x^2 + 1$ ; ferner giebt  $x-1=0$ ,  $x=1$ . Man hat daher  $U' = 2$ ,  $Q' = 2$ ; folglich, nach Formel (1),  $A = \frac{U'}{Q'} = 1$ .

Nach der Formel (2) ist daher

$$U_1 = \frac{U - 1 \cdot Q}{x-1} = 2x + 3;$$

mithin  $U_1 = 5$ , und folglich nach der Formel (3),  $A' = \frac{U_1}{Q'} = \frac{5}{2}$ .

Hieraus erhält man nach der Formel (4)

$$U_2 = \frac{U_1 - \frac{5}{2}Q}{x-1} = -\frac{5}{2}x - \frac{7}{2}$$

mithin  $U_2 = -3$ , und daher nach der Formel (5),  $A'' = \frac{U_2}{Q'} = -\frac{3}{2}$ .

Soll nun noch  $P$  bestimmt werden, so hat man nach der Formel (6)

$$U_3 = \frac{U_2 + \frac{1}{2}Q}{x-1} = \frac{1}{2}x - 1;$$

und daher  $P = U_3 = \frac{1}{2}x - 1$ .

Es ist demnach

$$\frac{U}{V} = \frac{1}{(x-1)^3} + \frac{5}{2(x-1)^2} - \frac{3}{2(x-1)} + \frac{3x-2}{2(x^2+1)}.$$

*Rechnung nach der zweiten Methode.*

Hier ist

$$\begin{aligned} \frac{U}{Q} &= \frac{5x^2 + x - 2}{x^2 + 1} \\ \frac{1}{1 \cdot \partial x} \partial \cdot \frac{U}{Q} &= \frac{-x^2 + 10x + 1}{(x^2 + 1)^2} \\ \frac{1}{1 \cdot 2 \cdot \partial x^2} \partial^2 \cdot \frac{U}{Q} &= \frac{(x^2 + 1)^2 (10 - 2x) + (x^2 - 10x - 1)(x^2 + 1) 4x}{2(x^2 + 1)^4}. \end{aligned}$$

Setzt man in den zweiten Theilen dieser Gleichungen  $x=1$ , so erhält man für  $A, A', A''$ , die nämlichen Werthe wie vorher.

### D r i t t e r F a l l

Es wird angenommen, daß der Nenner  $V$  des Bruches  $\frac{U}{V}$  den trinomischen Factor  $x^2 + ax + b$  habe, so daß  $V = (x^2 + ax + b)Q$  und  $Q$  eine ganze Function sey. Es wird ferner angenommen, daß das Trinom  $x^2 + ax + b$  sich nicht in zwey reelle Factoren von der Form  $x + a$  zerfallen lasse, welches Statt hat, wenn die Wurzeln der Gleichung  $x^2 + ax + b = 0$  imaginär sind, also, wenn  $a^2 - 4b$  negativ wird. Man verlangt den Bruch  $\frac{U}{V}$  in zwey andere so zu zerlegen, daß

$$\frac{U}{V} = \frac{A + Bx}{x^2 + ax + b} + \frac{P}{Q};$$

$A, B$ , sollen constante Größen, und  $P$  eine ganze Function von  $x$  seyn.

#### *Erste Methode.*

Die Gleichung  $x^2 + ax + b = 0$ ; wird, der Voraussetzung gemäß, zwey imaginäre Wurzeln haben; es mögen  $k \pm k\sqrt{-1}$  diese Wurzeln seyn. Man bilde die Function

$$U - (A + Bx)Q = Y;$$

setze hierauf in der Function  $Y$  durchgängig  $h + kV - 1$  für  $x$ , so wird dieselbe einen Werth von der Form  $M + NV - 1$  erhalten, und  $M$ ,  $N$ , werden zwey Constanten seyn, welche die gegenwärtig noch unbekannten Constanten  $A$ ,  $B$ , enthalten. Man mache hierauf die beiden Gleichungen

$$M = 0, \quad N = 0;$$

sie werden beide vom ersten Grade in Hinsicht auf  $A$  und  $B$  seyn. Man löse sie auf, so erhält man  $A$  und  $B$ .

Die Function  $P$  erhält man alsdann unmittelbar aus der Formel

$$P = \frac{U - (A + Bx)Q}{x^2 + ax + b}$$

wenn für  $A$ ,  $B$ , ihre gefundenen Werthe gesetzt, und die angezeigte Division wirklich ausgeführt wird.

### Zweite Methode.

Man setze in der Function  $Y$ ,  $\frac{1}{2kV-1} \cdot \frac{dV}{dx}$  anstatt  $Q$ , und verfähre hierauf wie bey der ersten Methode.

Anmerk. Die zweite Methode ist nur in gewissen Fällen, welche an ihrem Orte angezeigt werden sollen, der ersten vorzuziehen.

### Beispiel.

Es sey der zu zerlegende Bruch

$$\frac{U}{V} = \frac{2x + 1}{(x^2 + 2x + 5)(x^2 + x + 1)(x^2 + 1)}.$$

Für den Factor  $x^2 + 2x + 5$  ist  $Q = (x^2 + x + 1)(x^2 + 1)$ .

Da nun  $U = 2x + 1$ , so ist

$$Y = 2x + 1 - (A + Bx)(x^2 + x + 1)(x^2 + 1).$$

Die Gleichung  $x^2 + 2x + 5 = 0$  giebt  $x = -1 + 2V - 1$ .

Wird dieser Werth in  $Y$  substituirt, so erhält man

$$\begin{aligned} Y &= -1 + 4V - 1 - (A - B + 2BV - 1)(-2 + 16V - 1) \\ &= 2A + 30B - 1 + (20B - 16A + 4)V - 1; \end{aligned}$$

mithin ist  $M = 2A + 30B - 1$ ,  $N = 20B - 16A + 4$ . Man hat demnach die beiden Gleichungen:

$$2A + 30B - 1 = 0, \quad 20B - 16A + 4 = 0,$$

und diese geben  $A = \frac{7}{8}$ ,  $B = \frac{1}{8}$ .

Für den Factor  $x^2 + x + 1$  ist  $Q = (x^2 + 2x + 5)(x^2 + 1)$ ,  
und daher

$$Y = 2x + 1 - (A + Bx)(x^2 + 2x + 5)(x^2 + 1).$$

Die Gleichung  $x^2 + x + 1 = 0$  giebt  $x = -\frac{1}{2} + \frac{\sqrt{3}}{2} \sqrt{-1}$ .

Durch die Substitution dieses Werthes erhält man

$$\begin{aligned} Y &= \sqrt{3} \cdot \sqrt{-1} - (A - \frac{1}{2}B + \frac{B\sqrt{3}}{2}\sqrt{-1})(\frac{1}{2} - \frac{1}{2}\sqrt{3} \cdot \sqrt{-1}) \\ &= -\frac{1}{2}A - B + (\frac{1}{2}A - 2B + 1)\sqrt{3} \cdot \sqrt{-1} \end{aligned}$$

mithin ist  $M = -\frac{1}{2}A - B$ ,  $N = (\frac{1}{2}A - 2B + 1)\sqrt{3}$ . Man hat also die beiden Gleichungen:

$$\frac{1}{2}A + B = 0, \quad \frac{1}{2}A - 2B + 1 = 0$$

und diese geben  $A = -\frac{2}{5}$ ,  $B = \frac{1}{5}$ .

Für den dritten Factor  $x^2 + 1$  ist  $Q = (x^2 + 2x + 5)(x^2 + x + 1)$ ; also

$$Y = 2x + 1 - (A + Bx)(x^2 + 2x + 5)(x^2 + x + 1).$$

Die Gleichung  $x^2 + 1 = 0$  giebt  $x = \sqrt{-1}$ . Wird dieser Werth substituirt, so erhält man

$$\begin{aligned} Y &= 1 + 2\sqrt{-1} - (A + B\sqrt{-1})(-2 + 4\sqrt{-1}) \\ &= 2A + 4B + 1 + (2B - 4A + 2)\sqrt{-1} \end{aligned}$$

mithin ist  $M = 2A + 4B + 1$ ,  $N = 2B - 4A + 2$ . Man hat daher die beiden Gleichungen:

$$2A + 4B + 1 = 0, \quad 2B - 4A + 2 = 0,$$

und diese geben  $A = \frac{1}{10}$ ,  $B = -\frac{1}{5}$ .

Es ist demnach

$$\frac{U}{V} = \frac{\frac{1}{10} + \frac{1}{5}x}{x^2 + 2x + 5} + \frac{-\frac{1}{10} + \frac{1}{5}x}{x^2 + x + 1} + \frac{\frac{1}{10} - \frac{1}{5}x}{x^2 + 1}.$$

#### V i e r t e r F a l l

Es enthalte der Nenner  $V$  des Bruches  $\frac{U}{V}$  den Factor  $x^2 + ax + b$  mehrere Mal, so daß  $V = (x^2 + ax + b)^n Q$  und  $Q$  eine ganze Function von  $x$  sey. Es läßt sich alsdann dieser Bruch, auf eine ähnliche Art wie bey dem zweiten Falle, immer so zerlegen, daß



$$\frac{U}{V} = \frac{A + Bx}{(x^2 + ax + b)^n} + \frac{A' + B'x}{(x^2 + ax + b)^{n-1}} + \dots + \frac{A^{(n-1)'} + B^{(n-1)'}x}{x^2 + ax + b} + \frac{P}{Q};$$

und es kommt nunmehr nur darauf an, die constanten Größen  $A, B, A', B',$  etc. zu bestimmen.

### M e t h o d e.

Man bilde successive die Functionen  $U_1, U_2, U_3,$  etc. nach dem folgenden Schema:

$$(1) \quad U_1 = \frac{U - (A + Bx)Q}{x^2 + ax + b}$$

$$(2) \quad U_2 = \frac{U_1 - (A' + B'x)Q}{x^2 + ax + b}$$

$$(3) \quad U_3 = \frac{U_2 - (A'' + B''x)Q}{x^2 + ax + b}$$

$$(4) \quad U_4 = \frac{U_3 - (A''' + B'''x)Q}{x^2 + ax + b}$$

etc.

und bestimme aus  $U, U_1, U_2, U_3, U_4,$  etc., die Constanten  $A, B; A', B'; A'', B''; A''', B'''; A''', B''',$  etc., ganz nach der ersten Methode des dritten Falles.

Es werden nämlich zuerst die Constanten  $A, B,$  ganz auf die nämliche Art wie bey dem dritten Falle gefunden. Man substituïre nun ihre Werthe in der Formel (1), und verrichte die angezeigte Division durch  $x^2 + ax + b$  wirklich, so wird man für  $U_1$  eine ganze Function finden. Man verfare nun mit  $U_1$  gerade so wie vorher mit  $U$ , und bestimme dadurch die Constanten  $A', B'.$  Die Werthe derselben substituïre man in der Formel (2), so wird man für  $U_2$ , wenn die angezeigte Division wirklich ausgeführt wird, eine ganze Function finden, aus welcher nun wieder durch dasselbe Verfahren die Constanten  $A'', B'',$  bestimmt werden. Diese Operation setze man so lange fort, bis die sämmtlichen Constanten  $A, B; A', B'; A'', B''; A''', B''',$  .....  $A^{(n-1)'}, B^{(n-1)'}$  gefunden sind.

Verlangt man noch überdies  $P$  zu bestimmen, so suche man aus den nun bekannten Größen  $A^{(n-1)}$ ,  $B^{(n-1)}$ , die Function  $U_n$ ; es ist alsdann  $P = U_n$ .

### Beispiel.

Es sey der zu zerlegende Bruch

$$\frac{U}{V} = \frac{2x^5 + 7x^2 - 4x}{(x^2 + 1)^3 (2x^4 - 5)}$$

Hier ist  $U = 2x^5 + 7x^2 - 4x$ ,  $Q = 2x^4 - 5$ ; also

$$Y = 2x^5 + 7x^2 - 4x - (A + Bx)(2x^4 - 5).$$

Setzt man  $x^2 + 1 = 0$ , so erhält man  $x = V - 1$ , und die Substitution dieses Werthes verwandelt die Function  $Y$  in

$$-7 - 2V - 1 + 3(A + BV - 1)$$

Man hat daher die beiden Gleichungen  $3A - 7 = 0$ ,  $3B - 2 = 0$ , und diese geben  $A = \frac{7}{3}$ ,  $B = \frac{2}{3}$ .

Substituirt man diese Werthe in der Formel (1), so erhält man

$$\begin{aligned} U_1 &= \frac{2x^5 + 7x^2 - 4x - (\frac{7}{3} + \frac{2}{3}x)(2x^4 - 5)}{x^2 + 1} \\ &= \frac{1}{3}(2x^3 - 14x^2 - 2x + 35). \end{aligned}$$

Man behandle nun  $U_1$ , wie vorhin  $U$ ; so hat man

$$Y = \frac{1}{3}(2x^3 - 14x^2 - 2x + 35) - (A' + B'x)(2x^4 - 5)$$

und diese Function verwandelt sich, wenn  $x = V - 1$  gesetzt wird, in

$$\frac{4}{3} - \frac{4}{3}V - 1 + 3(A' + B'V - 1)$$

$$\text{oder } 3A' + \frac{4}{3} + (3B' - \frac{4}{3})V - 1.$$

Man hat daher die beiden Gleichungen:  $3A' + \frac{4}{3} = 0$ ,  $3B' - \frac{4}{3} = 0$ , und diese geben  $A' = -\frac{4}{9}$ ,  $B' = \frac{4}{9}$ .

Hieraus findet man nun wieder, vermittelst der Formel (2)

$$\begin{aligned} U_2 &= \frac{\frac{1}{3}(2x^3 - 14x^2 - 2x + 35) - (-\frac{4}{9} + \frac{4}{9}x)(2x^4 - 5)}{x^2 + 1} \\ &= \frac{1}{9}(-8x^3 + 98x^2 + 14x - 140) \end{aligned}$$

also hier

$$Y = \frac{1}{9}(-8x^3 + 98x^2 + 14x - 140) - (A'' + B''x)(2x^4 - 5)$$

und wenn  $V - 1$  für  $x$  gesetzt wird,

$$Y = -\frac{22}{9} + \frac{22}{9}V - 1 + 3(A'' + B''V - 1);$$

also  $3A'' - \frac{238}{9} = 0$ ,  $3B'' + \frac{22}{9} = 0$ ; mithin  $A'' = \frac{238}{27}$ ,  $B'' = -\frac{22}{27}$

Aus  $A''$  und  $B''$  erhält man endlich:

$$U_3 = \frac{\frac{1}{2}(-8x^3 + 98x^2 + 14x - 140) - (\frac{238}{27} - \frac{22}{27}x)(2x^4 - 5)}{x^2 + 1}$$

$$= \frac{1}{27}(44x^3 - 476x^2 - 68x + 770)$$

und dieses ist die Function  $P$ .

Es ist demnach

$$\frac{U}{V} = \frac{7 + 2x}{3(x^2 + 1)^3} + \frac{-49 + 4x}{9(x^2 + 1)^2} + \frac{238 - 22x}{27(x^2 + 1)}$$

$$+ \frac{44x^3 - 476x^2 - 68x + 770}{27(2x^4 - 5)}$$


---

# T a f e l n

d e r

Reductionsformeln für das Integral

$$\int x^{n-1} dx (a + bx^n + cx^{2n} + dx^{3n} + \text{etc.})^p.$$

---



Wenn man alle die verschiedenen Methoden, deren man sich in der Integralrechnung bedient, um das Integral einer vorgelegten Differentialfunction zu finden, einer näheren Prüfung unterwirft, so wird man einsehen, daß es hierbey einzig und allein auf die folgenden zwey Erfordernisse ankommt:

- 1) Auf die Kenntniß der Elementarintegrale, d. h. solcher Integrale, welche entweder wirklich in der einfachsten Gestalt erscheinen, deren sie fähig sind, oder wenigstens so angesehen werden, wie etwa  $\int x^m dx$ ,  $\int \frac{dx}{x}$ ,  $\int \frac{dx}{1+x^2}$ ,  $\int \frac{dx}{\sqrt{1+x^4}}$ , etc.

- 2) Auf die Methode ein vorgelegtes Integral auf eines oder das andere dieser Elementarintegrale zu reduciren.

Die Reduction eines vorgelegten Integrals auf andere kann auf mehrere Arten geschehen; nämlich:

- a) Durch die Zerlegung des gegebenen Differentials in andere; mithin durch partielle Integration.
- b) Durch die Einführung einer neuen veränderlichen Gröfse; also durch Substitution.
- c) Durch die Anwendung gewisser Formeln involutorischer Art, mit deren Hülfe ein vorgelegtes Integral, ohne eine Substitution, oder irgend ein anderes Mittel, auf ein einfacheres, dieses wieder auf ein einfacheres, u. s. w. zurückgeführt wird. Diese Formeln sollen ausschließend Reductionsformeln genannt werden.
- d) Durch die Anwendung einiger oder aller dieser Methoden zugleich.

Die allgemeinste Reductionsformel ist folgende:

$$(\odot) \int XY dx = XfY dx - \int dX fY dx$$

wo  $X, Y$ , zwey willkührliche algebraische oder transcendente Functionen von  $x$  bezeichnen. Aus derselben lassen sich mit Hülfe gewisser Kunstgriffe alle die Reductionsformeln ableiten, welche hier für algebraische Functionen gegeben werden.

Der Gebrauch dieser Formeln erstreckt sich auf das vielumfassende Integral  $\int x^{m-1} \partial x X^p$  ( $X = a + bx^n + cx^{2n} + dx^{3n} + \text{etc.}$ ), wo für  $m, n, p$ , alle mögliche Zahlen, positive oder negative, ganze oder gebrochene, angenommen werden können. Taf. I. giebt sie für das Binom  $a + bx^n$ ; Taf. II. für das Trinom  $a + bx^n + cx^{2n}$ ; Taf. III. für das Quadrinom  $a + bx^n + cx^{2n} + dx^{3n}$ ; Taf. IV. für das Polynom. Mit Hülfe derselben ist man im Stande die Exponenten  $m$  und  $p$  nach Belieben zu erhöhen oder zu erniedrigen, bis man zu solchen Integralen kommt, welche sich durch schickliche Substitutionen auf Elementarintegrale zurückführen lassen, die alsdann weiter entweder vollständig integrirt, oder durch Reihen ausgedruckt werden müssen.

Die gedachten Formeln sind übrigens nur so lange anwendbar, als die Nenner der Bruchcoefficienten, welche darin vorkommen, nicht verschwinden; wie dies z. B. bey den Formeln I. und V. Taf. I. geschieht, wenn  $m = 0$ , oder bey den Formeln III. und IV. Taf. I., wenn  $m + np = 0$  wird.

---

## T a f e l . I.

## Reductionsformeln für das Integral

$$\int x^{m-1} \partial x (a + bx^n)^p$$

---


$$\text{VZ. } a + bx^n = X$$


---

I.

$$\int x^{m-1} \partial x X^p = \frac{x^m X^p}{m} - \frac{pnb}{m} \int x^{m+n-1} \partial x X^{p-1}$$

II.

$$\int x^{m-1} \partial x X^p = \frac{x^{m-n} X^{p+1}}{(p+1)nb} - \frac{m-n}{(p+1)nb} \int x^{m-n-1} \partial x X^{p+1}$$

III.

$$\int x^{m-1} \partial x X^p = \frac{x^{m-n} X^{p+1}}{(m+np)b} - \frac{(m-n)a}{(m+np)b} \int x^{m-n-1} \partial x X^p$$

IV.

$$\int x^{m-1} \partial x X^p = \frac{x^m X^p}{m+np} + \frac{pna}{m+np} \int x^{m-1} \partial x X^{p-1}$$

V.

$$\int x^{m-1} \partial x X^p = \frac{x^m X^{p+1}}{ma} - \frac{(m+n+np)b}{ma} \int x^{m+n-1} \partial x X^p$$

VI.

$$\int x^{m-1} \partial x X^p = -\frac{x^m X^{p+1}}{(p+1)na} + \frac{m+n+np}{(p+1)na} \int x^{m-1} \partial x X^{p+1}$$



## T a f e l I.

Reductionsformeln für das Integral

$$\int x^{m-1} dx (a + bx^n)^2$$

---


$$\text{VZ. } a + bx^n = X$$


---

VII.

$$\begin{aligned} \int x^{m-1} dx X^i = \\ \left\{ \begin{aligned} &Ax^{m-1} - Bx^{m-2n} + Cx^{m-3n} - Dx^{m-4n} + \text{etc.} \end{aligned} \right\} X^{i+1} \\ + Kx^{m-(i-1)n} + Lx^{m-in} \\ + L(m-in)a \int x^{m-in-1} dx X^i \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{(m+np)b}, \quad B = \frac{(m-n)a}{(m-n+np)b} A, \quad C = \frac{(m-2n)a}{(m-2n+np)b} B, \\ D &= \frac{(m-3n)a}{(m-3n+np)b} C, \quad E = \frac{(m-4n)a}{(m-4n+np)b} D, \text{ etc.} \\ L &= \frac{[m-(i-1)n]a}{[m-(i-1)n+np]b} K \end{aligned}$$

VIII.

$$\begin{aligned} \int x^{m-1} dx X^i = \\ \left\{ \begin{aligned} &AX^i + BX^{i-1} + CX^{i-2} + DX^{i-3} + EX^{i-4} + \text{etc.} \end{aligned} \right\} x^m \\ + KX^{i-1} + LKX^{i-1} \\ + L(p-i+1)na \int x^{m-1} dx X^{i-1} \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{m+np}, \quad B = \frac{pna}{m-n+np} A, \quad C = \frac{(p-1)na}{m-2n+np} B, \\ D &= \frac{(p-2)na}{m-3n+np} C, \quad E = \frac{(p-3)na}{m-4n+np} D, \text{ etc.} \\ L &= \frac{(p-i+2)na'}{m-(i-1)n+np} K. \end{aligned}$$

## T a f e l I.

Reductionsformeln für das Integral

$$\int x^{m-1} dx (a + bx^n)^p$$

---


$$\text{VZ. } a + bx^n = X$$


---

IX.

$$\begin{aligned} \int x^{m-1} dx X^p = \\ \left\{ Ax^m - Bx^{m+1} + Cx^{m+2} - Dx^{m+3} + Ex^{m+4} - \text{etc.} \right\} X^{p+1} \\ + Kx^{m+(i-1)n} + Lx^{m+(i-1)n} \\ + L(m+in+np)b \int x^{m+i-1} dx X^p \end{aligned}$$

$$A = \frac{1}{ma}, \quad B = \frac{(m+n+np)b}{(m+n)a} A, \quad C = \frac{(m+2n+np)b}{(m+2n)a} B,$$

$$D = \frac{(m+3n+np)b}{(m+3n)a} C, \quad E = \frac{(m+4n+np)b}{(m+4n)a} D, \text{ etc.}$$

$$L = \frac{[m+(i-1)n+np]b}{[m+(i-1)n]a} K.$$

X.

$$\begin{aligned} \int x^{m-1} dx X^p = \\ - \left\{ AX^{p+1} + BX^{p+2} + CX^{p+3} + DX^{p+4} + EX^{p+5} + \text{etc.} \right\} x^m \\ + KX^{p+i-1} + LX^{p+i} \\ + L(m+in+np) \int x^{m-1} dx X^{p+1} \end{aligned}$$

$$A = \frac{1}{(p+1)na}, \quad B = \frac{m+n+np}{(p+2)na} A, \quad C = \frac{m+2n+np}{(p+3)na} B,$$

$$D = \frac{m+3n+np}{(p+4)na} C, \quad E = \frac{m+4n+np}{(p+5)na} D, \text{ etc.}$$

$$L = \frac{m+(i-1)n+np}{(p+i)na} K.$$


---

## T a f e l I I.

Reductionsformeln für das Integral

$$\int x^{m-1} dx (a + bx^n + cx^{2n})^p$$

---


$$\text{VZ. } a + bx^n + cx^{2n} = X$$


---

I.

$$\begin{aligned} \int x^{m-1} dx X^p &= \\ \frac{x^m X^p}{m} - \frac{pnb}{m} \int x^{m+n-1} dx X^{p-1} \\ &- \frac{2pnc}{m} \int x^{m+2n-1} dx X^{p-1} \end{aligned}$$

II.

$$\begin{aligned} \int x^{m-1} dx X^p &= \\ \frac{x^{m-2n} X^{p+1}}{(m+2pn)c} - \frac{(m-2n)a}{(m+2pn)c} \int x^{m-2n-1} dx X^p \\ &- \frac{(m-n+pn)b}{(m+2pn)c} \int x^{m-n-1} dx X^p \end{aligned}$$

III.

$$\begin{aligned} \int x^{m-1} dx X^p &= \\ \frac{x^m X^p}{m+2pn} + \frac{2pna}{m+2pn} \int x^{m-1} dx X^{p-1} \\ &+ \frac{pnb}{m+2pn} \int x^{m+n-1} dx X^{p-1} \end{aligned}$$

## T a f e l II.

Reductionsformeln für das Integral

$$\int x^{m-1} \partial x (a + bx^n + cx^{2n})^p$$

---


$$\text{VL. } a + bx^n + cx^{2n} = X$$


---

IV.

$$\begin{aligned} \int x^{m-1} \partial x X^p &= \\ \frac{x^m X^{p+1}}{ma} &\rightarrow \frac{(m+n+pn)b}{ma} \int x^{m+n-1} \partial x X^p \\ &- \frac{(m+2n+2pn)c}{ma} \int x^{m+2n-1} \partial x X^p \end{aligned}$$

V.

$$\begin{aligned} \int x^{m-1} \partial x X^p &= \\ \frac{Ax^m + Bx^{m+n}}{K} X^{p+1} &+ \frac{1}{K} \int (Cx^{m-1} + Dx^{m+n-1}) \partial x X^{p+1} \\ A &= 2ac - b^2 \\ B &= -bc \\ C &= n(p+1)(b^2 - 4ac) - m(2ac - b^2) \\ D &= (2pn + 3n + m)bc \\ K &= (p+1)(b^2 - 4ac)na \end{aligned}$$


---

## T a f e l III.

Reductionsformeln für das Integral

$$\int x^{m-1} \partial x (a + bx^2 + cx^2 + dx^2)^p$$

---


$$\text{VZ. } a + bx^2 + cx^2 + dx^2 = X$$


---

I.

$$\begin{aligned} \int x^{m-1} \partial x X^p &= \\ \frac{x^m X^p}{m} - \frac{pnb}{m} \int x^{m+2-1} \partial x X^{p-1} & \\ - \frac{2pnc}{m} \int x^{m+2-1} \partial x X^{p-1} - \frac{3pnd}{m} \int x^{m+2-1} \partial x X^{p-1} \end{aligned}$$

II.

$$\begin{aligned} \int x^{m-1} \partial x X^p &= \\ \frac{x^{m-2n} X^{p+1}}{(m+3pn)d} - \frac{(m-3n)a}{(m+3pn)d} \int x^{m-2n-1} \partial x X^p & \\ - \frac{(m-2n+pn)b}{(m+3pn)d} \int x^{m-2n-1} \partial x X^p - \frac{(m-n+2pn)c}{(m+3pn)d} \int x^{m-2n-1} \partial x X^p \end{aligned}$$

III.

$$\begin{aligned} \int x^{m-1} \partial x X^p &= \\ \frac{x^m X^p}{m+3pn} + \frac{3pna}{m+3pn} \int x^{m-1} \partial x X^{p-1} & \\ + \frac{2pnb}{m+3pn} \int x^{m+2-1} \partial x X^{p-1} + \frac{pnc}{m+3pn} \int x^{m+2-1} \partial x X^{p-1} \end{aligned}$$

## T a f e l. III.

Reductionsformeln für das Integral

$$\int x^{m-1} dx (a + bx^n + cx^{2n} + dx^{3n})^p$$

---


$$\text{VZ. } a + bx^n + cx^{2n} + dx^{3n} = X$$


---

IV.

$$\begin{aligned} \int x^{m-1} dx X^p = & \frac{x^m X^{p+1}}{ma} - \frac{(m+n+pn)b}{ma} \int x^{m+n-1} dx X^p \\ & - \frac{(m+2n+2pn)c}{ma} \int x^{m+2n-1} dx X^p - \frac{(m+3n+3pn)d}{ma} \int x^{m+3n-1} dx X^p \end{aligned}$$

V.

$$\begin{aligned} \int x^{m-1} dx X^p = & (Ax^n + Bx^{2n} + Cx^{3n}) X^{p+1} + \\ & \int (Dx^{m-1} + Ex^{m+n-1} + Fx^{m+2n-1}) X^{p+1} dx \end{aligned}$$

die Coefficienten  $A, B, C$ , sind durch die drey Gleichungen

$$\begin{aligned} bdA - 3adB + acC &= \frac{-bd}{(p+1)na} \\ (bc - 3ad)A - 2acB + 2abC &= \frac{ad - bc}{(p+1)na} \\ (b^2 - 2ac)A - abB + 3a^2C &= \frac{ac - b^2}{(p+1)na} \end{aligned}$$

gegeben, und aus diesen Coefficienten erhält man

$$\begin{aligned} D &= \frac{1}{a} - mA \\ E &= -\frac{(p+1)nb}{a}A - (m+n)B - \frac{b}{a^2} \\ F &= -(m+5n+3pn)C. \end{aligned}$$


---

## T a f e l I V.

## Reductionsformeln für das Integral

$$\int x^{m-1} \partial x (a + bx^n + cx^{2n} + \dots + tx^{kn})^p$$

*Verkürzungszeichen*

$$a + bx^n + cx^{2n} + dx^{3n} + \text{etc.} + sx^{(k-1)n} + tx^{kn} = X$$

$$\int x^{m+n-1} \partial x X^{p-1} = S'$$

$$\int x^{m-n-1} \partial x X^p = S'_x$$

$$\int x^{m+2n-1} \partial x X^{p-1} = S''$$

$$\int x^{m-2n-1} \partial x X^p = S''_x$$

$$\int x^{m+3n-1} \partial x X^{p-1} = S'''$$

$$\int x^{m-3n-1} \partial x X^p = S'''_x$$

$$\int x^{m+4n-1} \partial x X^{p-1} = S''''$$

$$\int x^{m-4n-1} \partial x X^p = S''''_x$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$\int x^{m+kn-1} \partial x X^{p-1} = S^p$$

$$\int x^{m-kn-1} \partial x X^p = S^p_x$$

$$\int x^{m-1} \partial x X^{p-1} = S_2$$

$$\int x^{m+n-1} \partial x X^p = S'_2$$

$$\int x^{m+n-1} \partial x X^{p-1} = S'_2$$

$$\int x^{m+2n-1} \partial x X^p = S''_2$$

$$\int x^{m+2n-1} \partial x X^{p-1} = S''_2$$

$$\int x^{m+3n-1} \partial x X^p = S'''_2$$

$$\int x^{m+3n-1} \partial x X^{p-1} = S'''_2$$

$$\int x^{m+4n-1} \partial x X^p = S''''_2$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$\int x^{m+kn-1} \partial x X^{p-1} = S^p_2$$

$$\int x^{m-kn-1} \partial x X^p = S^p_2$$

## T a f e l I V.

Reductionsformeln für das Integral

$$\int x^{m-1} dx (a + bx^n + cx^{2n} + \dots + tx^{kn})^t$$


---

I.

$$m \int x^{m-1} dx X^t = \\ x^m X^t - pnbS' - 2pncS'' - 3pndS''' - 4pneS'''' \\ - 5pnfS''''' - \text{etc.} - kpn t S^k.$$

II.

$$(m + kpn) t \int x^{m-1} dx X^t = \\ x^{m-kn} X^{t+1} - (m - kn) a S_1^t - [m - (k-1)n + pn] b S_1^{t-1} \\ - [m - (k-2)n + 2pn] c S_1^{t-2} - [m - (k-3)n + 3pn] d S_1^{t-3} \\ - \text{etc.} - [m - 2n + (k-2)pn] t S_1^{t-1} - [m - n + (k-1)pn] t S_1^t.$$

III.

$$(m + kpn) \int x^{m-1} dx X^t = \\ x^m X^t + kpnaS_2 + (k-1)pnbS_2' + (k-2)pncS_2'' \\ + (k-3)pndS_2''' + \text{etc.} + pntS_2^{k-1}.$$



## T a f e l I V.

Reductionsformeln für das Integral

$$\int x^{m-1} dx (a + bx^n + cx^{2n} + \dots + tx^{kn})^p$$


---

IV.

$$ma \int x^{m-1} dx X^p =$$

$$\begin{aligned} x^m X^{p+1} - (m + n + pn) b S'_3 - (m + 2n + 2pn) c S''_3 \\ - (m + 3n + 3pn) d S'''_3 - (m + 4n + 4pn) e S''''_3 \\ - \text{etc.} - (m + kn + kpn) t S^k_3. \end{aligned}$$

V.

$$\int x^{m-1} dx X^p =$$

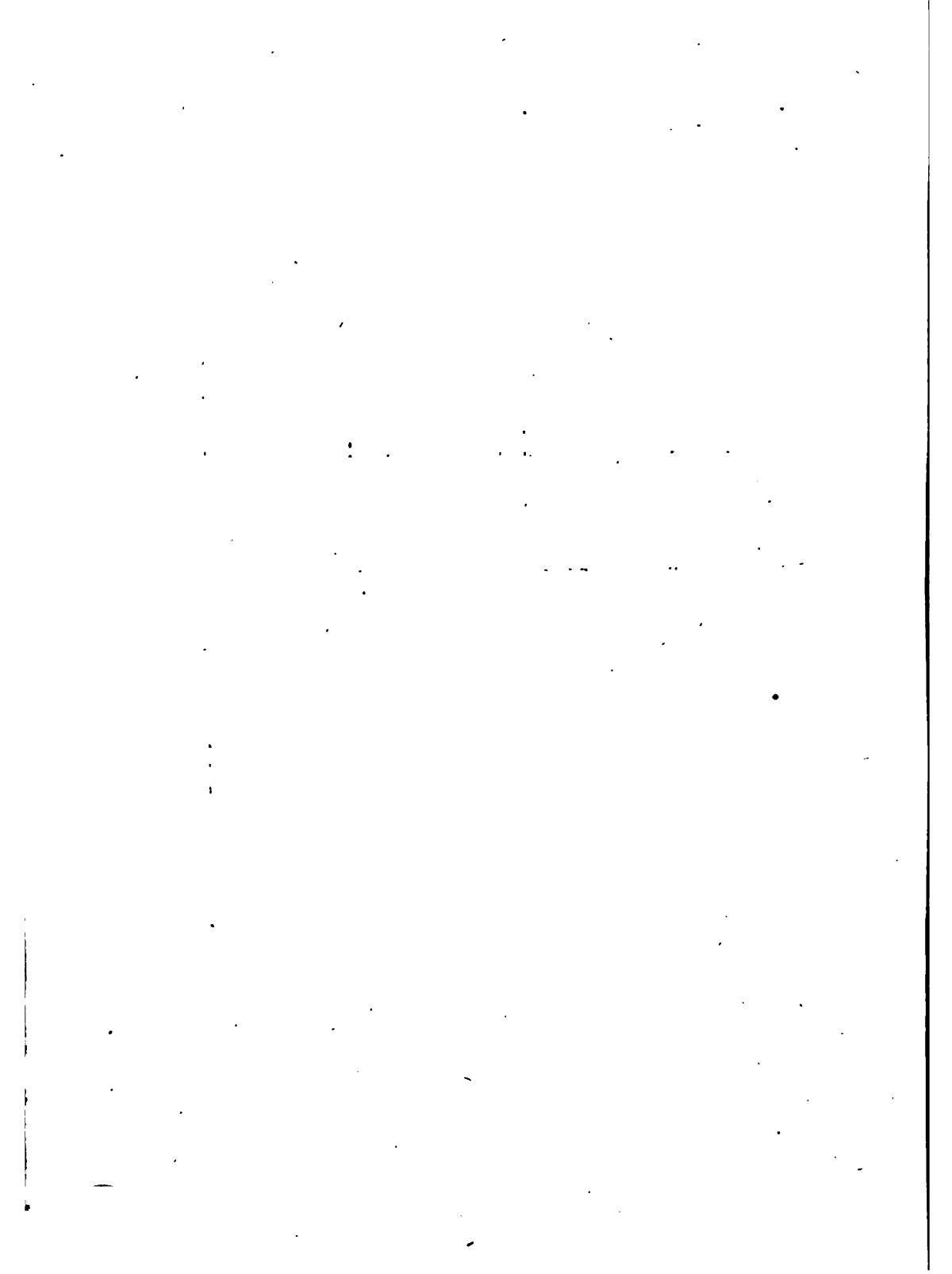
$$\begin{aligned} (Ax^m + Bx^{m+n} + Cx^{m+2n} + \text{etc.} + Tx^{m+(k-1)n}) X^{p+1} + \\ \int (A'x^{m-1} + B'x^{m+n-1} + C'x^{m+2n-1} + \text{etc.} + T'x^{m+(k-1)n-1}) X^{p+1} dx \end{aligned}$$

{ Aus dieser Gleichung können mit Hülfe des Differentials die Coefficienten  $A, B, C, \dots, T; A', B', C', \dots, T'$ , bestimmt werden. Die allgemeinen Werthe derselben lassen sich zwar angeben, aber nicht wohl in der Kürze darstellen. }

---

**I n t e g r a l t a f e l n**  
**f ü r**  
**r a t i o n a l e D i f f e r e n t i a l e .**

---



Die Integrale der rationalen Differentialfunctionen können, in Hinsicht auf die Operationen, durch welche sie gefunden werden, füglich in die nachstehenden drey Classen geordnet werden:

- 1) Integrale von  $Xdx$ , wenn die Function  $X$  entweder an sich schon eine begränzte Reihe von der Form  $a + bx^i + cx^k + dx^l + \text{etc.}$  ist, oder durch die Entwicklung der darin vorhandenen binomischen und polynomischen Potenzen und ihrer Producte unmittelbar darauf gebracht werden kann.
- 2) Integrale von  $Xdx$ , wenn  $X$  eine solche Function von  $x$  ist, bey welcher dieses nicht Statt hat.
- 3) Integrale von  $Xdx$ , wenn dieses Differential aus zwey Theilen  $Ydx$ ,  $Zdx$ , zusammen gesetzt ist, von welchen der eine zur ersten, der andere zur zweiten Classe gehört.

Die dritte Classe hat nichts Eigenes, und kann daher ganz übergangen werden; denn es ist immer  $\int Xdx = \int Ydx + \int Zdx$ .

### Erste Classe der Integrale.

1) Wenn  $k$  die willkürliche und nach Willkühr zu bildende Constante bezeichnet, so ist,  $m$  mag positiv oder negativ seyn,

$$\int ax^m dx = \frac{ax^{m+1}}{m+1} + k.$$

Ausgenommen hiervon ist der Fall, wo  $m = -1$ ; denn alsdann ist

$$\begin{aligned} \int ax^{-1} dx &= \int \frac{a dx}{x} = a \log x + k = a \log x + a \log k \\ &= a \log kx = \log k^a x^a = \log kx^a. \end{aligned}$$

2) Hieraus ergibt sich

$$\int (a + bx + cx^2 + dx^3 + \text{etc.}) dx = ax + \frac{1}{2}bx^2 + \frac{1}{3}cx^3 + \frac{1}{4}dx^4 + \text{etc.}$$

$$\int \left( \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{d}{x^4} + \text{etc.} \right) dx = a \log x - \frac{b}{x} - \frac{c}{2x^2} - \frac{d}{3x^3} - \text{etc.}$$

$$\int (ax^i + bx^k + cx^l + \text{etc.}) dx = \frac{ax^{i+1}}{i+1} + \frac{bx^{k+1}}{k+1} + \frac{cx^{l+1}}{l+1} + \text{etc.}$$

$$\int \left( \frac{a}{x^i} + \frac{b}{x^k} + \frac{c}{x^l} + \text{etc.} \right) dx = -\frac{a}{(i-1)x^{i-1}} - \frac{b}{(k-1)x^{k-1}} - \frac{c}{(l-1)x^{l-1}} - \text{etc.}$$

3) Hierher gehören auch alle Integrale von der Form  $\int X^m dx$ ,  $\int X^m Y^n dx$ ,  $\int X^m Y^n Z^p dx$ , etc.,  $\int \frac{X^m dx}{x^k}$ ,  $\int \frac{X^m Y^n dx}{x^k}$ , etc., wenn  $X$ ,  $Y$ ,  $Z$ , Functionen von der Form  $a + bx^i + cx^k + dx^l + \text{etc.}$ , und  $m$ ,  $n$ ,  $p$ , etc., ganze positive Zahlen sind; denn jede solche Potenz wie  $X^m$ ,  $Y^n$ ,  $Z^p$ , etc., läßt sich mit Hülfe des binomischen oder polynomischen Satzes in eine begränzte Reihe verwandeln, welche aus lauter Gliedern von der Form  $ax^m$  besteht, und das Product mehrerer derselben wird daher auch aus solchen Gliedern zusammengesetzt seyn. So z. B. ist

$$\int (a + bx)^2 dx = a^2 x + abx^2 + \frac{1}{3}b^2 x^3$$

$$\int (ax + bx^2)^3 dx = \frac{1}{4}a^3 x^4 + \frac{3}{2}a^2 b x^5 + \frac{3}{2}ab^2 x^6 + \frac{1}{4}b^3 x^7$$

$$\int \left( ax^2 + \frac{b}{x^3} \right)^2 dx = \frac{1}{5}a^2 x^5 + 2ab \log x - \frac{b^2}{5x^5}$$

$$\int \left( a + bx + \frac{c}{x} \right)^2 dx = a^2 x + abx^2 + 2ac \log x + \frac{1}{3}b^2 x^3 + 2bcx - \frac{c^2}{x}$$

$$\int (a^2 + x^2)(a + x)^2 x^3 dx = \frac{1}{4}a^4 x^4 + \frac{2}{3}a^3 x^5 + \frac{1}{3}a^2 x^6 + \frac{2}{7}a x^7 + \frac{1}{8}x^8$$

$$\int \frac{(a-x)^2(a+x)dx}{x^5} = -\frac{a^3}{(h-1)x^{h-1}} + \frac{a}{(h-3)x^{h-3}} + \frac{a^2}{(h-2)x^{h-2}} - \frac{1}{(h-4)x^{h-4}}$$

Da die Integrale, welche zu dieser Classe gehören, sehr leicht zu finden sind, so bedarf es dazu keiner Hülftafeln.

## Zweite Classe der Integrale.

Zu dieser Classe gehören diejenigen Integrale, welche unter der allgemeinen Form

$$\int \frac{Ax^m + Bx^{m-1} + Cx^{m-2} + Dx^{m-3} + \text{etc.}}{ax^k + bx^{k-1} + cx^{k-2} + dx^{k-3} + \text{etc.}} dx$$

begriffen sind; oder wenn Zähler und Nenner durch  $a$  dividirt wird, unter folgender:

$$\frac{1}{a} \int \frac{Ax^m + Bx^{m-1} + Cx^{m-2} + Dx^{m-3} + \text{etc.}}{x^k + ax^{k-1} + bx^{k-2} + cx^{k-3} + \text{etc.}} dx.$$

Man kann der Sache unbeschadet annehmen, daß  $m < k$  sey; denn wenn  $m > k$  oder  $m = k$ , so wird man durch die Division den Bruchcoefficienten in zwey Theile, nämlich in eine ganze Function  $Y$  und in eine Bruchfunction  $\frac{U}{V}$  auflösen können, so daß der höchste

Exponent des  $x$  im Zähler  $U$  kleiner ist, als im Nenner  $V$ , und es braucht daher, da  $\int Y dx$  sich finden läßt, nur  $\int \frac{U}{V} dx$  gefunden zu werden. Wie dieses mit Hülfe der im Vorhergehenden gegebenen Methoden durch die Zerlegung des Bruches  $\frac{U}{V}$  in Partialbrüche ge-

schiehet, wird in allen Lehrbüchern gezeigt. Da aber die hierzu erforderlichen Rechnungen für einzelne Integrale sehr beschwerlich sind, so hat es der Verf. für nützlich gehalten, die hier folgenden Tafeln zu verfertigen, wo man die Formeln für alle die Integrale, welche muthmaßlich in der Ausübung vorkommen können, sogleich vollständig berechnet, und in der einfachsten Gestalt, deren sie fähig sind, vorfindet. Die allgemeineren Formeln, nebst den bey der successiven Berechnung gebrauchten Reductionsformeln, sind am Ende angehängt, weil sich in den Tafeln selbst kein schicklicher Platz dazu fand.

1. 1.

$$\int \frac{1}{x} \frac{1}{1+x} \frac{1}{1+x^2}$$

$$\log x - \log(1+x) = X$$

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$$\int \frac{x^m \partial x}{(a + bx)^2}$$

Taf. II.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{X^2} = -\frac{1}{bX}$$

$$\int \frac{x \partial x}{X^2} = \frac{a}{b^2 X} + \frac{1}{b^2} \log X$$

$$\int \frac{x^2 \partial x}{X^2} = \left( \frac{x^2}{b} - \frac{2a^2}{b^3} \right) \frac{1}{X} - \frac{2a}{b^3} \log X$$

$$\int \frac{x^3 \partial x}{X^2} = \left( \frac{x^3}{2b} - \frac{3ax^2}{2b^2} + \frac{3a^2}{b^4} \right) \frac{1}{X} + \frac{3a^2}{b^4} \log X$$

$$\int \frac{x^4 \partial x}{X^2} = \left( \frac{x^4}{3b} - \frac{2ax^3}{3b^2} + \frac{2a^2x^2}{b^3} - \frac{4a^4}{b^5} \right) \frac{1}{X} - \frac{4a^3}{b^5} \log X$$

$$\int \frac{x^5 \partial x}{X^2} = \left( \frac{x^5}{4b} - \frac{5ax^4}{12b^2} + \frac{5a^2x^3}{6b^3} - \frac{5a^3x^2}{2b^4} + \frac{5a^5}{b^6} \right) \frac{1}{X} + \frac{5a^4}{b^6} \log X$$

$$\int \frac{x^6 \partial x}{X^2} = \left( \frac{x^6}{5b} - \frac{3ax^5}{10b^2} + \frac{a^2x^4}{2b^3} - \frac{a^3x^3}{b^4} + \frac{3a^4x^2}{b^5} - \frac{6a^6}{b^7} \right) \frac{1}{X} - \frac{6a^5}{b^7} \log X$$

$$\int \frac{x^7 \partial x}{X^2} = \left( \frac{x^7}{6b} - \frac{7ax^6}{30b^2} + \frac{7a^2x^5}{20b^3} - \frac{7a^3x^4}{12b^4} + \frac{7a^4x^3}{6b^5} - \frac{7a^5x^2}{2b^6} + \frac{7a^7}{b^8} \right) \frac{1}{X} + \frac{7a^6}{b^8} \log X$$

$$\int \frac{x^8 \partial x}{X^2} = \left( \frac{x^8}{7b} - \frac{4ax^7}{21b^2} + \frac{4a^2x^6}{15b^3} - \frac{2a^3x^5}{5b^4} + \frac{2a^4x^4}{3b^5} - \frac{4a^5x^3}{3b^6} + \frac{4a^6x^2}{b^7} - \frac{8a^8}{b^9} \right) \frac{1}{X} - \frac{8a^7}{b^9} \log X$$

$$\int \frac{x^9 \partial x}{X^2} = \left( \frac{x^9}{8b} - \frac{9ax^8}{56b^2} + \frac{3a^2x^7}{14b^3} - \frac{3a^3x^6}{10b^4} + \frac{9a^4x^5}{20b^5} - \frac{3a^5x^4}{4b^6} + \frac{3a^6x^3}{2b^7} - \frac{9a^7x^2}{2b^8} + \frac{9a^9}{b^{10}} \right) \frac{1}{X} + \frac{9a^8}{b^{10}} \log X$$



$$\frac{x^2}{1-x^2}$$

$$\frac{x^2}{1-x^2}$$

$$1-x^2=1$$

$$\int \frac{x^2}{1-x^2} = -\frac{x}{1-x^2}$$

$$\int \frac{x^3}{1-x^2} = -\frac{x^2}{1-x^2} - \frac{x}{1-x^2}$$

$$\int \frac{x^{4-2x}}{1-x^2} = \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{5-2x}}{1-x^2} = \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{6-2x}}{1-x^2} = \frac{x^5}{1-x^2} - \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{7-2x}}{1-x^2} = \frac{x^6}{1-x^2} - \frac{x^5}{1-x^2} - \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{8-2x}}{1-x^2} = \frac{x^7}{1-x^2} - \frac{x^6}{1-x^2} - \frac{x^5}{1-x^2} - \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{9-2x}}{1-x^2} = \frac{x^8}{1-x^2} - \frac{x^7}{1-x^2} - \frac{x^6}{1-x^2} - \frac{x^5}{1-x^2} - \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{10-2x}}{1-x^2} = \frac{x^9}{1-x^2} - \frac{x^8}{1-x^2} - \frac{x^7}{1-x^2} - \frac{x^6}{1-x^2} - \frac{x^5}{1-x^2} - \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^{11-2x}}{1-x^2} = \frac{x^{10}}{1-x^2} - \frac{x^9}{1-x^2} - \frac{x^8}{1-x^2} - \frac{x^7}{1-x^2} - \frac{x^6}{1-x^2} - \frac{x^5}{1-x^2} - \frac{x^4}{1-x^2} - \frac{x^3}{1-x^2} - \frac{x^2}{1-x^2} - \frac{x}{1-x^2} - \frac{1}{1-x^2} \log x$$

$$\int \frac{x^m dx}{(a + bx)^4}$$

Taf. IV.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^4} = -\frac{1}{3bX^3}$$

$$\int \frac{x dx}{X^4} = -\left(\frac{x}{2b} + \frac{a}{6b^2}\right) \frac{1}{X^3}$$

$$\int \frac{x^2 dx}{X^4} = -\left(\frac{x^2}{b} + \frac{ax}{b^2} + \frac{a^2}{3b^3}\right) \frac{1}{X^3}$$

$$\int \frac{x^3 dx}{X^4} = \left(\frac{3ax^2}{b^2} + \frac{9a^2x}{2b^3} + \frac{11a^3}{6b^4}\right) \frac{1}{X^3} + \frac{1}{b^4} \log X$$

$$\int \frac{x^4 dx}{X^4} = \left(\frac{x^4}{b} - \frac{12a^2x^2}{b^3} - \frac{18a^3x}{b^4} - \frac{22a^4}{3b^5}\right) \frac{1}{X^3} - \frac{4a}{b^5} \log X$$

$$\int \frac{x^5 dx}{X^4} = \left(\frac{x^5}{2b} - \frac{5ax^4}{2b^2} + \frac{30a^2x^2}{b^4} + \frac{45a^4x}{b^5} + \frac{55a^5}{3b^6}\right) \frac{1}{X^3} + \frac{10a^2}{b^6} \log X$$

$$\int \frac{x^6 dx}{X^4} = \left(\frac{x^6}{3b} - \frac{ax^5}{b^2} + \frac{5a^2x^4}{b^3} - \frac{60a^4x^2}{b^5} - \frac{90a^5x}{b^6} - \frac{110a^6}{3b^7}\right) \frac{1}{X^3} - \frac{20a^3}{b^7} \log X$$

$$\int \frac{x^7 dx}{X^4} = \left(\frac{x^7}{4b} - \frac{7ax^6}{12b^2} + \frac{7a^2x^5}{4b^3} - \frac{35a^3x^4}{4b^4} + \frac{105a^5x^2}{b^6} + \frac{315a^6x}{2b^7} + \frac{385a^7}{6b^8}\right) \frac{1}{X^3} + \frac{35a^4}{b^8} \log X$$

$$\int \frac{x^8 dx}{X^4} = \left(\frac{x^8}{5b} - \frac{2ax^7}{5b^2} + \frac{14a^2x^6}{15b^3} - \frac{14a^3x^5}{5b^4} + \frac{14a^4x^4}{b^5} - \frac{168a^6x^2}{b^7} - \frac{252a^7x}{b^8} - \frac{308a^8}{3b^9}\right) \frac{1}{X^3} - \frac{56a^5}{b^9} \log X$$



$$\int \frac{x^n dx}{(a+bx)^6}$$

Taf. VI.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^6} = -\frac{1}{5bX^5}$$

$$\int \frac{x dx}{X^6} = -\left(\frac{x}{4b} + \frac{a}{20b^2}\right) \frac{1}{X^5}$$

$$\int \frac{x^2 dx}{X^6} = -\left(\frac{x^2}{3b} + \frac{ax}{6b^2} + \frac{a^2}{30b^3}\right) \frac{1}{X^4}$$

$$\int \frac{x^3 dx}{X^6} = -\left(\frac{x^3}{2b} + \frac{ax^2}{2b^2} + \frac{a^2x}{4b^3} + \frac{a^3}{20b^4}\right) \frac{1}{X^3}$$

$$\int \frac{x^4 dx}{X^6} = -\left(\frac{x^4}{b} + \frac{2ax^3}{b^2} + \frac{2a^2x^2}{b^3} + \frac{a^3x}{b^4} + \frac{a^4}{5b^5}\right) \frac{1}{X^2}$$

$$\int \frac{x^5 dx}{X^6} = \left(\frac{5ax^4}{b^2} + \frac{15a^2x^3}{b^3} + \frac{55a^3x^2}{3b^4} + \frac{125a^4x}{12b^5} + \frac{137a^5}{60b^6}\right) \frac{1}{X} + \frac{1}{b^6} \log X$$

$$\int \frac{x^6 dx}{X^6} = \left(\frac{x^6}{b} - \frac{30a^2x^4}{b^3} - \frac{90a^3x^3}{b^4} - \frac{110a^4x^2}{b^5} - \frac{125a^5x}{2b^6} - \frac{137a^6}{10b^7}\right) \frac{1}{X^5} - \frac{6a}{b^7} \log X$$

$$\int \frac{x^7 dx}{X^6} = \left(\frac{x^7}{2b} - \frac{7ax^6}{2b^2} + \frac{105a^3x^4}{b^4} + \frac{315a^4x^3}{b^5} + \frac{385a^5x^2}{b^6} + \frac{875a^6x}{4b^7} + \frac{959a^7}{20b^8}\right) \frac{1}{X^4} + \frac{21a^2}{b^8} \log X$$

$$\int \frac{x^8 dx}{X^6} = \left(\frac{x^8}{3b} - \frac{4ax^7}{3b^2} + \frac{28a^2x^6}{3b^3} - \frac{280a^4x^4}{b^5} - \frac{840a^5x^3}{b^6} - \frac{3080a^6x^2}{3b^7} - \frac{1750a^7x}{3b^8} - \frac{1918a^8}{15b^9}\right) \frac{1}{X^3} - \frac{56a^3}{b^9} \log X$$

Taf. VII

$$\int \frac{dx}{x^2(a + bx)}$$

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$$\text{VL } a + bx = X$$


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$$\int \frac{dx}{xX} = \frac{1}{a} \log \frac{x}{X} = -\frac{1}{a} \log \frac{X}{x}^*)$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} + \frac{b}{a^2} \log \frac{X}{x}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} + \frac{b}{a^2x} - \frac{b^2}{a^3} \log \frac{X}{x}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2}{a^3x} + \frac{b^3}{a^4} \log \frac{X}{x}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{3a^2x^3} - \frac{b^2}{2a^3x^2} + \frac{b^3}{a^4x} - \frac{b^4}{a^5} \log \frac{X}{x}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{4a^2x^4} - \frac{b^2}{3a^3x^3} + \frac{b^3}{2a^4x^2} - \frac{b^4}{a^5x} + \frac{b^5}{a^6} \log \frac{X}{x}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{5a^2x^5} - \frac{b^2}{4a^3x^4} + \frac{b^3}{3a^4x^3} - \frac{b^4}{2a^5x^2} + \frac{b^5}{a^6x} - \frac{b^6}{a^7} \log \frac{X}{x}$$

$$\int \frac{dx}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{6a^2x^6} - \frac{b^2}{5a^3x^5} + \frac{b^3}{4a^4x^4} - \frac{b^4}{3a^5x^3} + \frac{b^5}{2a^6x^2} - \frac{b^6}{a^7x} + \frac{b^7}{a^8} \log \frac{X}{x}$$

$$\int \frac{dx}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{7a^2x^7} - \frac{b^2}{6a^3x^6} + \frac{b^3}{5a^4x^5} - \frac{b^4}{4a^5x^4} + \frac{b^5}{3a^6x^3} - \frac{b^6}{2a^7x^2} + \frac{b^7}{a^8x} - \frac{b^8}{a^9} \log \frac{X}{x}$$

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$$*) \log \frac{x}{X} + k = \log \frac{kx}{X} = -\log \frac{X}{kx} = -\log \frac{kX}{x}$$

$$\int \frac{\partial x}{x^n(a+bx)^2}.$$

Taf. VIII.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{xX^2} = \frac{1}{aX} - \frac{1}{a^2} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{2b}{a^2}\right) \frac{1}{X} + \frac{2b}{a^3} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{a^3}\right) \frac{1}{X} - \frac{3b^2}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \frac{2b^2}{a^3x} - \frac{4b^3}{a^4}\right) \frac{1}{X} + \frac{4b^3}{a^5} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{5b}{12a^2x^3} - \frac{5b^2}{6a^3x^2} + \frac{5b^3}{2a^4x} + \frac{5b^4}{a^5}\right) \frac{1}{X} - \frac{5b^4}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4} - \frac{b^2}{2a^3x^3} + \frac{b^3}{a^4x^2} - \frac{3b^4}{a^5x} - \frac{6b^5}{a^6}\right) \frac{1}{X} + \frac{6b^5}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \frac{7b^2}{20a^3x^4} + \frac{7b^3}{12a^4x^3} - \frac{7b^4}{6a^5x^2} + \frac{7b^5}{2a^6x} + \frac{7b^6}{a^7}\right) \frac{1}{X} - \frac{7b^6}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{4b}{21a^2x^6} - \frac{4b^2}{15a^3x^5} + \frac{2b^3}{5a^4x^4} - \frac{2b^4}{3a^5x^3} + \frac{4b^5}{3a^6x^2} - \frac{4b^6}{a^7x} - \frac{8b^7}{a^8}\right) \frac{1}{X} + \frac{8b^7}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{9b}{56a^2x^7} - \frac{3b^2}{14a^3x^6} + \frac{3b^3}{10a^4x^5} - \frac{9b^4}{20a^5x^4} + \frac{3b^5}{4a^6x^3} - \frac{3b^6}{2a^7x^2} + \frac{9b^7}{2a^8x} + \frac{9b^8}{a^9}\right) \frac{1}{X} - \frac{9b^8}{a^{10}} \log \frac{X}{x}$$

Taf. IX.

$$\int \frac{dx}{x^n(a+bx)^3}$$

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$$\text{VZ. } a+bx=X$$


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$$\int \frac{dx}{xX^3} = \frac{5}{2a} + \frac{bx}{a^2} \frac{1}{X^2} - \frac{1}{a^3} \log \frac{X}{x}$$

$$\int \frac{dx}{x^2X^3} = \left( -\frac{1}{ax} - \frac{9b}{2a^2} - \frac{5b^2x}{a^3} \right) \frac{1}{X^2} + \frac{5b}{a^4} \log \frac{X}{x}$$

$$\int \frac{dx}{x^3X^3} = \left( -\frac{1}{2ax^2} + \frac{2b}{a^2x} + \frac{9b^2}{a^3} + \frac{6b^3x}{a^4} \right) \frac{1}{X^2} - \frac{6b^3}{a^5} \log \frac{X}{x}$$

$$\int \frac{dx}{x^4X^3} = \left( -\frac{1}{3ax^3} + \frac{5b}{6a^2x^2} - \frac{17b^2}{3a^3x} - \frac{15b^3}{a^4} - \frac{10b^4x}{a^5} \right) \frac{1}{X^2} \\ + \frac{10b^4}{a^6} \log \frac{X}{x}$$

$$\int \frac{dx}{x^5X^3} = \left( -\frac{1}{4ax^4} + \frac{b}{2a^2x^3} - \frac{5b^2}{4a^3x^2} + \frac{5b^3}{a^4x} - \frac{45b^4}{2a^5} + \frac{15b^5x}{a^6} \right) \frac{1}{X^2} \\ - \frac{15b^5}{a^7} \log \frac{X}{x}$$

$$\int \frac{dx}{x^6X^3} = \left( -\frac{1}{5ax^5} + \frac{7b}{20a^2x^4} - \frac{7b^2}{10a^3x^3} - \frac{7b^3}{4a^4x^2} - \frac{7b^4}{a^5x} - \frac{65b^5}{2a^6} \right. \\ \left. - \frac{21b^6x}{a^7} \right) \frac{1}{X^2} + \frac{21b^6}{a^8} \log \frac{X}{x}$$

$$\int \frac{dx}{x^7X^3} = \left( -\frac{1}{6ax^6} + \frac{4b}{15a^2x^5} - \frac{7b^2}{15a^3x^4} - \frac{12b^3}{15a^4x^3} - \frac{7b^4}{5a^5x^2} \right. \\ \left. + \frac{55b^5}{3a^6x} + \frac{42b^6}{a^7} + \frac{25b^7x}{a^8} \right) \frac{1}{X^2} - \frac{25b^7}{a^9} \log \frac{X}{x}$$

$$\int \frac{dx}{x^8X^3} = \left( -\frac{1}{7ax^7} + \frac{3b}{14a^2x^6} - \frac{12b^2}{35a^3x^5} + \frac{5b^3}{5a^4x^4} - \frac{6b^4}{5a^5x^3} \right. \\ \left. - \frac{5b^5}{a^6x^2} - \frac{12b^6}{a^7x} - \frac{5b^7}{a^8} - \frac{7b^8x}{a^9} \right) \frac{1}{X^2} - \frac{7b^8}{a^{10}} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^m(a+bx)^4}$$

Taf. X.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{xX^4} = \left( \frac{11}{6a} + \frac{5bx}{2a^2} + \frac{b^2x^2}{a^3} \right) \frac{1}{X^3} - \frac{1}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2X^4} = \left( -\frac{1}{ax} - \frac{22b}{3a^2} - \frac{10b^2x}{a^3} - \frac{4b^3x^2}{a^4} \right) \frac{1}{X^3} + \frac{4b}{a^3} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3X^4} = \left( -\frac{1}{2ax^2} + \frac{5b}{2a^2x} + \frac{55b^2}{3a^3} + \frac{25b^3x}{a^4} + \frac{10b^4x^2}{a^5} \right) \frac{1}{X^3} - \frac{10b^2}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4X^4} = \left( -\frac{1}{3ax^3} + \frac{b}{a^2x^2} - \frac{5b^2}{a^3x} - \frac{110b^3}{3a^4} - \frac{50b^4x}{a^5} - \frac{20b^5x^2}{a^6} \right) \frac{1}{X^3} + \frac{20b^3}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5X^4} = \left( -\frac{1}{4ax^4} + \frac{7b}{12a^2x^3} - \frac{7b^2}{4a^3x^2} + \frac{35b^3}{4a^4x} + \frac{385b^4}{6a^5} + \frac{175b^5x}{2a^6} + \frac{35b^6x^2}{a^7} \right) \frac{1}{X^3} - \frac{35b^4}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6X^4} = \left( -\frac{1}{5ax^5} + \frac{2b}{5a^2x^4} - \frac{14b^2}{15a^3x^3} + \frac{14b^3}{5a^4x^2} - \frac{14b^4}{a^5x} - \frac{308b^5}{3a^6} - \frac{140b^6x}{a^7} - \frac{56b^7x^2}{a^8} \right) \frac{1}{X^3} + \frac{56b^5}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7X^4} = \left( -\frac{1}{6ax^6} + \frac{3b}{10a^2x^5} - \frac{3b^2}{5a^3x^4} + \frac{7b^3}{5a^4x^3} - \frac{21b^4}{5a^5x^2} + \frac{21b^5}{a^6x} + \frac{154b^6}{a^7} + \frac{210b^7x}{a^8} + \frac{84b^8x^2}{a^9} \right) \frac{1}{X^3} - \frac{84b^6}{a^{10}} \log \frac{X}{x}$$



Taf. V.

$$\int \frac{x^n dx}{(a + bx)^5}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^5} = -\frac{1}{4bX^4}$$

$$\int \frac{x dx}{X^5} = -\left(\frac{x}{3b} + \frac{a}{12b^2}\right) \frac{1}{X^4}$$

$$\int \frac{x^2 dx}{X^5} = -\left(\frac{x^2}{2b} + \frac{ax}{3b^2} + \frac{a^2}{12b^3}\right) \frac{1}{X^4}$$

$$\int \frac{x^3 dx}{X^5} = -\left(\frac{x^3}{b} + \frac{3ax^2}{2b^2} + \frac{a^2x}{b^3} + \frac{a^3}{4b^4}\right) \frac{1}{X^4}$$

$$\int \frac{x^4 dx}{X^5} = \left(\frac{4ax^3}{b^2} + \frac{9a^2x^2}{b^3} + \frac{22a^3x}{3b^4} + \frac{25a^4}{12b^5}\right) \frac{1}{X^4} + \frac{1}{b^5} \log X$$

$$\int \frac{x^5 dx}{X^5} = \left(\frac{x^5}{b} - \frac{20a^2x^3}{b^3} - \frac{45a^3x^2}{b^4} - \frac{110a^4x}{3b^5} - \frac{125a^5}{12b^6}\right) \frac{1}{X^4} - \frac{5a}{b^6} \log X$$

$$\int \frac{x^6 dx}{X^5} = \left(\frac{x^6}{2b} - \frac{3ax^5}{b^2} + \frac{60a^3x^3}{b^4} + \frac{135a^4x^2}{b^5} + \frac{110a^5x}{b^6} + \frac{125a^6}{4b^7}\right) \frac{1}{X^4} + \frac{15a^2}{b^7} \log X$$

$$\int \frac{x^7 dx}{X^5} = \left(\frac{x^7}{3b} - \frac{7ax^6}{6b^2} + \frac{7a^2x^5}{b^3} - \frac{140a^4x^3}{b^5} - \frac{315a^5x^2}{b^6} - \frac{770a^6x}{3b^7} - \frac{875a^7}{12b^8}\right) \frac{1}{X^4} - \frac{35a^3}{b^8} \log X$$

$$\int \frac{x^8 dx}{X^5} = \left(\frac{x^8}{4b} - \frac{2ax^7}{3b^2} + \frac{7a^2x^6}{3b^3} - \frac{14a^3x^5}{b^4} + \frac{280a^5x^3}{b^6} + \frac{630a^6x^2}{b^7} + \frac{1540a^7x}{3b^8} + \frac{875a^8}{6b^9}\right) \frac{1}{X^4} + \frac{70a^4}{b^9} \log X$$

$$\int \frac{x^n dx}{(a+bx)^6}$$

Taf. VI.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{X^6} = -\frac{1}{5bX^5}$$

$$\int \frac{x \partial x}{X^6} = -\left(\frac{x}{4b} + \frac{a}{20b^2}\right) \frac{1}{X^5}$$

$$\int \frac{x^2 \partial x}{X^6} = -\left(\frac{x^2}{3b} + \frac{ax}{6b^2} + \frac{a^2}{30b^3}\right) \frac{1}{X^4}$$

$$\int \frac{x^3 \partial x}{X^6} = -\left(\frac{x^3}{2b} + \frac{ax^2}{2b^2} + \frac{a^2x}{4b^3} + \frac{a^3}{20b^4}\right) \frac{1}{X^3}$$

$$\int \frac{x^4 \partial x}{X^6} = -\left(\frac{x^4}{b} + \frac{2ax^3}{b^2} + \frac{2a^2x^2}{b^3} + \frac{a^3x}{b^4} + \frac{a^4}{5b^5}\right) \frac{1}{X^2}$$

$$\int \frac{x^5 \partial x}{X^6} = \left(\frac{5ax^4}{b^2} + \frac{15a^2x^3}{b^3} + \frac{55a^3x^2}{3b^4} + \frac{125a^4x}{12b^5} + \frac{137a^5}{60b^6}\right) \frac{1}{X} + \frac{1}{b^6} \log X$$

$$\int \frac{x^6 \partial x}{X^6} = \left(\frac{x^6}{b} - \frac{30a^2x^4}{b^3} - \frac{90a^3x^3}{b^4} - \frac{110a^4x^2}{b^5} - \frac{125a^5x}{2b^6} - \frac{137a^6}{10b^7}\right) \frac{1}{X^5} - \frac{6a}{b^7} \log X$$

$$\int \frac{x^7 \partial x}{X^6} = \left(\frac{x^7}{2b} - \frac{7ax^6}{2b^2} + \frac{105a^3x^4}{b^4} + \frac{315a^4x^3}{b^5} + \frac{385a^5x^2}{b^6} + \frac{875a^6x}{4b^7} + \frac{959a^7}{20b^8}\right) \frac{1}{X^3} + \frac{21a^2}{b^8} \log X$$

$$\int \frac{x^8 \partial x}{X^6} = \left(\frac{x^8}{3b} - \frac{4ax^7}{3b^2} + \frac{28a^2x^6}{3b^3} - \frac{280a^4x^4}{b^5} - \frac{840a^5x^3}{b^6} - \frac{3080a^6x^2}{3b^7} - \frac{1750a^7x}{3b^8} - \frac{1918a^8}{15b^9}\right) \frac{1}{X} - \frac{56a^3}{b^9} \log X$$



$$\int \frac{\partial x}{x^n(a+bx)^2}$$

Taf. VIII.

$$\text{VZ. } a + bx = X$$

$$\int \frac{\partial x}{xX^2} = \frac{1}{aX} - \frac{1}{a^2} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{2b}{a^2}\right) \frac{1}{X} + \frac{2b}{a^3} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{a^3}\right) \frac{1}{X} - \frac{3b^2}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \frac{2b^2}{a^3x} - \frac{4b^3}{a^4}\right) \frac{1}{X} + \frac{4b^3}{a^5} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{5b}{12a^2x^3} - \frac{5b^2}{6a^3x^2} + \frac{5b^3}{2a^4x} + \frac{5b^4}{a^5}\right) \frac{1}{X} - \frac{5b^4}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4} - \frac{b^2}{2a^3x^3} + \frac{b^3}{a^4x^2} - \frac{3b^4}{a^5x} - \frac{6b^5}{a^6}\right) \frac{1}{X} + \frac{6b^5}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \frac{7b^2}{20a^3x^4} + \frac{7b^3}{12a^4x^3} - \frac{7b^4}{6a^5x^2} + \frac{7b^5}{2a^6x} + \frac{7b^6}{a^7}\right) \frac{1}{X} - \frac{7b^6}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{4b}{21a^2x^6} - \frac{4b^2}{15a^3x^5} + \frac{2b^3}{5a^4x^4} - \frac{2b^4}{3a^5x^3} + \frac{4b^5}{3a^6x^2} - \frac{4b^6}{a^7x} - \frac{8b^7}{a^8}\right) \frac{1}{X} + \frac{8b^7}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{9b}{56a^2x^7} - \frac{3b^2}{14a^3x^6} + \frac{3b^3}{10a^4x^5} - \frac{9b^4}{20a^5x^4} + \frac{3b^5}{4a^6x^3} - \frac{3b^6}{2a^7x^2} + \frac{9b^7}{2a^8x} + \frac{9b^8}{a^9}\right) \frac{1}{X} - \frac{9b^8}{a^{10}} \log \frac{X}{x}$$

Taf. IX.

$$\int \frac{dx}{x^m(a+bx)^3}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{xX^3} = \left( \frac{3}{2a} + \frac{bx}{a^2} \right) \frac{1}{X^2} - \frac{1}{a^3} \log \frac{X}{x}$$

$$\int \frac{dx}{x^2X^3} = \left( -\frac{1}{ax} - \frac{9b}{2a^2} - \frac{3b^2x}{a^3} \right) \frac{1}{X^2} + \frac{3b}{a^4} \log \frac{X}{x}$$

$$\int \frac{dx}{x^3X^3} = \left( -\frac{1}{2ax^2} + \frac{2b}{a^2x} + \frac{9b^2}{a^3} + \frac{6b^3x}{a^4} \right) \frac{1}{X^2} - \frac{6b^2}{a^5} \log \frac{X}{x}$$

$$\int \frac{dx}{x^4X^3} = \left( -\frac{1}{3ax^3} + \frac{5b}{6a^2x^2} - \frac{10b^2}{3a^3x} - \frac{15b^3}{a^4} - \frac{10b^4x}{a^5} \right) \frac{1}{X^2} + \frac{10b^3}{a^6} \log \frac{X}{x}$$

$$\int \frac{dx}{x^5X^3} = \left( -\frac{1}{4ax^4} + \frac{b}{2a^2x^3} - \frac{5b^2}{4a^3x^2} + \frac{5b^3}{a^4x} + \frac{45b^4}{2a^5} + \frac{15b^5x}{a^6} \right) \frac{1}{X^2} - \frac{15b^4}{a^7} \log \frac{X}{x}$$

$$\int \frac{dx}{x^6X^3} = \left( -\frac{1}{5ax^5} + \frac{7b}{20a^2x^4} - \frac{7b^2}{10a^3x^3} + \frac{7b^3}{4a^4x^2} - \frac{7b^4}{a^5x} - \frac{63b^5}{2a^6} - \frac{21b^6x}{a^7} \right) \frac{1}{X^2} + \frac{21b^5}{a^8} \log \frac{X}{x}$$

$$\int \frac{dx}{x^7X^3} = \left( -\frac{1}{6ax^6} + \frac{4b}{15a^2x^5} - \frac{7b^2}{15a^3x^4} + \frac{14b^3}{15a^4x^3} - \frac{7b^4}{5a^5x^2} + \frac{28b^5}{3a^6x} + \frac{42b^6}{a^7} + \frac{28b^7x}{a^8} \right) \frac{1}{X^2} - \frac{28b^6}{a^9} \log \frac{X}{x}$$

$$\int \frac{dx}{x^8X^3} = \left( -\frac{1}{7ax^7} + \frac{3b}{14a^2x^6} - \frac{12b^2}{35a^3x^5} + \frac{3b^3}{5a^4x^4} - \frac{6b^4}{5a^5x^3} + \frac{3b^5}{a^6x^2} - \frac{12b^6}{a^7x} - \frac{54b^7}{a^8} - \frac{36b^8x}{a^9} \right) \frac{1}{X^2} + \frac{56b^7}{a^{10}} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^m(a+bx)^4}$$

Taf. X.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{xX^4} = \left( \frac{11}{6a} + \frac{5bx}{2a^2} + \frac{b^2x^2}{a^3} \right) \frac{1}{X^3} - \frac{1}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2X^4} = \left( -\frac{1}{ax} - \frac{22b}{3a^2} - \frac{10b^2x}{a^3} - \frac{4b^3x^2}{a^4} \right) \frac{1}{X^3} + \frac{4b}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3X^4} = \left( -\frac{1}{2ax^2} + \frac{5b}{2a^2x} + \frac{55b^2}{3a^3} + \frac{25b^3x}{a^4} + \frac{10b^4x^2}{a^5} \right) \frac{1}{X^3} - \frac{10b^2}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4X^4} = \left( -\frac{1}{3ax^3} + \frac{b}{a^2x^2} - \frac{5b^2}{a^3x} - \frac{110b^3}{3a^4} - \frac{50b^4x}{a^5} - \frac{20b^5x^2}{a^6} \right) \frac{1}{X^3} + \frac{20b^3}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5X^4} = \left( -\frac{1}{4ax^4} + \frac{7b}{12a^2x^3} - \frac{7b^2}{4a^3x^2} + \frac{35b^3}{4a^4x} + \frac{385b^4}{6a^5} + \frac{175b^5x}{2a^6} + \frac{35b^6x^2}{a^7} \right) \frac{1}{X^3} - \frac{35b^4}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6X^4} = \left( -\frac{1}{5ax^5} + \frac{2b}{5a^2x^4} - \frac{14b^2}{15a^3x^3} + \frac{14b^3}{5a^4x^2} - \frac{14b^4}{a^5x} - \frac{308b^5}{3a^6} - \frac{140b^6x}{a^7} - \frac{56b^7x^2}{a^8} \right) \frac{1}{X^3} + \frac{56b^5}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7X^4} = \left( -\frac{1}{6ax^6} + \frac{3b}{10a^2x^5} - \frac{3b^2}{5a^3x^4} + \frac{7b^3}{5a^4x^3} - \frac{21b^4}{5a^5x^2} + \frac{21b^5}{a^6x} + \frac{154b^6}{a^7} + \frac{210b^7x}{a^8} + \frac{84b^8x^2}{a^9} \right) \frac{1}{X^3} - \frac{84b^6}{a^{10}} \log \frac{X}{x}$$

Type III

$$\int \frac{dx}{x^2(a+bx^2)}$$

$$IX. a + bx^2 = X$$

$$\int \frac{dx}{x^2 X} = \frac{1}{ax} - \frac{bx}{ax^2} - \frac{b^2 x^3}{4a^3} - \frac{b^3 x^5}{8a^4} - \frac{1}{a^2} \log \frac{X}{x}$$

$$\int \frac{dx}{x^3 X} = \left( -\frac{1}{ax^2} - \frac{bx}{ax^3} - \frac{6b^2 x}{4a^3} - \frac{3b^3 x^3}{8a^4} - \frac{3b^2 x^5}{8a^5} \right) \frac{1}{X^2} - \frac{3b}{a^2} \log \frac{X}{x}$$

$$\int \frac{dx}{x^4 X} = \left( -\frac{1}{ax^3} - \frac{bx}{a^2 x^2} - \frac{15b^2 x}{4a^3} - \frac{6b^3 x}{a^4} - \frac{15b^2 x^3}{8a^5} \right) \frac{1}{X^3} - \frac{15b^2}{a^3} \log \frac{X}{x}$$

$$\int \frac{dx}{x^5 X} = \left( -\frac{1}{5ax^4} - \frac{bx}{6a^2 x^3} - \frac{7b^2}{4a^3 x} - \frac{7b^3 x}{12a^4} - \frac{45b^2 x^3}{32a^5} - \frac{21b^3 x^5}{8a^6} \right) \frac{1}{X^4} - \frac{15b^2}{a^4} \log \frac{X}{x}$$

$$\int \frac{dx}{x^6 X} = \left( -\frac{1}{6ax^5} - \frac{bx}{5a^2 x^4} - \frac{7b^2}{5a^3 x^2} - \frac{11b^3}{4a^4 x} - \frac{7b^2 x^3}{6a^5} - \frac{31b^3 x^5}{32a^6} \right) \frac{1}{X^5} - \frac{15b^2}{a^5} \log \frac{X}{x}$$

$$\int \frac{dx}{x^7 X} = \left( -\frac{1}{7ax^6} - \frac{bx}{6a^2 x^5} - \frac{6b^2}{5a^3 x^3} - \frac{21b^3}{5a^4 x} - \frac{12b^2 x^3}{5a^5} - \frac{53b^3 x^5}{80a^6} \right) \frac{1}{X^6} - \frac{15b^2}{a^6} \log \frac{X}{x}$$

$$\int \frac{dx}{x^8 X} = \left( -\frac{1}{8ax^7} - \frac{bx}{7a^2 x^6} - \frac{3b^2}{4a^3 x^4} - \frac{21b^3}{4a^4 x^2} - \frac{7b^2 x^4}{a^5} - \frac{41b^3 x^6}{64a^6} \right) \frac{1}{X^7} - \frac{15b^2}{a^7} \log \frac{X}{x}$$

$$\int \frac{dx}{x^m(a+bx)^6}$$

Taf. XII.

$$\text{VZ. } a + bx = X$$

$$\int \frac{dx}{xX^6} = \left( \frac{137}{60a} + \frac{77bx}{12a^2} + \frac{47b^2x^2}{6a^3} + \frac{9b^3x^3}{2a^4} + \frac{b^4x^4}{a^5} \right) \frac{1}{X^5} - \frac{1}{a^6} \log \frac{X}{x}$$

$$\int \frac{dx}{x^2X^6} = \left( -\frac{1}{ax} - \frac{137b}{10a^2} - \frac{77b^2x}{2a^3} - \frac{47b^3x^2}{a^4} - \frac{27b^4x^3}{a^5} - \frac{6b^5x^4}{a^6} \right) \frac{1}{X^4} + \frac{6b}{a^7} \log \frac{X}{x}$$

$$\int \frac{dx}{x^3X^6} = \left( -\frac{1}{2ax^2} + \frac{7b}{2a^2x} + \frac{959b^2}{20a^3} + \frac{539b^3x}{4a^4} + \frac{329b^4x^2}{2a^5} + \frac{189b^5x^3}{a^6} + \frac{21b^6x^4}{a^7} \right) \frac{1}{X^3} - \frac{21b^2}{a^8} \log \frac{X}{x}$$

$$\int \frac{dx}{x^4X^6} = \left( -\frac{1}{3ax^3} + \frac{4b}{3a^2x^2} - \frac{28b^2}{3a^3x} - \frac{1918b^3}{15a^4} - \frac{1078b^4x}{3a^5} - \frac{1316b^5x^2}{3a^6} - \frac{504b^6x^3}{a^7} - \frac{56b^7x^4}{a^8} \right) \frac{1}{X^2} + \frac{56b^3}{a^9} \log \frac{X}{x}$$

$$\int \frac{dx}{x^5X^6} = \left( -\frac{1}{4ax^4} + \frac{3b}{4a^2x^3} - \frac{3b^2}{a^3x^2} + \frac{21b^3}{a^4x} + \frac{2877b^4}{10a^5} + \frac{1617b^5x}{2a^6} + \frac{987b^6x^2}{a^7} + \frac{1134b^7x^3}{a^8} + \frac{126b^8x^4}{a^9} \right) \frac{1}{X} - \frac{126b^4}{a^{10}} \log \frac{X}{x}$$

$$\int \frac{dx}{x^6X^6} = \left( -\frac{1}{5ax^5} + \frac{b}{2a^2x^4} - \frac{3b^2}{2a^3x^3} + \frac{6b^3}{a^4x^2} - \frac{42b^4}{a^5x} - \frac{2877b^5}{5a^6} - \frac{1617b^6x}{a^7} - \frac{1974b^7x^2}{a^8} - \frac{2268b^8x^3}{a^9} - \frac{252b^9x^4}{a^{10}} \right) \frac{1}{X^5} + \frac{252b^5}{a^{11}} \log \frac{X}{x}$$



$$\int \frac{x^n dx}{a + bx^2}$$

Taf. XIV.

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{dx}{X} = \int \frac{dx}{X} \text{ (S. 47.)}$$

$$\int \frac{x dx}{X} = \frac{1}{2b} \log X$$

$$\int \frac{x^2 dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^3 dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X}$$

$$\int \frac{x^4 dx}{X} = \frac{x^3}{3b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{x^4}{4b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2} \int \frac{x dx}{X}$$

$$\int \frac{x^6 dx}{X} = \frac{x^5}{5b} - \frac{ax^3}{3b^2} + \frac{a^2 x}{b^3} - \frac{a^3}{b^3} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^6}{6b} - \frac{ax^4}{4b^2} + \frac{a^2 x^2}{2b^3} - \frac{a^3}{b^3} \int \frac{x dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^7}{7b} - \frac{ax^5}{5b^2} + \frac{a^2 x^3}{5b^3} - \frac{a^3 x}{b^4} + \frac{a^4}{b^4} \int \frac{dx}{X}$$

$$\int \frac{x^9 dx}{X} = \frac{x^8}{8b} - \frac{ax^6}{6b^2} + \frac{a^2 x^4}{4b^3} - \frac{a^3 x^2}{2b^4} + \frac{a^4}{b^4} \int \frac{x dx}{X}$$

$$\int \frac{x^{10} dx}{X} = \frac{x^9}{9b} - \frac{ax^7}{7b^2} + \frac{a^2 x^5}{5b^3} - \frac{a^3 x^3}{5b^4} + \frac{a^4 x}{b^5} - \frac{a^5}{b^5} \int \frac{dx}{X}$$

$$\int \frac{x^{11} dx}{X} = \frac{x^{10}}{10b} - \frac{ax^8}{8b^2} + \frac{a^2 x^6}{6b^3} - \frac{a^3 x^4}{4b^4} + \frac{a^4 x^2}{2b^5} - \frac{a^5}{b^5} \int \frac{x dx}{X}$$

$$\int \frac{x^{12} dx}{X} = \frac{x^{11}}{11b} - \frac{ax^9}{9b^2} + \frac{a^2 x^7}{7b^3} - \frac{a^3 x^5}{5b^4} + \frac{a^4 x^3}{5b^5} - \frac{a^5 x}{b^6} + \frac{a^6}{b^6} \int \frac{dx}{X}$$

$$\int \frac{x^n \partial x}{(a + bx^2)^2}.$$

Taf. XV.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{X^2} = \frac{x}{2aX} + \frac{1}{2a} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^2} = -\frac{1}{2bX}$$

$$\int \frac{x^2 \partial x}{X^2} = -\frac{x}{2bX} + \frac{1}{2b} \int \frac{\partial x}{X}$$

$$\int \frac{x^3 \partial x}{X^2} = \frac{a}{2b^2 X} + \frac{1}{2b^2} \log X$$

$$\int \frac{x^4 \partial x}{X^2} = \left( \frac{x^3}{b} + \frac{3ax}{2b^2} \right) \frac{1}{X} - \frac{3a}{2b^2} \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^2} = \left( \frac{x^4}{2b} - \frac{a^2}{b^3} \right) \frac{1}{X} - \frac{a}{b^3} \log X$$

$$\int \frac{x^6 \partial x}{X^2} = \left( \frac{x^5}{3b} - \frac{5ax^3}{3b^2} - \frac{5a^2 x}{2b^3} \right) \frac{1}{X} + \frac{5a^2}{2b^3} \int \frac{\partial x}{X}$$

$$\int \frac{x^7 \partial x}{X^2} = \left( \frac{x^6}{4b} - \frac{3ax^4}{4b^2} + \frac{3a^2}{2b^4} \right) \frac{1}{X} + \frac{3a^2}{2b^4} \log X$$

$$\int \frac{x^8 \partial x}{X^2} = \left( \frac{x^7}{5b} - \frac{7ax^5}{15b^2} + \frac{7a^2 x^3}{3b^3} + \frac{7a^3 x}{2b^4} \right) \frac{1}{X} - \frac{7a^3}{2b^4} \int \frac{\partial x}{X}$$

$$\int \frac{x^9 \partial x}{X^2} = \left( \frac{x^8}{6b} - \frac{ax^6}{3b^2} + \frac{a^2 x^4}{b^3} - \frac{2a^3}{b^5} \right) \frac{1}{X} - \frac{2a^3}{b^5} \log X$$

$$\int \frac{x^{10} \partial x}{X^2} = \left( \frac{x^9}{7b} - \frac{9ax^7}{35b^2} + \frac{3a^2 x^5}{5b^3} - \frac{3a^3 x^3}{b^4} - \frac{9a^4 x}{2b^5} \right) \frac{1}{X} + \frac{9a^4}{2b^5} \int \frac{\partial x}{X}$$

$$\int \frac{x^{11} \partial x}{X^2} = \left( \frac{x^{10}}{8b} - \frac{5ax^8}{24b^2} + \frac{5a^2 x^6}{12b^3} - \frac{5a^3 x^4}{4b^4} + \frac{5a^5}{2b^6} \right) \frac{1}{X} + \frac{5a^4}{2b^6} \log X$$

Taf. XVI

$$\int \frac{x^n dx}{(a + bx^2)^3}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx}{X^3} = \left( \frac{5bx^3}{8a^2} + \frac{5x}{8a} \right) \frac{1}{X^2} + \frac{5}{8a^2} \int \frac{dx}{X}$$

$$\int \frac{xdx}{X^3} = -\frac{1}{4bX^2}$$

$$\int \frac{x^2 dx}{X^3} = \left( \frac{x^3}{8a} - \frac{x}{8b} \right) \frac{1}{X^2} + \frac{1}{8ab} \int \frac{dx}{X}$$

$$\int \frac{x^3 dx}{X^3} = \left( -\frac{x^2}{2b} - \frac{a}{4b^2} \right) \frac{1}{X^2}$$

$$\int \frac{x^4 dx}{X^3} = \left( -\frac{5x^3}{8b} - \frac{5ax}{8b^2} \right) \frac{1}{X^2} + \frac{5}{8b^2} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X^3} = \left( \frac{ax^2}{b^2} + \frac{3a^2}{4b^3} \right) \frac{1}{X^2} + \frac{1}{2b^3} \log X$$

$$\int \frac{x^6 dx}{X^3} = \left( \frac{x^5}{b} + \frac{25ax^3}{8b^2} + \frac{15a^2x}{8b^3} \right) \frac{1}{X^2} - \frac{15a}{8b^3} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X^3} = \left( \frac{x^6}{2b} - \frac{3a^2x^2}{b^3} - \frac{9a^3}{4b^4} \right) \frac{1}{X^2} - \frac{5a}{2b^4} \log X$$

$$\int \frac{x^8 dx}{X^3} = \left( \frac{x^7}{3b} - \frac{7ax^5}{5b^2} - \frac{175a^2x^3}{24b^3} - \frac{55a^3x}{8b^4} \right) \frac{1}{X^2} + \frac{55a^2}{8b^4} \int \frac{dx}{X}$$

$$\int \frac{x^9 dx}{X^3} = \left( \frac{x^8}{4b} - \frac{ax^6}{b^2} + \frac{6a^2x^4}{b^4} + \frac{9a^4}{2b^5} \right) \frac{1}{X^2} + \frac{5a^2}{b^5} \log X$$

$$\int \frac{x^{10} dx}{X^3} = \left( \frac{x^9}{5b} - \frac{3ax^7}{5b^2} + \frac{21a^2x^5}{5b^3} + \frac{105a^3x^3}{8b^4} + \frac{63a^4x}{8b^5} \right) \frac{1}{X^2} - \frac{63a^3}{8b^5} \int \frac{dx}{X}$$

$$\int \frac{x^m dx}{(a + bx^2)^4}$$

Taf. XVII.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{X^4} = \left( \frac{5b^2x^5}{16a^3} + \frac{5bx^3}{6a^2} + \frac{11x}{16a} \right) \frac{1}{X^3} + \frac{5}{16a^3} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^4} = -\frac{1}{6bX^3}$$

$$\int \frac{x^2 \partial x}{X^4} = \left( \frac{bx^5}{16a^2} + \frac{x^3}{6a} - \frac{x}{16b} \right) \frac{1}{X^3} + \frac{1}{16a^2b} \int \frac{\partial x}{X}$$

$$\int \frac{x^3 \partial x}{X^4} = \left( -\frac{x^2}{4b} - \frac{a}{12b^2} \right) \frac{1}{X^3}$$

$$\int \frac{x^4 \partial x}{X^4} = \left( \frac{x^5}{16a} - \frac{x^3}{6b} - \frac{ax}{16b^2} \right) \frac{1}{X^3} + \frac{1}{16ab^2} \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^4} = \left( -\frac{x^4}{2b} - \frac{ax^2}{2b^2} - \frac{a^2}{6b^3} \right) \frac{1}{X^3}$$

$$\int \frac{x^6 \partial x}{X^4} = \left( -\frac{11x^5}{16b} - \frac{5ax^3}{6b^2} - \frac{5a^2x}{16b^3} \right) \frac{1}{X^3} + \frac{5}{16b^3} \int \frac{\partial x}{X}$$

$$\int \frac{x^7 \partial x}{X^4} = \left( \frac{3ax^4}{2b^2} + \frac{9a^2x^2}{4b^3} + \frac{11a^3}{12b^4} \right) \frac{1}{X^3} + \frac{1}{2b^4} \log X$$

$$\int \frac{x^8 \partial x}{X^4} = \left( \frac{x^7}{b} + \frac{77ax^5}{16b^2} + \frac{35a^2x^3}{6b^3} + \frac{35a^3x}{16b^4} \right) \frac{1}{X^3} - \frac{35a}{16b^4} \int \frac{\partial x}{X}$$

$$\int \frac{x^9 \partial x}{X^4} = \left( \frac{x^8}{2b} - \frac{6a^2x^4}{b^3} - \frac{9a^3x^2}{b^4} - \frac{11a^4}{3b^5} \right) \frac{1}{X^3} - \frac{2a}{b^5} \log X$$

$$\int \frac{x^{10} \partial x}{X^4} = \left( \frac{x^9}{3b} - \frac{3ax^7}{b^2} - \frac{231a^2x^5}{16b^3} - \frac{35a^3x^3}{2b^4} - \frac{105a^4x}{16b^5} \right) \frac{1}{X^3} + \frac{105a^2}{16b^5} \int \frac{\partial x}{X}$$

$$\int \frac{x^{11} \partial x}{X^4} = \left( \frac{x^{10}}{4b} - \frac{5ax^8}{4b^2} + \frac{15a^3x^4}{b^4} + \frac{45a^4x^2}{2b^5} + \frac{55a^5}{6b^6} \right) \frac{1}{X^3} + \frac{5a^2}{b^6} \log X$$

Taf. XVIII

$$\int \frac{x^2 x}{(x + bx^2)^2}$$

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$$\text{VZ } x + bx^2 = X$$


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$$\int \frac{dx}{X} = \frac{35x^5}{128a^4} + \frac{55x^4}{32a^3} + \frac{51x^3}{32a^2} + \frac{65x^2}{128a} + \frac{35}{128a} \sqrt{X}$$

$$\int \frac{x^2 dx}{X} = -\frac{1}{8aX}$$

$$\int \frac{x^3 dx}{X} = \frac{3x^5}{128a^4} + \frac{55x^4}{32a^3} - \frac{7x^3}{32a^2} - \frac{5x^2}{128a} + \frac{5}{128a} \sqrt{X}$$

$$\int \frac{x^4 dx}{X} = \left( -\frac{x^2}{6a} - \frac{a}{24a^2} \right) \frac{1}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{3x^5}{128a^4} + \frac{11x^4}{128a} - \frac{11x^3}{128a} - \frac{5x^2}{128a^2} + \frac{3}{128a^2} \sqrt{X}$$

$$\int \frac{x^6 dx}{X} = \left( -\frac{x^4}{4a} - \frac{ax^2}{6a^2} - \frac{a^2}{24a^3} \right) \frac{1}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{3x^5}{128a^4} - \frac{7x^4}{32a^3} - \frac{55x^3}{32a^2} - \frac{5x^2}{128a} + \frac{5}{128a} \sqrt{X}$$

$$\int \frac{x^8 dx}{X} = \left( -\frac{x^6}{2a} - \frac{5ax^4}{4a^2} - \frac{a^2x^2}{2a^3} - \frac{a^3}{6a^4} \right) \frac{1}{X}$$

$$\int \frac{x^9 dx}{X} = \left( \frac{35x^5}{128a^4} - \frac{511x^4}{32a^3} - \frac{553x^3}{32a^2} - \frac{55x^2}{128a} + \frac{35}{128a^2} \right) \sqrt{X}$$

$$\int \frac{x^{10} dx}{X} = \left( \frac{7x^6}{3a} - \frac{9ax^4}{2a^2} - \frac{11x^2}{3a^3} - \frac{25a}{24a^4} \right) \frac{1}{X} - \frac{1}{2a^2} \log X$$

$$\int \frac{x^{11} dx}{X} = \frac{x^8}{3a} + \frac{35ax^6}{128a^2} - \frac{1553x^5}{128a^2} - \frac{1155x^4}{128a^2} - \frac{55x^3}{128a^2} \frac{1}{X} - \frac{35x^2}{128a^2} \sqrt{X}$$

$$\int \frac{x^{12} dx}{X} = \left( \frac{x^{10}}{2a} - \frac{11x^8}{2a^2} - \frac{55x^6}{24a^3} - \frac{55x^4}{32a^3} - \frac{11x^2}{24a^3} \right) \frac{1}{X} - \frac{7x}{2a^2} \log X$$

$$\int \frac{x^n dx}{(a + bx^2)^6}$$

Taf. XIX.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx}{X^6} = \left( \frac{63b^4x^2}{256a^5} + \frac{147b^3x^7}{128a^4} + \frac{21b^2x^5}{10a^3} + \frac{237bx^3}{128a^2} + \frac{193x}{256a} \right) \frac{1}{X^5} + \frac{63}{256a^5} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^6} = -\frac{1}{10bX^5}$$

$$\int \frac{x^2 dx}{X^6} = \left( \frac{7b^3x^9}{256a^4} + \frac{49b^2x^7}{384a^3} + \frac{7bx^5}{30a^2} + \frac{79x^3}{384a} - \frac{7x}{256b} \right) \frac{1}{X^5} + \frac{7}{256a^4b} \int \frac{dx}{X}$$

$$\int \frac{x^3 dx}{X^6} = \left( -\frac{x^2}{8b} - \frac{a}{40b^2} \right) \frac{1}{X^5}$$

$$\int \frac{x^4 dx}{X^6} = \left( \frac{3b^2x^9}{256a^3} + \frac{7bx^7}{128a^2} + \frac{x^5}{10a} - \frac{7x^3}{128b} - \frac{3ax}{256b^2} \right) \frac{1}{X^5} + \frac{3}{256a^3b^2} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X^6} = \left( -\frac{x^4}{6b} - \frac{ax^2}{12b^2} - \frac{a^2}{60b^3} \right) \frac{1}{X^5}$$

$$\int \frac{x^6 dx}{X^6} = \left( \frac{3bx^9}{256a^2} + \frac{7x^7}{128a} - \frac{x^5}{10b} - \frac{7ax^3}{128b^2} - \frac{3a^2x}{256b^3} \right) \frac{1}{X^5} + \frac{3}{256a^2b^3} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X^6} = \left( -\frac{x^6}{4b} - \frac{ax^4}{4b^2} - \frac{a^2x^2}{8b^3} - \frac{a^3}{40b^4} \right) \frac{1}{X^5}$$

$$\int \frac{x^8 dx}{X^6} = \left( \frac{7x^9}{256a} - \frac{79x^7}{384b} - \frac{7ax^5}{30b^2} - \frac{49a^2x^3}{384b^3} - \frac{7a^3x}{256b^4} \right) \frac{1}{X^5} + \frac{7}{256ab^4} \int \frac{dx}{X}$$

$$\int \frac{x^9 dx}{X^6} = \left( -\frac{x^8}{2b} - \frac{ax^6}{b^2} - \frac{a^2x^4}{b^3} - \frac{a^3x^2}{2b^4} - \frac{a^4}{10b^5} \right) \frac{1}{X^5}$$

T. f. XX.

$$\int \frac{dx}{x^2(a + bx^2)}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx}{x^2 X} = \frac{1}{2a} \log \frac{x^2}{X} = \frac{1}{a} \log \frac{x}{\sqrt{X}} = -\frac{1}{2a} \log \frac{X}{x^2} = -\frac{1}{a} \log \frac{\sqrt{X}}{x}$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4 X} = -\frac{1}{3ax^2} - \frac{b}{a} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^5 X} = -\frac{1}{4ax^3} - \frac{b}{a^2 x} - \frac{b^2}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^6 X} = -\frac{1}{5ax^4} - \frac{b}{2a^2 x^2} - \frac{b^2}{a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^7 X} = -\frac{1}{6ax^5} - \frac{b}{3a^2 x^3} - \frac{b^2}{a^2 x} - \frac{b^3}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^8 X} = -\frac{1}{7ax^6} - \frac{b}{4a^2 x^4} - \frac{b^2}{2a^2 x^2} - \frac{b^3}{a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^9 X} = -\frac{1}{8ax^7} - \frac{b}{5a^2 x^5} - \frac{b^2}{3a^2 x^3} - \frac{b^3}{a^2 x} - \frac{b^4}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^{10} X} = -\frac{1}{9ax^8} - \frac{b}{6a^2 x^6} - \frac{b^2}{4a^2 x^4} - \frac{b^3}{2a^2 x^2} - \frac{b^4}{a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^{11} X} = -\frac{1}{10ax^9} - \frac{b}{7a^2 x^7} - \frac{b^2}{5a^2 x^5} - \frac{b^3}{3a^2 x^3} - \frac{b^4}{a^2 x} - \frac{b^5}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^{12} X} = -\frac{1}{11ax^{10}} - \frac{b}{8a^2 x^8} - \frac{b^2}{6a^2 x^6} - \frac{b^3}{4a^2 x^4} - \frac{b^4}{2a^2 x^2} - \frac{b^5}{a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^{13} X} = -\frac{1}{12ax^{11}} - \frac{b}{9a^2 x^9} - \frac{b^2}{7a^2 x^7} - \frac{b^3}{5a^2 x^5} - \frac{b^4}{3a^2 x^3} - \frac{b^5}{a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^{14} X} = -\frac{1}{13ax^{12}} - \frac{b}{10a^2 x^{10}} - \frac{b^2}{8a^2 x^8} - \frac{b^3}{6a^2 x^6} - \frac{b^4}{4a^2 x^4} - \frac{b^5}{2a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{dx}{x^{15} X} = -\frac{1}{14ax^{13}} - \frac{b}{11a^2 x^{11}} - \frac{b^2}{10a^2 x^9} - \frac{b^3}{8a^2 x^7} - \frac{b^4}{6a^2 x^5} - \frac{b^5}{4a^2} \int \frac{dx}{x^2 X}$$

$$\int \frac{\partial x}{x^n(a+bx^2)^2}$$

Taf. XXI.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{xX^2} = \frac{1}{2aX} + \frac{1}{a} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{3bx}{2a^2}\right) \frac{1}{X} - \frac{3b}{2a^2} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{b}{a^2}\right) \frac{1}{X} - \frac{2b}{a^2} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x} + \frac{5b^2x}{2a^3}\right) \frac{1}{X} + \frac{5b^2}{2a^3} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2} + \frac{3b^2}{2a^3}\right) \frac{1}{X} + \frac{3b^2}{a^3} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{7b}{15a^2x^3} - \frac{7b^2}{3a^3x} - \frac{7b^3x}{2a^4}\right) \frac{1}{X} - \frac{7b^3}{2a^4} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{3a^2x^4} - \frac{b^2}{a^3x^2} - \frac{2b^3}{a^4}\right) \frac{1}{X} - \frac{4b^3}{a^4} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5} - \frac{3b^2}{5a^3x^3} + \frac{3b^3}{a^4x} + \frac{9b^4x}{2a^5}\right) \frac{1}{X} + \frac{9b^4}{2a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{5b}{24a^2x^6} - \frac{5b^2}{12a^3x^4} + \frac{5b^3}{4a^4x^2} + \frac{5b^4}{2a^5}\right) \frac{1}{X} + \frac{5b^4}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10}X^2} = \left(-\frac{1}{9ax^9} + \frac{11b}{63a^2x^7} - \frac{11b^2}{35a^3x^5} + \frac{11b^3}{15a^4x^3} - \frac{11b^4}{3a^5x} - \frac{11b^5x}{2a^6}\right) \frac{1}{X} - \frac{11b^5}{2a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^{11}X^2} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{20a^2x^8} - \frac{b^2}{4a^3x^6} + \frac{b^3}{2a^4x^4} - \frac{3b^4}{2a^5x^2} - \frac{3b^5}{a^6}\right) \frac{1}{X} - \frac{6b^5}{a^6} \int \frac{\partial x}{xX}$$



Taf. XXII.

$$\int \frac{\partial x}{x^m(a + bx^2)^2}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\begin{aligned} \int \frac{\partial x}{xX^3} &= \left(\frac{3}{4a} + \frac{bx^2}{2a^2}\right) \frac{1}{X^2} + \frac{1}{a^2} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^2X^3} &= \left(-\frac{1}{ax} - \frac{25bx}{8a^2} - \frac{15b^2x^3}{8a^3}\right) \frac{1}{X^2} - \frac{15b}{8a^3} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^3X^3} &= \left(-\frac{1}{2ax^2} - \frac{9b}{4a^2} - \frac{3b^2x^2}{2a^3}\right) \frac{1}{X^2} - \frac{3b}{a^3} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^4X^3} &= \left(-\frac{1}{3ax^3} + \frac{7b}{3a^2x} + \frac{175b^2x}{24a^3} + \frac{35b^3x^3}{8a^4}\right) \frac{1}{X^2} + \frac{35b^2}{8a^4} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^5X^3} &= \left(-\frac{1}{4ax^4} + \frac{b}{a^2x^2} + \frac{9b^2}{2a^3} + \frac{3b^3x^2}{a^4}\right) \frac{1}{X^2} + \frac{6b^2}{a^4} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^6X^3} &= \left(-\frac{1}{5ax^4} + \frac{3b}{5a^2x^3} - \frac{21b^2}{5a^3x} - \frac{105b^3x}{8a^4} - \frac{63b^4x^3}{8a^5}\right) \frac{1}{X^2} \\ &\quad - \frac{63b^3}{8a^5} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^7X^3} &= \left(-\frac{1}{6ax^6} + \frac{5b}{12a^2x^4} - \frac{5b^2}{3a^3x^2} - \frac{15b^3}{2a^4} - \frac{5b^4x^2}{a^5}\right) \frac{1}{X^2} \\ &\quad - \frac{10b^3}{a^5} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^8X^3} &= \left(-\frac{1}{7ax^7} + \frac{11b}{35a^2x^5} - \frac{33b^2}{35a^3x^3} + \frac{33b^3}{5a^4x} + \frac{165b^4x}{8a^5}\right. \\ &\quad \left.+ \frac{99b^5x^3}{8a^6}\right) \frac{1}{X^2} + \frac{99b^4}{8a^6} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^9X^3} &= \left(-\frac{1}{8ax^8} + \frac{b}{4a^2x^6} - \frac{5b^2}{8a^3x^4} + \frac{5b^3}{2a^4x^2} + \frac{45b^4}{4a^5}\right. \\ &\quad \left.+ \frac{15b^5x^2}{2a^6}\right) \frac{1}{X^2} + \frac{15b^4}{a^6} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^{10}X^3} &= \left(-\frac{1}{9ax^9} + \frac{13b}{63a^2x^7} - \frac{143b^2}{315a^3x^5} + \frac{143b^3}{105a^4x^3} - \frac{143b^4}{15a^5x}\right. \\ &\quad \left.- \frac{715b^5x}{24a^6} - \frac{143b^6x^3}{8a^7}\right) \frac{1}{X^2} - \frac{143b^5}{8a^7} \int \frac{\partial x}{X} \end{aligned}$$

$$\int \frac{\partial x}{x^m (a + bx^2)^4}$$

Taf. XXIII.

$$\text{VZ. } a + bx^2 = X$$

$$\int \frac{\partial x}{xX^4} = \left( \frac{11}{12a} + \frac{5bx^2}{4a^2} + \frac{b^2x^4}{2a^3} \right) \frac{1}{X^3} + \frac{1}{a^3} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^4} = \left( -\frac{1}{ax} - \frac{77bx}{16a^2} - \frac{35b^2x^3}{6a^3} - \frac{55b^3x^5}{16a^4} \right) \frac{1}{X^3} - \frac{35b^5}{16a^4} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^4} = \left( -\frac{1}{2ax^2} - \frac{11b}{3a^2} - \frac{5b^2x^2}{a^3} - \frac{2b^3x^4}{a^4} \right) \frac{1}{X^3} - \frac{4b}{a^4} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^4} = \left( -\frac{1}{3ax^3} + \frac{3b}{a^2x} + \frac{231b^2x}{16a^3} + \frac{35b^3x^3}{2a^4} + \frac{105b^4x^5}{16a^5} \right) \frac{1}{X^3} + \frac{105b^5}{16a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^4} = \left( -\frac{1}{4ax^4} + \frac{5b}{4a^2x^2} + \frac{55b^2}{6a^3} + \frac{25b^3x^2}{2a^4} + \frac{5b^4x^4}{a^5} \right) \frac{1}{X^3} + \frac{10b^5}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^4} = \left( -\frac{1}{5ax^5} + \frac{11b}{15a^2x^3} - \frac{33b^2}{5a^3x} - \frac{2541b^3x}{80a^4} - \frac{77b^4x^3}{2a^5} - \frac{231b^5x^5}{16a^6} \right) \frac{1}{X^3} - \frac{231b^5}{16a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^4} = \left( -\frac{1}{6ax^6} + \frac{b}{2a^2x^4} - \frac{5b^2}{2a^3x^2} - \frac{55b^3}{3a^4} - \frac{25b^4x^2}{a^5} - \frac{10b^5x^4}{a^6} \right) \frac{1}{X^3} - \frac{20b^5}{a^6} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^4} = \left( -\frac{1}{7ax^7} + \frac{13b}{35a^2x^5} - \frac{143b^2}{105a^3x^3} + \frac{429b^3}{35a^4x} + \frac{4719b^4x}{80a^5} + \frac{143b^5x^3}{2a^6} + \frac{429b^6x^5}{16a^7} \right) \frac{1}{X^3} + \frac{429b^4}{16a^7} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^4} = \left( -\frac{1}{8ax^8} + \frac{7b}{24a^2x^6} - \frac{7b^2}{8a^3x^4} + \frac{35b^3}{8a^4x^2} + \frac{385b^4}{12a^5} + \frac{175b^5x^2}{4a^6} + \frac{35b^6x^4}{2a^7} \right) \frac{1}{X^3} + \frac{35b^5}{a^7} \int \frac{\partial x}{xX}$$

T. j. XXIV.

$$\int \frac{dx}{x^2 + bx^2 + c}$$

$$\text{VL } a + bx^2 = X$$

$$\int \frac{dx}{xX'} = \left( \frac{15}{24a^2} + \frac{15bx^2}{6a^2} + \frac{7b^2x^4}{4a^2} + \frac{b^4x^6}{2a^2} \right) \frac{1}{X^2} + \frac{1}{a^2} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^2X'} = \left( -\frac{1}{ax} - \frac{85bx}{128a^2} - \frac{155b^2x^2}{128a^3} - \frac{1155b^3x^3}{128a^4} \right. \\ \left. - \frac{315b^4x^4}{128a^5} \right) \frac{1}{X^2} - \frac{315b}{128a^5} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^3X'} = \left( -\frac{1}{2ax^2} - \frac{125b}{24a^2} - \frac{65b^2x^2}{6a^3} - \frac{55b^3x^4}{4a^4} - \frac{5b^4x^6}{2a^5} \right) \frac{1}{X^2} \\ - \frac{5b}{a^5} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^4X'} = \left( -\frac{1}{3ax^3} + \frac{11b}{3a^2x} + \frac{56b^2x^2}{128a^3} + \frac{5621b^3x^3}{128a^4} + \frac{4255b^4x^5}{128a^5} \right. \\ \left. + \frac{1155b^5x^7}{128a^6} \right) \frac{1}{X^2} + \frac{1155b^2}{128a^6} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X'} = \left( -\frac{1}{4ax^4} + \frac{3b}{2a^2x^2} + \frac{125b^2}{8a^3} + \frac{65b^3x^2}{2a^4} + \frac{105b^4x^4}{4a^5} \right. \\ \left. + \frac{15b^5x^6}{2a^6} \right) \frac{1}{X^2} + \frac{15b^2}{a^6} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X'} = \left( -\frac{1}{5ax^5} + \frac{15b}{15a^2x^3} - \frac{145b^2}{15a^2x} - \frac{5989b^3x}{640a^4} - \frac{73075b^4x^3}{640a^5} \right. \\ \left. - \frac{11011b^5x^5}{128a^6} - \frac{3005b^6x^7}{128a^7} \right) \frac{1}{X^2} - \frac{3005b^3}{128a^7} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X'} = \left( -\frac{1}{6ax^6} + \frac{7b}{12a^2x^4} - \frac{7b^2}{2a^3x^2} - \frac{85b^3}{24a^4} - \frac{455b^4x^2}{6a^5} \right. \\ \left. - \frac{245b^5x^4}{4a^6} - \frac{55b^6x^6}{2a^7} \right) \frac{1}{X^2} - \frac{55b^3}{a^7} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^8X'} = \left( -\frac{1}{7ax^7} + \frac{3b}{7a^2x^5} - \frac{15b^2}{7a^3x^3} + \frac{145b^3}{7a^4x} + \frac{119691b^4x}{896a^5} \right. \\ \left. + \frac{31517b^5x^3}{128a^6} + \frac{25595b^6x^5}{128a^7} + \frac{6455b^7x^7}{128a^8} \right) \frac{1}{X^2} + \frac{6455b^4}{128a^8} \int \frac{dx}{X}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^6} \quad \text{Taf. XXV.}$$

$$\text{VZ. } a + bx^2 = X$$

$$\int \frac{\partial x}{xX^6} = \left( \frac{137}{120a} + \frac{77bx^2}{24a^2} + \frac{47b^2x^4}{12a^3} + \frac{9b^3x^6}{4a^4} + \frac{b^4x^8}{2a^5} \right) \frac{1}{X^5} + \frac{1}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^6} = \left( -\frac{1}{ax} - \frac{2123bx}{256a^2} - \frac{2607b^2x^2}{128a^3} - \frac{231b^3x^4}{10a^4} - \frac{1617b^4x^6}{128a^5} - \frac{693b^5x^8}{256a^6} \right) \frac{1}{X^5} - \frac{693b}{256a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^6} = \left( -\frac{1}{2ax^2} - \frac{137b}{20a^2} - \frac{77b^2x^2}{4a^3} - \frac{47b^3x^4}{2a^4} - \frac{27b^4x^6}{2a^5} - \frac{3b^5x^8}{a^6} \right) \frac{1}{X^5} - \frac{6b}{a^6} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^6} = \left( -\frac{1}{3ax^3} + \frac{13b}{3a^2x} + \frac{27599b^2x}{768a^3} + \frac{11297b^3x^3}{128a^4} + \frac{1001b^4x^5}{10a^5} + \frac{7007b^5x^7}{128a^6} + \frac{3003b^6x^9}{256a^7} \right) \frac{1}{X^5} + \frac{3003b^2}{256a^7} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^6} = \left( -\frac{1}{4ax^4} + \frac{7b}{4a^2x^2} + \frac{959b^2}{40a^3} + \frac{539b^3x^2}{8a^4} + \frac{329b^4x^4}{4a^5} + \frac{189b^5x^6}{4a^6} + \frac{21b^6x^8}{2a^7} \right) \frac{1}{X^5} + \frac{21b^2}{a^7} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^6} = \left( -\frac{1}{5ax^5} + \frac{b}{a^2x^3} - \frac{13b^2}{a^3x} - \frac{27599b^3x}{256a^4} - \frac{33891b^4x^3}{128a^5} - \frac{3003b^5x^5}{10a^6} - \frac{21021b^6x^7}{128a^7} - \frac{9009b^7x^9}{256a^8} \right) \frac{1}{X^5} - \frac{9009b^3}{256a^8} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^6} = \left( -\frac{1}{6ax^6} + \frac{2b}{3a^2x^4} - \frac{14b^2}{5a^3x^2} - \frac{959b^3}{15a^4} - \frac{539b^4x^2}{3a^5} - \frac{658b^5x^4}{3a^6} - \frac{126b^6x^6}{a^7} - \frac{28b^7x^8}{a^8} \right) \frac{1}{X^5} - \frac{56b^3}{a^8} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^6} = \left( -\frac{1}{7ax^7} + \frac{17b}{55a^2x^5} - \frac{17b^2}{7a^3x^3} + \frac{221b^3}{7a^4x} + \frac{469183b^4x}{1792a^5} + \frac{576147b^5x^3}{896a^6} + \frac{7293b^6x^5}{10a^7} + \frac{51051b^7x^7}{128a^8} + \frac{21879b^8x^9}{256a^9} \right) \frac{1}{X^5} + \frac{21879b^4}{256a^9} \int \frac{\partial x}{X}$$

Taf. XXVI

$$\int \frac{dx}{(a + bx + cx^2)^n}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \frac{dx}{X} = \int \frac{dx}{X} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{X^2} = \frac{2cx + b}{kX} + \frac{2c}{k} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^3} = \left( \frac{1}{2kX^2} + \frac{3c}{k^2X} \right) (2cx + b) + \frac{6c^2}{k^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^4} = \left( \frac{1}{3kX^3} + \frac{5c}{3k^2X^2} + \frac{10c^2}{k^3X} \right) (2cx + b) + \frac{20c^3}{k^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^5} = \left( \frac{1}{4kX^4} + \frac{7c}{6k^2X^3} + \frac{35c^2}{6k^3X^2} + \frac{35c^3}{k^4X} \right) (2cx + b) + \frac{70c^4}{k^4} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^6} = \left( \frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X} \right) (2cx + b) + \frac{252c^5}{k^5} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^7} = \left( \frac{1}{6kX^6} + \frac{11c}{15k^2X^5} + \frac{55c^2}{10k^3X^4} + \frac{77c^3}{5k^4X^3} + \frac{77c^4}{k^5X^2} + \frac{462c^5}{k^6X} \right) \times$$

$$(2cx + b) + \frac{924c^6}{k^6} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^8} = \left( \frac{1}{7kX^7} + \frac{13c}{21k^2X^6} + \frac{286c^2}{105k^3X^5} + \frac{858c^3}{70k^4X^4} + \frac{286c^4}{5k^5X^3} + \frac{286c^5}{k^6X^2} \right.$$

$$\left. + \frac{1716c^6}{k^7X} \right) (2cx + b) + \frac{5452c^7}{k^7} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^9} = \left( \frac{1}{8kX^8} + \frac{15c}{28k^2X^7} + \frac{65c^2}{28k^3X^6} + \frac{145c^3}{14k^4X^5} + \frac{1287c^4}{28k^5X^4} + \frac{429c^5}{2k^6X^3} \right.$$

$$\left. + \frac{2145c^6}{2k^7X^2} + \frac{6435c^7}{k^8X} \right) (2cx + b) + \frac{12870c^8}{k^8} \int \frac{dx}{X}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen, wenn  $X$  seine Bedeutung auf der vorigen Seite behält,

$$\begin{aligned}\int \frac{\partial x}{X} &= \frac{2}{\sqrt{4ac - b^2}} \text{Arc Tang} \frac{2cx + b}{\sqrt{4ac - b^2}} \\ &= \frac{1}{\sqrt{b^2 - 4ac}} \log \frac{2cx + b - \sqrt{b^2 - 4ac}}{2cx + b + \sqrt{b^2 - 4ac}}.\end{aligned}$$

Die erste Form wird reell, wenn  $4ac - b^2$  positiv, die zweite wird es, wenn  $4ac - b^2$  negativ ist. Hieraus ergeben sich zwey Fälle:

I.  $4ac - b^2$  positiv. (VZ.  $4ac - b^2 = k$ .)

$$\begin{aligned}\int \frac{\partial x}{X} &= \frac{2}{\sqrt{k}} \text{Arc Tang} \frac{2cx + b}{\sqrt{k}} = \frac{2}{\sqrt{k}} \text{Arc Cot} \frac{\sqrt{k}}{2cx + b} = \frac{2}{\sqrt{k}} \text{Arc Sec} \frac{2\sqrt{cX}}{\sqrt{k}} \\ &= \frac{2}{\sqrt{k}} \text{Arc Cosec} \frac{2\sqrt{cX}}{2cx + b} = \frac{2}{\sqrt{k}} \text{Arc Cos} \frac{\sqrt{k}}{2\sqrt{cX}} = \frac{2}{\sqrt{k}} \text{Arc Sin} \frac{2cx + b}{2\sqrt{cX}} \\ &= \frac{1}{\sqrt{k}} \text{Arc Sin} \frac{(2cx + b)\sqrt{k}}{2cX} = \frac{1}{\sqrt{k}} \text{Arc Cos} \left( \frac{k}{2cX} - 1 \right) \\ &= \frac{1}{\sqrt{k}} \text{Arc Sin vers} \frac{(2cx + b)^2}{2cX}.\end{aligned}$$

und wenn  $\int \frac{\partial x}{X}$  für  $x = 0$  verschwinden soll,

$$\begin{aligned}\int \frac{\partial x}{X} &= \frac{2}{\sqrt{k}} \text{Arc Tang} \frac{x\sqrt{k}}{2a + bx} = \frac{2}{\sqrt{k}} \text{Arc Cot} \frac{2a + bx}{x\sqrt{k}} = \frac{2}{\sqrt{k}} \text{Arc Sec} \frac{2\sqrt{aX}}{2a + bx} \\ &= \frac{2}{\sqrt{k}} \text{Arc Cosec} \frac{2\sqrt{aX}}{x\sqrt{k}} = \frac{2}{\sqrt{k}} \text{Arc Sin} \frac{x\sqrt{k}}{2\sqrt{aX}} = \frac{2}{\sqrt{k}} \text{Arc Cos} \frac{2a + bx}{2\sqrt{aX}} \\ &= \frac{1}{\sqrt{k}} \text{Arc Sin} \frac{(2ax + bx^2)\sqrt{k}}{2aX} = \frac{1}{\sqrt{k}} \text{Arc Sin vers} \frac{kx^2}{2aX}.\end{aligned}$$

II.  $4ac - b^2$  negativ. (VZ.  $b^2 - 4ac = k'$ .)

$$\int \frac{\partial x}{X} = \frac{1}{\sqrt{k'}} \log \frac{2cx + b - \sqrt{k'}}{2cx + b + \sqrt{k'}} = \frac{2}{\sqrt{k'}} \log \frac{2cx + b - \sqrt{k'}}{2\sqrt{cX}}$$

und wenn das Integral für  $x = 0$  verschwinden soll,

$$\int \frac{\partial x}{X} = \frac{1}{\sqrt{k'}} \log \frac{(b + \sqrt{k'})(2cx + b - \sqrt{k'})}{(b - \sqrt{k'})(2cx + b + \sqrt{k'})}.$$

In beiden Arten von Integralen kann  $\sqrt{k}$  und  $\sqrt{k'}$  sowohl positiv als negativ angenommen werden.

Taf. XX.

$$\int \frac{dx}{x^n(a + bx^2)}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{dx}{xX} = \frac{1}{2a} \log \frac{x^2}{X} = \frac{1}{a} \log \frac{x}{\sqrt{X}} = -\frac{1}{2a} \log \frac{X}{x^2} = -\frac{1}{a} \log \frac{\sqrt{X}}{x}$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{dx}{X} \quad 147$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^4 X} = -\frac{1}{3ax^3} + \frac{b}{a^2 x} + \frac{b^2}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^5 X} = -\frac{1}{4ax^4} + \frac{b}{2a^2 x^2} + \frac{b^2}{a^2} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6 X} = -\frac{1}{5ax^5} + \frac{b}{3a^2 x^3} - \frac{b^2}{a^3 x} - \frac{b^3}{a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7 X} = -\frac{1}{6ax^6} + \frac{b}{4a^2 x^4} - \frac{b^2}{2a^3 x^2} - \frac{b^3}{a^3} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^8 X} = -\frac{1}{7ax^7} + \frac{b}{5a^2 x^5} - \frac{b^2}{3a^3 x^3} + \frac{b^3}{a^4 x} + \frac{b^4}{a^4} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^9 X} = -\frac{1}{8ax^8} + \frac{b}{6a^2 x^6} - \frac{b^2}{4a^3 x^4} + \frac{b^3}{2a^4 x^2} + \frac{b^4}{a^4} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^{10} X} = -\frac{1}{9ax^9} + \frac{b}{7a^2 x^7} - \frac{b^2}{5a^3 x^5} + \frac{b^3}{3a^4 x^3} - \frac{b^4}{a^5 x} - \frac{b^5}{a^5} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^{11} X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2 x^8} - \frac{b^2}{6a^3 x^6} + \frac{b^3}{4a^4 x^4} - \frac{b^4}{2a^5 x^2} - \frac{b^5}{a^5} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^{12} X} = -\frac{1}{11ax^{11}} + \frac{b}{9ax^9} - \frac{b^2}{7a^3 x^7} + \frac{b^3}{5a^4 x^5} - \frac{b^4}{3a^5 x^3} + \frac{b^5}{a^6 x} + \frac{b^6}{a^6} \int \frac{dx}{X}$$

$$\int \frac{\partial x}{x^n(a + bx^2)^2}$$

Taf. XXI.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{xX^2} = \frac{1}{2aX} + \frac{1}{a} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{3bx}{2a^2}\right) \frac{1}{X} - \frac{3b}{2a^2} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{b}{a^2}\right) \frac{1}{X} - \frac{2b}{a^2} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x} + \frac{5b^2x}{2a^3}\right) \frac{1}{X} + \frac{5b^2}{2a^3} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2} + \frac{3b^2}{2a^3}\right) \frac{1}{X} + \frac{3b^2}{a^3} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{7b}{15a^2x^3} - \frac{7b^2}{3a^3x} - \frac{7b^3x}{2a^4}\right) \frac{1}{X} - \frac{7b^3}{2a^4} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{3a^2x^4} - \frac{b^2}{a^3x^2} - \frac{2b^3}{a^4}\right) \frac{1}{X} - \frac{4b^3}{a^4} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5} - \frac{3b^2}{5a^3x^3} + \frac{3b^3}{a^4x} + \frac{9b^4x}{2a^5}\right) \frac{1}{X} + \frac{9b^4}{2a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{5b}{24a^2x^6} - \frac{5b^2}{12a^3x^4} + \frac{5b^3}{4a^4x^2} + \frac{5b^4}{2a^5}\right) \frac{1}{X} + \frac{5b^4}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10}X^2} = \left(-\frac{1}{9ax^9} + \frac{11b}{63a^2x^7} - \frac{11b^2}{35a^3x^5} + \frac{11b^3}{15a^4x^3} - \frac{11b^4}{3a^5x} - \frac{11b^5x}{2a^6}\right) \frac{1}{X} - \frac{11b^5}{2a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^{11}X^2} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{20a^2x^8} - \frac{b^2}{4a^3x^6} + \frac{b^3}{2a^4x^4} - \frac{3b^4}{2a^5x^2} - \frac{3b^5}{a^6}\right) \frac{1}{X} - \frac{6b^5}{a^6} \int \frac{\partial x}{xX}$$



Taf. XX.

$$\int \frac{\partial x}{x^n(a + bx^2)}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{\partial x}{xX} = \frac{1}{2a} \log \frac{x^2}{X} = \frac{1}{a} \log \frac{x}{\sqrt{X}} = -\frac{1}{2a} \log \frac{X}{x^2} = -\frac{1}{a} \log \frac{\sqrt{X}}{x}$$

$$\int \frac{\partial x}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{\partial x}{X} \quad 147$$

$$\int \frac{\partial x}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4 X} = -\frac{1}{3ax^3} + \frac{b}{a^2 x} + \frac{b^2}{a^2} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5 X} = -\frac{1}{4ax^4} + \frac{b}{2a^2 x^2} + \frac{b^2}{a^2} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6 X} = -\frac{1}{5ax^5} + \frac{b}{3a^2 x^3} - \frac{b^2}{a^3 x} - \frac{b^3}{a^3} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7 X} = -\frac{1}{6ax^6} + \frac{b}{4a^2 x^4} - \frac{b^2}{2a^3 x^2} - \frac{b^3}{a^3} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8 X} = -\frac{1}{7ax^7} + \frac{b}{5a^2 x^5} - \frac{b^2}{3a^3 x^3} + \frac{b^3}{a^4 x} + \frac{b^4}{a^4} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9 X} = -\frac{1}{8ax^8} + \frac{b}{6a^2 x^6} - \frac{b^2}{4a^3 x^4} + \frac{b^3}{2a^4 x^2} + \frac{b^4}{a^4} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10} X} = -\frac{1}{9ax^9} + \frac{b}{7a^2 x^7} - \frac{b^2}{5a^3 x^5} + \frac{b^3}{3a^4 x^3} - \frac{b^4}{a^5 x} - \frac{b^5}{a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^{11} X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2 x^8} - \frac{b^2}{6a^3 x^6} + \frac{b^3}{4a^4 x^4} - \frac{b^4}{2a^5 x^2} - \frac{b^5}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{12} X} = -\frac{1}{11ax^{11}} + \frac{b}{9a^2 x^9} - \frac{b^2}{7a^3 x^7} + \frac{b^3}{5a^4 x^5} - \frac{b^4}{3a^5 x^3} + \frac{b^5}{a^6 x} + \frac{b^6}{a^6} \int \frac{\partial x}{X}$$

$$\int \frac{dx}{x^m(a+bx^2)^2}$$

Taf. XXI.

$$\text{VZ. } a + bx^2 = X$$

$$\int \frac{dx}{xX^2} = \frac{1}{2aX} + \frac{1}{a} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{3bx}{2a^2}\right) \frac{1}{X} - \frac{3b}{2a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{b}{a^2}\right) \frac{1}{X} - \frac{2b}{a^2} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x} + \frac{5b^2x}{2a^3}\right) \frac{1}{X} + \frac{5b^2}{2a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2} + \frac{3b^2}{2a^3}\right) \frac{1}{X} + \frac{3b^2}{a^3} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{7b}{15a^2x^3} - \frac{7b^2}{3a^3x} - \frac{7b^3x}{2a^4}\right) \frac{1}{X} - \frac{7b^3}{2a^4} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{3a^2x^4} - \frac{b^2}{a^3x^2} - \frac{2b^3}{a^4}\right) \frac{1}{X} - \frac{4b^3}{a^4} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5} - \frac{3b^2}{5a^3x^3} + \frac{3b^3}{a^4x} + \frac{9b^4x}{2a^5}\right) \frac{1}{X} + \frac{9b^4}{2a^5} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{5b}{24a^2x^6} - \frac{5b^2}{12a^3x^4} + \frac{5b^3}{4a^4x^2} + \frac{5b^4}{2a^5}\right) \frac{1}{X} + \frac{5b^4}{a^5} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^{10}X^2} = \left(-\frac{1}{9ax^9} + \frac{11b}{63a^2x^7} - \frac{11b^2}{35a^3x^5} + \frac{11b^3}{15a^4x^3} - \frac{11b^4}{3a^5x} - \frac{11b^5x}{2a^6}\right) \frac{1}{X} - \frac{11b^5}{2a^6} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^{11}X^2} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{20a^2x^8} - \frac{b^2}{4a^3x^6} + \frac{b^3}{2a^4x^4} - \frac{3b^4}{2a^5x^2} - \frac{3b^5}{a^6}\right) \frac{1}{X} - \frac{6b^5}{a^6} \int \frac{dx}{xX}$$

Taf. XXII.

$$\int \frac{\partial x}{x^m(a + bx^2)^3}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{xX^3} = \left( \frac{3}{4a} + \frac{bx^2}{2a^2} \right) \frac{1}{X^2} + \frac{1}{a^2} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^3} = \left( -\frac{1}{ax} - \frac{25bx}{8a^2} - \frac{15b^2x^3}{8a^3} \right) \frac{1}{X^2} - \frac{15b}{8a^3} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^3} = \left( -\frac{1}{2ax^2} - \frac{9b}{4a^2} - \frac{3b^2x^2}{2a^3} \right) \frac{1}{X^2} - \frac{3b}{a^3} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^3} = \left( -\frac{1}{3ax^3} + \frac{7b}{3a^2x} + \frac{175b^2x}{24a^3} + \frac{35b^3x^3}{8a^4} \right) \frac{1}{X^2} + \frac{35b^2}{8a^4} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^3} = \left( -\frac{1}{4ax^4} + \frac{b}{a^2x^2} + \frac{9b^2}{2a^3} + \frac{3b^3x^2}{a^4} \right) \frac{1}{X^2} + \frac{6b^2}{a^4} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^3} = \left( -\frac{1}{5ax^5} + \frac{3b}{5a^2x^3} - \frac{21b^2}{5a^3x} - \frac{105b^3x}{8a^4} - \frac{63b^4x^3}{8a^5} \right) \frac{1}{X^2} - \frac{63b^3}{8a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^3} = \left( -\frac{1}{6ax^6} + \frac{5b}{12a^2x^4} - \frac{5b^2}{3a^3x^2} - \frac{15b^3}{2a^4} - \frac{5b^4x^2}{a^5} \right) \frac{1}{X^2} - \frac{10b^3}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^3} = \left( -\frac{1}{7ax^7} + \frac{11b}{35a^2x^5} - \frac{33b^2}{35a^3x^3} + \frac{33b^3}{5a^4x} + \frac{165b^4x}{8a^5} + \frac{99b^5x^3}{8a^6} \right) \frac{1}{X^2} + \frac{99b^4}{8a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^3} = \left( -\frac{1}{8ax^8} + \frac{b}{4a^2x^6} - \frac{5b^2}{8a^3x^4} + \frac{5b^3}{2a^4x^2} + \frac{45b^4}{4a^5} + \frac{15b^5x^2}{2a^6} \right) \frac{1}{X^2} + \frac{15b^4}{a^6} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10}X^3} = \left( -\frac{1}{9ax^9} + \frac{13b}{63a^2x^7} - \frac{143b^2}{315a^3x^5} + \frac{143b^3}{105a^4x^3} - \frac{143b^4}{15a^5x} - \frac{715b^5x}{24a^6} - \frac{143b^6x^3}{8a^7} \right) \frac{1}{X^2} - \frac{143b^5}{8a^7} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^n(a+bx^2)^4}$$

Taf. XXIII.

$$\text{VZ. } a + bx^2 = X$$

$$\begin{aligned} \int \frac{\partial x}{xX^4} &= \left( \frac{11}{12a} + \frac{5bx^2}{4a^2} + \frac{b^2x^4}{2a^3} \right) \frac{1}{X^3} + \frac{1}{a^3} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^2X^4} &= \left( -\frac{1}{ax} - \frac{77bx}{16a^2} - \frac{35b^2x^3}{6a^3} - \frac{55b^3x^5}{16a^4} \right) \frac{1}{X^3} - \frac{35b^3}{16a^4} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^3X^4} &= \left( -\frac{1}{2ax^2} - \frac{11b}{3a^2} - \frac{5b^2x^2}{a^3} - \frac{2b^3x^4}{a^4} \right) \frac{1}{X^3} - \frac{4b}{a^4} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^4X^4} &= \left( -\frac{1}{3ax^3} + \frac{3b}{a^2x} + \frac{231b^2x}{16a^3} + \frac{35b^3x^3}{2a^4} + \frac{105b^4x^5}{16a^5} \right) \frac{1}{X^3} \\ &\quad + \frac{105b^2}{16a^5} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^5X^4} &= \left( -\frac{1}{4ax^4} + \frac{5b}{4a^2x^2} + \frac{55b^2}{6a^3} + \frac{25b^3x^2}{2a^4} + \frac{5b^4x^4}{a^5} \right) \frac{1}{X^3} \\ &\quad + \frac{10b^2}{a^5} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^6X^4} &= \left( -\frac{1}{5ax^5} + \frac{11b}{15a^2x^3} - \frac{33b^2}{5a^3x} - \frac{2541b^3x}{80a^4} - \frac{77b^4x^3}{2a^5} \right. \\ &\quad \left. - \frac{231b^5x^5}{16a^6} \right) \frac{1}{X^3} - \frac{231b^3}{16a^6} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^7X^4} &= \left( -\frac{1}{6ax^6} + \frac{b}{2a^2x^4} - \frac{5b^2}{2a^3x^2} - \frac{55b^3}{3a^4} - \frac{25b^4x^2}{a^5} \right. \\ &\quad \left. - \frac{10b^5x^4}{a^6} \right) \frac{1}{X^3} - \frac{20b^2}{a^6} \int \frac{\partial x}{xX} \\ \int \frac{\partial x}{x^8X^4} &= \left( -\frac{1}{7ax^7} + \frac{13b}{35a^2x^5} - \frac{143b^2}{105a^3x^3} + \frac{429b^3}{35a^4x} + \frac{4719b^4x}{80a^5} \right. \\ &\quad \left. + \frac{143b^5x^3}{2a^6} + \frac{429b^6x^5}{16a^7} \right) \frac{1}{X^3} + \frac{429b^4}{16a^7} \int \frac{\partial x}{X} \\ \int \frac{\partial x}{x^9X^4} &= \left( -\frac{1}{8ax^8} + \frac{7b}{24a^2x^6} - \frac{7b^2}{8a^3x^4} + \frac{35b^3}{8a^4x^2} + \frac{385b^4}{12a^5} \right. \\ &\quad \left. + \frac{175b^5x^2}{4a^6} + \frac{35b^6x^4}{2a^7} \right) \frac{1}{X^3} + \frac{35b^3}{a^7} \int \frac{\partial x}{xX} \end{aligned}$$

Taf. XXIV.

$$\int \frac{\partial x}{x^m(a+bx^2)^5}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{xX^5} = \left( \frac{25}{24a} + \frac{13bx^2}{6a^2} + \frac{7b^2x^4}{4a^3} + \frac{b^3x^6}{2a^4} \right) \frac{1}{X^4} + \frac{1}{a^4} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^5} = \left( -\frac{1}{ax} - \frac{837bx}{128a^2} - \frac{1533b^2x^3}{128a^3} - \frac{1155b^3x^5}{128a^4} - \frac{315b^4x^7}{128a^5} \right) \frac{1}{X^4} - \frac{315b}{128a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^5} = \left( -\frac{1}{2ax^2} - \frac{125b}{24a^2} - \frac{65b^2x^2}{6a^3} - \frac{35b^3x^4}{4a^4} - \frac{5b^4x^6}{2a^5} \right) \frac{1}{X^4} - \frac{5b}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^5} = \left( -\frac{1}{3ax^3} + \frac{11b}{3a^2x} + \frac{3069b^2x}{128a^3} + \frac{5621b^3x^3}{128a^4} + \frac{4235b^4x^5}{128a^5} + \frac{1155b^5x^7}{128a^6} \right) \frac{1}{X^4} + \frac{1155b^2}{128a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^5} = \left( -\frac{1}{4ax^4} + \frac{3b}{2a^2x^2} + \frac{125b^2}{8a^3} + \frac{65b^3x^2}{2a^4} + \frac{105b^4x^4}{4a^5} + \frac{15b^5x^6}{2a^6} \right) \frac{1}{X^4} + \frac{15b^2}{a^6} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^5} = \left( -\frac{1}{5ax^5} + \frac{13b}{15a^2x^3} - \frac{143b^2}{15a^3x} - \frac{39897b^3x}{640a^4} - \frac{73073b^4x^3}{640a^5} - \frac{11011b^5x^5}{128a^6} - \frac{3003b^6x^7}{128a^7} \right) \frac{1}{X^4} - \frac{3003b^3}{128a^7} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^5} = \left( -\frac{1}{6ax^6} + \frac{7b}{12a^2x^4} - \frac{7b^2}{2a^3x^2} - \frac{875b^3}{24a^4} - \frac{455b^4x^2}{6a^5} - \frac{245b^5x^4}{4a^6} - \frac{35b^6x^6}{2a^7} \right) \frac{1}{X^4} - \frac{35b^3}{a^7} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^5} = \left( -\frac{1}{7ax^7} + \frac{3b}{7a^2x^5} - \frac{13b^2}{7a^3x^3} + \frac{143b^3}{7a^4x} + \frac{119691b^4x}{896a^5} + \frac{31317b^5x^3}{128a^6} + \frac{23595b^6x^5}{128a^7} + \frac{6435b^7x^7}{128a^8} \right) \frac{1}{X^4} + \frac{6435b^4}{128a^8} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^m(a + bx^2)^6}$$

Taf. XXV.

$$\text{VZ. } a + bx^2 = X$$

$$\int \frac{\partial x}{xX^6} = \left( \frac{137}{120a} + \frac{77bx^2}{24a^2} + \frac{47b^2x^4}{12a^3} + \frac{9b^3x^6}{4a^4} + \frac{b^4x^8}{2a^5} \right) \frac{1}{X^5} + \frac{1}{a^5} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^6} = \left( -\frac{1}{ax} - \frac{2123bx}{256a^2} - \frac{2607b^2x^3}{128a^3} - \frac{231b^3x^5}{10a^4} - \frac{1617b^4x^7}{128a^5} - \frac{693b^5x^9}{256a^6} \right) \frac{1}{X^5} - \frac{693b}{256a^6} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^6} = \left( -\frac{1}{2ax^2} - \frac{137b}{20a^2} - \frac{77b^2x^2}{4a^3} - \frac{47b^3x^4}{2a^4} - \frac{27b^4x^6}{2a^5} - \frac{3b^5x^8}{a^6} \right) \frac{1}{X^5} - \frac{6b}{a^6} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^6} = \left( -\frac{1}{3ax^3} + \frac{13b}{3a^2x} + \frac{27599b^2x}{768a^3} + \frac{11297b^3x^3}{128a^4} + \frac{1001b^4x^5}{10a^5} + \frac{7007b^5x^7}{128a^6} + \frac{3003b^6x^9}{256a^7} \right) \frac{1}{X^5} + \frac{3003b^2}{256a^7} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^6} = \left( -\frac{1}{4ax^4} + \frac{7b}{4a^2x^2} + \frac{959b^2}{40a^3} + \frac{539b^3x^2}{8a^4} + \frac{329b^4x^4}{4a^5} + \frac{189b^5x^6}{4a^6} + \frac{21b^6x^8}{2a^7} \right) \frac{1}{X^5} + \frac{21b^2}{a^7} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^6} = \left( -\frac{1}{5ax^5} + \frac{b}{a^2x^3} - \frac{13b^2}{a^3x} - \frac{27599b^3x}{256a^4} - \frac{33891b^4x^3}{128a^5} - \frac{3003b^5x^5}{10a^6} - \frac{21021b^6x^7}{128a^7} - \frac{9009b^7x^9}{256a^8} \right) \frac{1}{X^5} - \frac{9009b^3}{256a^8} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^6} = \left( -\frac{1}{6ax^6} + \frac{2b}{3a^2x^4} - \frac{14b^2}{5a^3x^2} - \frac{959b^3}{15a^4} - \frac{539b^4x^2}{3a^5} - \frac{658b^5x^4}{3a^6} - \frac{126b^6x^6}{a^7} - \frac{28b^7x^8}{a^8} \right) \frac{1}{X^5} - \frac{56b^3}{a^8} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^6} = \left( -\frac{1}{7ax^7} + \frac{17b}{55a^2x^5} - \frac{17b^2}{7a^3x^3} + \frac{221b^3}{7a^4x} + \frac{469183b^4x}{1792a^5} + \frac{576147b^5x^3}{896a^6} + \frac{7293b^6x^5}{10a^7} + \frac{51051b^7x^7}{128a^8} + \frac{21879b^8x^9}{256a^9} \right) \frac{1}{X^5} + \frac{21879b^4}{256a^9} \int \frac{\partial x}{X}$$

Taf. XXVI.

$$\int \frac{dx}{(a + bx + cx^2)^2}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \frac{dx}{X} = \int \frac{dx}{X} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{X^2} = \frac{2cx + b}{kX} + \frac{2c}{k} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^3} = \left( \frac{1}{2kX^2} + \frac{3c}{k^2X} \right) (2cx + b) + \frac{6c^2}{k^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^4} = \left( \frac{1}{3kX^3} + \frac{5c}{3k^2X^2} + \frac{10c^2}{k^3X} \right) (2cx + b) + \frac{20c^3}{k^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^5} = \left( \frac{1}{4kX^4} + \frac{7c}{6k^2X^3} + \frac{35c^2}{6k^3X^2} + \frac{35c^3}{k^4X} \right) (2cx + b) + \frac{70c^4}{k^4} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^6} = \left( \frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X} \right) (2cx + b) + \frac{252c^5}{k^5} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^7} = \left( \frac{1}{6kX^6} + \frac{11c}{15k^2X^5} + \frac{33c^2}{10k^3X^4} + \frac{77c^3}{5k^4X^3} + \frac{77c^4}{k^5X^2} + \frac{462c^5}{k^6X} \right) \times \\ (2cx + b) + \frac{924c^6}{k^6} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^8} = \left( \frac{1}{7kX^7} + \frac{13c}{21k^2X^6} + \frac{286c^2}{105k^3X^5} + \frac{858c^3}{70k^4X^4} + \frac{286c^4}{5k^5X^3} + \frac{286c^5}{k^6X^2} + \frac{1716c^6}{k^7X} \right) (2cx + b) + \frac{3432c^7}{k^7} \int \frac{dx}{X}$$

$$\int \frac{dx}{X^9} = \left( \frac{1}{8kX^8} + \frac{15c}{28k^2X^7} + \frac{65c^2}{28k^3X^6} + \frac{143c^3}{14k^4X^5} + \frac{1287c^4}{28k^5X^4} + \frac{429c^5}{2k^6X^3} + \frac{2145c^6}{2k^7X^2} + \frac{6435c^7}{k^8X} \right) (2cx + b) + \frac{12870c^8}{k^8} \int \frac{dx}{X}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen, wenn  $X$  seine Bedeutung auf der vorigen Seite behält,

$$\begin{aligned}\int \frac{dx}{X} &= \frac{2}{\sqrt{(4ac - b^2)}} \operatorname{Arc Tang} \frac{2cx + b}{\sqrt{(4ac - b^2)}} \\ &= \frac{1}{\sqrt{(b^2 - 4ac)}} \log \frac{2cx + b - \sqrt{(b^2 - 4ac)}}{2cx + b + \sqrt{(b^2 - 4ac)}}.\end{aligned}$$

Die erste Form wird reell, wenn  $4ac - b^2$  positiv, die zweite wird es, wenn  $4ac - b^2$  negativ ist. Hieraus ergeben sich zwey Fälle:

I.  $4ac - b^2$  positiv. (VZ.  $4ac - b^2 = k$ .)

$$\begin{aligned}\int \frac{dx}{X} &= \frac{2}{\sqrt{k}} \operatorname{Arc Tang} \frac{2cx + b}{\sqrt{k}} = \frac{2}{\sqrt{k}} \operatorname{Arc Cot} \frac{\sqrt{k}}{2cx + b} = \frac{2}{\sqrt{k}} \operatorname{Arc Sec} \frac{2\sqrt{cX}}{\sqrt{k}} \\ &= \frac{2}{\sqrt{k}} \operatorname{Arc Cosc} \frac{2\sqrt{cX}}{2cx + b} = \frac{2}{\sqrt{k}} \operatorname{Arc Cos} \frac{\sqrt{k}}{2\sqrt{cX}} = \frac{2}{\sqrt{k}} \operatorname{Arc Sin} \frac{2cx + b}{2\sqrt{cX}} \\ &= \frac{1}{\sqrt{k}} \operatorname{Arc Sin} \frac{(2cx + b)\sqrt{k}}{2cX} = \frac{1}{\sqrt{k}} \operatorname{Arc Cos} \left( \frac{k}{2cX} - 1 \right) \\ &= \frac{1}{\sqrt{k}} \operatorname{Arc Sin} \operatorname{vers} \frac{(2cx + b)^2}{2cX}.\end{aligned}$$

und wenn  $\int \frac{dx}{X}$  für  $x = 0$  verschwinden soll,

$$\begin{aligned}\int \frac{dx}{X} &= \frac{2}{\sqrt{k}} \operatorname{Arc Tang} \frac{x\sqrt{k}}{2a + bx} = \frac{2}{\sqrt{k}} \operatorname{Arc Cot} \frac{2a + bx}{x\sqrt{k}} = \frac{2}{\sqrt{k}} \operatorname{Arc Sec} \frac{2\sqrt{aX}}{2a + bx} \\ &= \frac{2}{\sqrt{k}} \operatorname{Arc Cosec} \frac{2\sqrt{aX}}{x\sqrt{k}} = \frac{2}{\sqrt{k}} \operatorname{Arc Sin} \frac{x\sqrt{k}}{2\sqrt{aX}} = \frac{2}{\sqrt{k}} \operatorname{Arc Cos} \frac{2a + bx}{2\sqrt{aX}} \\ &= \frac{1}{\sqrt{k}} \operatorname{Arc Sin} \frac{(2ax + bx^2)\sqrt{k}}{2aX} = \frac{1}{\sqrt{k}} \operatorname{Arc Sin} \operatorname{vers} \frac{kx^2}{2aX}.\end{aligned}$$

II.  $4ac - b^2$  negativ. (VZ.  $b^2 - 4ac = k'$ .)

$$\int \frac{dx}{X} = \frac{1}{\sqrt{k'}} \log \frac{2cx + b - \sqrt{k'}}{2cx + b + \sqrt{k'}} = \frac{2}{\sqrt{k'}} \log \frac{2cx + b - \sqrt{k'}}{2\sqrt{cX}}$$

und wenn das Integral für  $x = 0$  verschwinden soll,

$$\int \frac{dx}{X} = \frac{1}{\sqrt{k'}} \log \frac{(b + \sqrt{k'})(2cx + b - \sqrt{k'})}{(b - \sqrt{k'})(2cx + b + \sqrt{k'})}.$$

In beiden Arten von Integralen kann  $\sqrt{k}$  und  $\sqrt{k'}$  sowohl positiv als negativ angenommen werden.



Taf. XXVII.

$$\int \frac{x^m dx}{a + bx + cx^2}$$

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$$\text{VZ. } a + bx + cx^2 = X$$


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$$\int \frac{\partial x}{X} = \int \frac{\partial x}{X} \quad [\text{Man s. die vorhergehende Seite.}]$$

$$\int \frac{x \partial x}{X} = \frac{1}{2c} \log X - \frac{b}{2c} \int \frac{\partial x}{X}$$

$$\int \frac{x^2 \partial x}{X} = \frac{x}{c} - \frac{b}{2c^2} \log X + \left( \frac{b^2}{2c^2} - \frac{a}{c} \right) \int \frac{\partial x}{X}$$

$$\int \frac{x^3 \partial x}{X} = \frac{x^2}{2c} - \frac{bx}{c^2} + \left( \frac{b^2}{2c^2} - \frac{a}{2c^2} \right) \log X - \left( \frac{b^3}{2c^3} - \frac{3ab}{2c^2} \right) \int \frac{\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X} = \frac{x^3}{3c} - \frac{bx^2}{2c^2} + \left( \frac{b^2}{c^2} - \frac{a}{c^2} \right) x - \left( \frac{b^3}{2c^4} - \frac{ab}{c^3} \right) \log X + \left( \frac{b^4}{2c^4} - \frac{2ab^2}{c^3} + \frac{a^2}{c^2} \right) \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^4}{4c} - \frac{b}{c} \int \frac{x^4 \partial x}{X} - \frac{a}{c} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^5}{5c} - \frac{bx^4}{4c^2} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^4 \partial x}{X} + \frac{ab}{c^2} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^6}{6c} - \frac{bx^5}{5c^2} + \left( \frac{b^2}{4c^3} - \frac{a}{4c^2} \right) x^4 - \left( \frac{b^2}{c^3} - \frac{2ab}{c^2} \right) \int \frac{x^4 \partial x}{X} - \left( \frac{ab^2}{c^3} - \frac{a^2}{c^2} \right) \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^7}{7c} - \frac{b}{c} \int \frac{x^7 \partial x}{X} - \frac{a}{c} \int \frac{x^6 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^8}{8c} - \frac{bx^7}{7c^2} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^7 \partial x}{X} + \frac{ab}{c^2} \int \frac{x^6 \partial x}{X}$$

$$\int \frac{x^m dx}{(a + bx + cx^2)^2} \quad \text{Taf. XXVIII.}$$

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$$\text{VL. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \frac{dx}{X^2} = \frac{2cx + b}{kX} + \frac{2c}{k} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^2} = -\frac{1}{2cX} - \frac{b}{2c} \int \frac{dx}{X^2}$$

$$\int \frac{x^2 dx}{X^2} = -\frac{x}{cX} + \frac{a}{c} \int \frac{dx}{X^2}$$

$$\int \frac{x^3 dx}{X^2} = \left(\frac{bx}{c^2} + \frac{a}{2c^2}\right) \frac{1}{X} + \frac{1}{2c^2} \log X - \frac{ab}{2c^2} \int \frac{dx}{X^2} - \frac{b}{2c^2} \int \frac{dx}{X}$$

$$\int \frac{x^4 dx}{X^2} = \frac{x^3}{cX} - \frac{2b}{c} \int \frac{x^3 dx}{X^2} - \frac{3a}{c} \int \frac{x^2 dx}{X^2}$$

$$\int \frac{x^5 dx}{X^2} = \left(\frac{x^4}{2c} - \frac{3bx^3}{2c^2}\right) \frac{1}{X} + \left(\frac{3b^2}{c^2} - \frac{2a}{c}\right) \int \frac{x^3 dx}{X^2} + \frac{9ab}{2c^2} \int \frac{x^2 dx}{X^2}$$

$$\int \frac{x^6 dx}{X^2} = \left[\frac{x^5}{3c} - \frac{2bx^4}{3c^2} + \left(\frac{2b^2}{c^2} - \frac{5a}{3c^2}\right) x^3\right] \frac{1}{X} - \left(\frac{4b^3}{c^3} - \frac{6ab}{c^2}\right) \times \\ \int \frac{x^3 dx}{X^2} - \left(\frac{6ab^2}{c^3} - \frac{5a^2}{c^2}\right) \int \frac{x^2 dx}{X^2}$$

$$\int \frac{x^7 dx}{X^2} = \left[\frac{x^6}{4c} - \frac{5bx^5}{12c^2} + \left(\frac{5b^2}{6c^2} - \frac{3a}{4c^2}\right) x^4 - \left(\frac{5b^3}{2c^4} - \frac{13ab}{3c^3}\right) x^3\right] \frac{1}{X} \\ + \left(\frac{5b^4}{c^4} - \frac{12ab^2}{c^3} + \frac{3a^2}{c^2}\right) \int \frac{x^3 dx}{X^2} + \left(\frac{15ab^3}{2c^4} - \frac{13a^2b}{c^3}\right) \int \frac{x^2 dx}{X^2}$$

$$\int \frac{x^8 dx}{X^2} = \frac{x^7}{5cX} - \frac{6b}{5c} \int \frac{x^7 dx}{X^2} - \frac{7a}{5c} \int \frac{x^6 dx}{X^2}$$

Taf. XXIX.

$$\int \frac{x^n dx}{(a + bx + cx^2)^3}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \frac{dx}{X^3} = \left( \frac{1}{2kX^2} + \frac{3c}{k^2 X} \right) (2cx + b) + \frac{6c^2}{k^2} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^3} = -\frac{1}{4cX^2} - \frac{b}{2c} \int \frac{dx}{X^3}$$

$$\int \frac{x^2 dx}{X^3} = \left( -\frac{x}{3c} + \frac{b}{12c^2} \right) \frac{1}{X^2} + \left( \frac{b^2}{6c^2} + \frac{a}{3c} \right) \int \frac{dx}{X^3}$$

$$\int \frac{x^3 dx}{X^3} = \left( -\frac{x^2}{2c} - \frac{a}{4c^2} \right) \frac{1}{X^2} - \frac{ab}{2c^2} \int \frac{dx}{X^3}$$

$$\int \frac{x^4 dx}{X^3} = \left( -\frac{x^3}{c} - \frac{bx^2}{2c^2} - \frac{ax}{c^2} \right) \frac{1}{X^2} + \frac{a^2}{c^2} \int \frac{dx}{X^3}$$

$$\int \frac{x^5 dx}{X^3} = \frac{1}{c} \int \frac{x^3 dx}{X^2} - \frac{a}{c} \int \frac{x^3 dx}{X^3} - \frac{b}{c} \int \frac{x^4 dx}{X^2}$$

$$\int \frac{x^6 dx}{X^3} = \frac{x^5}{cX^2} - \frac{3b}{c} \int \frac{x^5 dx}{X^3} - \frac{5a}{c} \int \frac{x^4 dx}{X^2}$$

$$\int \frac{x^7 dx}{X^3} = \left( \frac{x^6}{2c} - \frac{2bx^5}{c^2} \right) \frac{1}{X^2} + \left( \frac{6b^2}{c^2} - \frac{3a}{c} \right) \int \frac{x^5 dx}{X^3} + \frac{10ab}{c^2} \int \frac{x^4 dx}{X^2}$$

$$\int \frac{x^8 dx}{X^3} = \left[ \frac{x^7}{3c} - \frac{5bx^6}{6c^2} + \left( \frac{10b^2}{3c^3} - \frac{7a}{3c^2} \right) x^5 \right] \frac{1}{X^2} \\ - \left( \frac{10b^3}{c^3} - \frac{12ab}{c^2} \right) \int \frac{x^5 dx}{X^3} - \left( \frac{50ab^2}{3c^3} - \frac{35a^2}{3c^2} \right) \int \frac{x^4 dx}{X^2}$$

$$\int \frac{x^9 dx}{X^3} = \frac{x^8}{4cX^2} - \frac{3b}{2c} \int \frac{x^8 dx}{X^2} - \frac{2a}{c} \int \frac{x^7 dx}{X^2}$$

$$\int \frac{x^m dx}{(a+bx+cx^2)^4}$$

Taf. XXX.

$$\text{VL. } a+bx+cx^2=X, 4ac-b^2=k$$

$$\int \frac{dx}{X^4} = \left( \frac{1}{3kX^3} + \frac{5c}{3k^2X^2} + \frac{10c^2}{k^3X} \right) (2cx+b) + \frac{20c^3}{k^3} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^4} = -\frac{1}{6cX^3} - \frac{b}{2c} \int \frac{dx}{X^4}$$

$$\int \frac{x^2 dx}{X^4} = \left( -\frac{x}{5c} + \frac{b}{15c^2} \right) \frac{1}{X^3} + \left( \frac{b^2}{5c^2} + \frac{1}{5c} \right) \int \frac{dx}{X^4}$$

$$\int \frac{x^3 dx}{X^4} = \left[ -\frac{x^2}{4c} + \frac{bx}{20c^2} - \left( \frac{b^2}{60c^3} + \frac{a}{12c^2} \right) \right] \frac{1}{X^3} - \left( \frac{b^3}{20c^3} + \frac{3ab}{10c^2} \right) \int \frac{dx}{X^4}$$

$$\int \frac{x^4 dx}{X^4} = \left( -\frac{x^3}{3c} - \frac{ax}{5c^2} + \frac{ab}{15c^3} \right) \frac{1}{X^3} + \left( \frac{ab^2}{5c^3} + \frac{a^2}{5c^2} \right) \int \frac{dx}{X^4}$$

$$\int \frac{x^5 dx}{X^4} = \left( -\frac{x^4}{2c} - \frac{bx^3}{6c^2} - \frac{ax^2}{2c^2} - \frac{a^2}{6c^3} \right) \frac{1}{X^3} - \frac{a^2b}{2c^3} \int \frac{dx}{X^4}$$

$$\int \frac{x^6 dx}{X^4} = \left[ -\frac{x^5}{c} - \frac{bx^4}{c^2} - \left( \frac{b^2}{3c^3} + \frac{5a}{3c^2} \right) x^3 - \frac{abx^2}{c^3} - \frac{a^2x}{c^3} \right] \frac{1}{X^3} + \frac{a^3}{c^3} \int \frac{dx}{X^4}$$

$$\int \frac{x^7 dx}{X^4} = \frac{1}{c} \int \frac{x^5 dx}{X^3} - \frac{a}{c} \int \frac{x^5 dx}{X^4} - \frac{b}{c} \int \frac{x^6 dx}{X^4}$$

$$\int \frac{x^8 dx}{X^4} = \frac{x^7}{cX^3} - \frac{4b}{c} \int \frac{x^7 dx}{X^4} - \frac{7a}{c} \int \frac{x^6 dx}{X^4}$$

$$\int \frac{x^9 dx}{X^4} = \left( \frac{x^8}{2c} - \frac{5bx^7}{2c^2} \right) \frac{1}{X^3} + \left( \frac{10b^2}{c^2} - \frac{4a}{c} \right) \int \frac{x^7 dx}{X^4} + \frac{55ab}{2c^2} \int \frac{x^6 dx}{X^4}$$

Taf. XXXI.

$$\int \frac{x^m dx}{(a + bx + cx^2)^5}$$

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$$\text{VZ. } a + bx + cx^2 = X, 4ac - b^2 = h$$


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$$\int \frac{dx}{X^5} = \left( \frac{1}{4kX^4} + \frac{7c}{6k^2X^3} + \frac{35c^2}{4k^3X^2} + \frac{35c^3}{k^4X} \right) (2cx + b) + \frac{70c^4}{k^4} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^5} = -\frac{1}{8cX^4} - \frac{b}{2c} \int \frac{dx}{X^5}$$

$$\int \frac{x^2 dx}{X^5} = \left( -\frac{x}{7c} + \frac{3b}{56c^2} \right) \frac{1}{X^4} + \left( \frac{3b^2}{14c^2} + \frac{a}{7c} \right) \int \frac{dx}{X^5}$$

$$\int \frac{x^3 dx}{X^5} = \left( -\frac{x^2}{6c} + \frac{bx}{21c^2} - \frac{b^2}{56c^3} - \frac{a}{24c^2} \right) \frac{1}{X^4} - \left( \frac{b^3}{14c^3} + \frac{3ab}{14c^2} \right) \int \frac{dx}{X^5}$$

$$\int \frac{x^4 dx}{X^5} = \left[ -\frac{x^3}{5c} + \frac{bx^2}{30c^2} - \left( \frac{b^2}{105c^3} + \frac{3a}{35c^2} \right) x + \frac{b^3}{280c^4} + \frac{17ab}{40c^3} \right] \frac{1}{X^4} + \left( \frac{b^4}{70c^4} + \frac{6ab^2}{35c^3} + \frac{3a^2}{35c^2} \right) \int \frac{dx}{X^5}$$

$$\int \frac{x^5 dx}{X^5} = \left( -\frac{x^4}{4c} - \frac{ax^2}{6c^2} + \frac{abx}{21c^3} - \frac{ab^2}{56c^4} - \frac{a^2}{24c^3} \right) \frac{1}{X^4} - \left( \frac{ab^3}{14c^4} + \frac{3a^2b}{14c^3} \right) \int \frac{dx}{X^5}$$

$$\int \frac{x^6 dx}{X^5} = -\frac{x^5}{3cX^4} + \frac{b}{3c} \int \frac{x^5 dx}{X^5} + \frac{5a}{3c} \int \frac{x^4 dx}{X^5}$$

$$\int \frac{x^7 dx}{X^5} = \left( -\frac{x^6}{2c} - \frac{bx^5}{3c^2} \right) \frac{1}{X^4} + \left( \frac{b^2}{3c^2} + \frac{3a}{c} \right) \int \frac{x^5 dx}{X^5} + \frac{5ab}{3c^2} \int \frac{x^4 dx}{X^5}$$

$$\int \frac{x^8 dx}{X^5} = \left[ -\frac{x^7}{c} - \frac{3bx^6}{2c^2} - \left( \frac{b^2}{c^3} + \frac{7a}{3c^2} \right) x^5 \right] \frac{1}{X^4} + \left( \frac{b^3}{c^3} + \frac{17ab}{3c^2} \right) \times \int \frac{x^5 dx}{X^5} + \left( \frac{5ab^2}{c^3} + \frac{35a^2}{3c^2} \right) \int \frac{x^4 dx}{X^5}$$

$$\int \frac{x^m dx}{(a + bx + cx^2)^6}$$

Taf. XXXII.

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \frac{dx}{X^6} = \left( \frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X} \right) (2cx + b) + \frac{252c^5}{k^5} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^6} = -\frac{1}{10cX^5} - \frac{b}{2c} \int \frac{dx}{X^6}$$

$$\int \frac{x^2 dx}{X^6} = \left( -\frac{x}{9c} + \frac{2b}{45c^2} \right) \frac{1}{X^5} + \left( \frac{2b^2}{9c^2} + \frac{a}{9c} \right) \int \frac{dx}{X^6}$$

$$\int \frac{x^3 dx}{X^6} = \left( -\frac{x^2}{8c} + \frac{bx}{24c^2} - \frac{b^2}{60c^3} - \frac{a}{40c^2} \right) \frac{1}{X^5} - \left( \frac{b^3}{12c^3} + \frac{ab}{6c^2} \right) \int \frac{dx}{X^6}$$

$$\int \frac{x^4 dx}{X^6} = \left[ -\frac{x^3}{7c} + \frac{bx^2}{28c^2} - \left( \frac{b^2}{84c^3} + \frac{a}{21c^2} \right) x + \frac{b^3}{210c^4} + \frac{11ab}{420c^3} \right] \frac{1}{X^5} + \left( \frac{b^4}{42c^4} + \frac{ab^2}{7c^3} + \frac{a^2}{21c^2} \right) \int \frac{dx}{X^6}$$

$$\int \frac{x^5 dx}{X^6} = -\frac{x^4}{6cX^5} - \frac{b}{6c} \int \frac{x^4 dx}{X^6} + \frac{2a}{3c} \int \frac{x^3 dx}{X^6}$$

$$\int \frac{x^6 dx}{X^6} = -\frac{x^5}{5cX^5} + \frac{a}{c} \int \frac{x^4 dx}{X^6}$$

$$\int \frac{x^7 dx}{X^6} = \left( -\frac{x^6}{4c} - \frac{bx^5}{20c^2} - \frac{ax^4}{4c^2} \right) \frac{1}{X^5} + \frac{a^2}{c^2} \int \frac{x^3 dx}{X^6}$$

$$\int \frac{x^8 dx}{X^6} = \left[ -\frac{x^7}{3c} - \frac{bx^6}{6c^2} - \left( \frac{b^2}{30c^3} + \frac{7a}{15c^2} \right) x^5 - \frac{abx^4}{6c^2} \right] \frac{1}{X^5} + \frac{7a^2}{3c^2} \int \frac{x^4 dx}{X^6} + \frac{2a^2b}{3c^3} \int \frac{x^3 dx}{X^6}$$

Taf. XXXIII.

$$\int \frac{\partial x}{x^m(a+bx+cx^2)}$$

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$$\text{VL } a+bx+cx^2 = X$$


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$$\int \frac{\partial x}{xX} = \frac{1}{2a} \log \frac{x^2}{X} - \frac{b}{2a} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} - \frac{b}{2a^2} \log \frac{x^2}{X} - \left(\frac{b^2}{2a^2} - \frac{c}{a}\right) \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} + \frac{b}{a^2x} - \left(\frac{b^2}{2a^2} - \frac{c}{2a^2}\right) \log \frac{x^2}{X} - \left(\frac{b^3}{2a^3} - \frac{5b^2c}{2a^2}\right) \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \left(\frac{b^2}{a^3} - \frac{c}{a^2}\right) \frac{1}{x} - \left(\frac{b^3}{2a^4} - \frac{bc}{a^3}\right) \log \frac{x^2}{X} + \left(\frac{b^4}{2a^4} - \frac{2b^2c}{a^3} + \frac{c^2}{a^2}\right) \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} - \frac{b}{a} \int \frac{\partial x}{x^4X} - \frac{c}{a} \int \frac{\partial x}{x^3X}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{4a^2x^4} + \left(\frac{b^2}{a^3} - \frac{c}{a}\right) \int \frac{\partial x}{x^4X} + \frac{bc}{a^2} \int \frac{\partial x}{x^3X}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{5a^2x^5} - \left(\frac{b^2}{4a^3} - \frac{c}{4a^2}\right) \frac{1}{x^4} - \left(\frac{b^3}{a^4} - \frac{2bc}{a^3}\right) \int \frac{\partial x}{x^4X} - \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right) \int \frac{\partial x}{x^3X}$$

$$\int \frac{\partial x}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{6a^2x^6} - \left(\frac{b^2}{5a^3} - \frac{c}{5a^2}\right) \frac{1}{x^5} + \left(\frac{b^3}{4a^4} - \frac{bc}{2a^3}\right) \frac{1}{x^4} + \left(\frac{b^4}{a^4} - \frac{3b^2c}{a^3} + \frac{c^2}{a^2}\right) \int \frac{\partial x}{x^4X} + \left(\frac{b^3c}{a^4} - \frac{2bc^2}{a^3}\right) \int \frac{\partial x}{x^3X}$$

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\*) Das Integral  $\int \frac{\partial x}{xX}$  kann für  $x=0$  nicht verschwinden, weil alsdann  $\log \frac{x^2}{X} = \log 0 = -\infty$  wird. Uebrigens ist  $\log \frac{x^2}{X} = -\log \frac{X}{x^2}$ .

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^2}$$

Taf. XXXIV.

$$\text{VZ. } a+bx+cx^2=X$$

$$\int \frac{\partial x}{xX^2} = \frac{1}{2aX} + \frac{1}{2a^2} \log \frac{x^2}{X} - \frac{b}{2a} \int \frac{\partial x}{X^2} - \frac{b}{2a^2} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{b}{a^2}\right) \frac{1}{X} - \frac{b}{a^3} \log \frac{x^2}{X} + \left(\frac{b^2}{a^2} - \frac{3c}{a}\right) \int \frac{\partial x}{X^2} + \frac{b^2}{a^3} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{2a^3} - \frac{c}{a^2}\right) \frac{1}{X} + \left(\frac{3b^3}{2a^4} - \frac{c}{a^3}\right) \log \frac{x^2}{X} - \left(\frac{3b^3}{2a^3} - \frac{11bc}{2a^2}\right) \int \frac{\partial x}{X^2} - \left(\frac{3b^3}{2a^4} - \frac{bc}{a^3}\right) \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^4X^2} = \left[-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \left(\frac{2b^2}{a^3} - \frac{5c}{3a^2}\right) \frac{1}{x} - \frac{2b^3}{a^4} + \frac{3bc}{a^3}\right] \frac{1}{X} - \left(\frac{2b^3}{a^5} - \frac{3bc}{a^4}\right) \log \frac{x^2}{X} + \left(\frac{2b^4}{a^4} - \frac{9b^2c}{a^3} + \frac{5c^2}{a^2}\right) \int \frac{\partial x}{X^2} + \left(\frac{2b^4}{a^5} - \frac{3b^2c}{a^4}\right) \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^2} = -\frac{1}{4ax^4X} - \frac{5b}{4a} \int \frac{\partial x}{x^4X^2} - \frac{3c}{2a} \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4}\right) \frac{1}{X} + \left(\frac{3b^2}{2a^2} - \frac{7c}{5a}\right) \int \frac{\partial x}{x^4X^2} + \frac{9bc}{5a^3} \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{\partial x}{x^7X^2} = \left[-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \left(\frac{7b^2}{20a^3} - \frac{c}{3a^2}\right) \frac{1}{x^4}\right] \frac{1}{X} - \left(\frac{7b^3}{4a^3} - \frac{33bc}{10a^2}\right) \int \frac{\partial x}{x^4X^2} - \left(\frac{21b^2c}{10a^3} - \frac{2c^2}{a^2}\right) \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{\partial x}{x^8X^2} = -\frac{1}{7ax^7X} - \frac{8b}{7a} \int \frac{\partial x}{x^7X^2} - \frac{9c}{7a} \int \frac{\partial x}{x^6X^2}$$



Taf. XXXV.

$$\int \frac{dx}{x^m(a + bx + cx^2)^3}$$

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$$\text{VZ. } a + bx + cx^2 = X$$


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$$\int \frac{dx}{xX^3} = \frac{1}{4aX^2} + \frac{1}{2a^2X} + \frac{1}{2a^3} \log \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X^3} - \frac{b}{2a^2} \int \frac{dx}{X^2} - \frac{b}{2a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^2X^3} = -\frac{1}{axX^2} - \frac{3b}{a} \int \frac{dx}{xX^3} - \frac{5c}{a} \int \frac{dx}{X^3}$$

$$\int \frac{dx}{x^3X^3} = \left(-\frac{1}{2ax^2} + \frac{2b}{a^2x}\right) \frac{1}{X^2} + \left(\frac{6b^2}{a^2} - \frac{3c}{a}\right) \int \frac{dx}{xX^3} + \frac{10bc}{a^2} \int \frac{dx}{X^3}$$

$$\int \frac{dx}{x^4X^3} = \left[-\frac{1}{3ax^3} + \frac{5b}{6a^2x^2} - \left(\frac{10b^2}{3a^3} - \frac{7c}{3a^2}\right) \frac{1}{x}\right] \frac{1}{X^2} - \left(\frac{10b^3}{a^3} - \frac{12bc}{a^2}\right) \int \frac{dx}{xX^3} - \left(\frac{50b^2c}{3a^3} - \frac{35c^2}{3a^2}\right) \int \frac{dx}{X^3}$$

$$\int \frac{dx}{x^5X^3} = -\frac{1}{4ax^4X^2} - \frac{3b}{2a} \int \frac{dx}{x^4X^3} - \frac{2c}{a} \int \frac{dx}{x^3X^3}$$

$$\int \frac{dx}{x^6X^3} = \left(-\frac{1}{5ax^5} + \frac{7b}{20a^2x^4}\right) \frac{1}{X^2} + \left(\frac{21b^2}{10a^2} - \frac{9c}{5a}\right) \int \frac{dx}{x^4X^3} + \frac{14bc}{5a^2} \int \frac{dx}{x^3X^3}$$

$$\int \frac{dx}{x^7X^3} = \left[-\frac{1}{6ax^6} + \frac{4b}{15a^2x^5} - \left(\frac{7b^2}{15a^3} - \frac{5c}{12a^2}\right)\right] \frac{1}{X^2} - \left(\frac{14b^3}{5a^3} - \frac{49bc}{10a^2}\right) \int \frac{dx}{x^4X^3} - \left(\frac{56b^2c}{15a^3} - \frac{10c^2}{3a^2}\right) \int \frac{dx}{x^3X^3}$$

$$\int \frac{dx}{x^8X^3} = -\frac{1}{7ax^7X^2} - \frac{9b}{7a} \int \frac{dx}{x^7X^3} - \frac{11c}{7a} \int \frac{dx}{x^6X^3}$$

$$\int \frac{dx}{x^n(a+bx+cx^2)^4}$$

Taf. XXXVI.

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$$\text{VZ. } a + bx + cx^2 = X$$


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$$\int \frac{dx}{xX^4} = \frac{1}{6aX^3} + \frac{1}{4a^2X^2} + \frac{1}{2a^3X} + \frac{1}{2a^4} \log \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X^4}$$

$$- \frac{b}{2a^2} \int \frac{dx}{X^3} - \frac{b}{2a^3} \int \frac{dx}{X^2} - \frac{b}{2a^4} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^2X^4} = -\frac{1}{axX^3} - \frac{4b}{a} \int \frac{dx}{xX^4} - \frac{7c}{a} \int \frac{dx}{X^4}$$

$$\int \frac{dx}{x^3X^4} = \left(-\frac{1}{2ax^2} + \frac{5b}{2a^2x}\right) \frac{1}{X^3} + \left(\frac{10b^2}{a^2} - \frac{4c}{a}\right) \int \frac{dx}{xX^4}$$

$$+ \frac{35bc}{2a^2} \int \frac{dx}{X^4}$$

$$\int \frac{dx}{x^4X^4} = \left[-\frac{1}{3ax^3} + \frac{b}{a^2x^2} - \left(\frac{5b^2}{a^3} - \frac{3c}{a^2}\right) \frac{1}{x}\right] \frac{1}{X^3}$$

$$- \left(\frac{20b^3}{a^3} - \frac{20bc}{a^2}\right) \int \frac{dx}{xX^4} - \left(\frac{35b^2c}{a^3} - \frac{21c^2}{a^2}\right) \int \frac{dx}{X^4}$$

$$\int \frac{dx}{x^5X^4} = -\frac{1}{4ax^4X^3} - \frac{7b}{4a} \int \frac{dx}{x^4X^4} - \frac{5c}{2a} \int \frac{dx}{x^3X^4}$$

$$\int \frac{dx}{x^6X^4} = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^4}\right) \frac{1}{X^3} + \left(\frac{14b^2}{5a^2} - \frac{11c}{5a}\right) \int \frac{dx}{x^4X^4}$$

$$+ \frac{4bc}{a^2} \int \frac{dx}{x^3X^4}$$

$$\int \frac{dx}{x^7X^4} = \left[-\frac{1}{6ax^6} + \frac{3b}{10a^2x^5} - \left(\frac{3b^2}{5a^3} - \frac{c}{2a^2}\right) \frac{1}{x^4}\right] \frac{1}{X^3}$$

$$- \left(\frac{21b^3}{5a^3} - \frac{34bc}{5a^2}\right) \int \frac{dx}{x^4X^4} - \left(\frac{6b^2c}{a^3} - \frac{5c^2}{a^2}\right) \int \frac{dx}{x^3X^4}$$

$$\int \frac{dx}{x^8X^4} = -\frac{1}{7ax^7X^3} - \frac{10b}{7a} \int \frac{dx}{x^7X^4} - \frac{13c}{7a} \int \frac{dx}{x^6X^4}$$

For example

$$\int \frac{x^2}{x^2 - 12x + 35} dx$$

$$x^2 - 12x + 35 = 0$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx + \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$= \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx + \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = \left[ -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx \right] \frac{1}{X^2}$$

$$= -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = \left( -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx + \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx \right) \frac{1}{X^4}$$

$$+ \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = \left[ -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx + \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx \right] \frac{1}{X^4}$$

$$- \left( \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx \right) \frac{1}{X^4}$$

$$\int \frac{x^2}{x^2 - 12x + 35} dx = -\frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx - \frac{1}{2} \int \frac{x^2}{x^2 - 12x + 35} dx$$

$$\int \frac{dx}{x^m(a+bx+cx^2)^6} \quad \text{Taf. XXXVIII.}$$

$$\text{VZ. } a+bx+cx^2=X$$

$$\begin{aligned} \int \frac{dx}{xX^6} = & \frac{1}{10aX^5} + \frac{1}{8a^2X^4} + \frac{1}{6a^3X^3} + \frac{1}{4a^4X^2} + \frac{1}{2a^5X} + \frac{1}{2a^6} \log \frac{x^2}{X} \\ & - \frac{b}{2a} \int \frac{dx}{X^6} - \frac{b}{2a^2} \int \frac{dx}{X^5} - \frac{b}{2a^3} \int \frac{dx}{X^4} - \frac{b}{2a^4} \int \frac{dx}{X^3} \\ & - \frac{b}{2a^5} \int \frac{dx}{X^2} - \frac{b}{2a^6} \int \frac{dx}{X} \end{aligned}$$

$$\int \frac{dx}{x^2X^6} = -\frac{1}{axX^5} - \frac{6b}{a} \int \frac{dx}{xX^6} - \frac{11c}{a} \int \frac{dx}{X^6}$$

$$\int \frac{dx}{x^3X^6} = \left(-\frac{1}{2ax^2} + \frac{7b}{2a^2x}\right) \frac{1}{X^5} + \left(\frac{21b^2}{a^2} - \frac{6c}{a}\right) \int \frac{dx}{xX^6} + \frac{77bc}{a^2} \int \frac{dx}{X^6}$$

$$\begin{aligned} \int \frac{dx}{x^4X^6} = & \left[-\frac{1}{3ax^3} + \frac{4b}{3a^2x^2} - \left(\frac{28b^2}{3a^3} - \frac{13c}{3a^2}\right) \frac{1}{x}\right] \frac{1}{X^5} \\ & - \left(\frac{56b^3}{a^3} - \frac{42bc}{a^2}\right) \int \frac{dx}{xX^6} - \left(\frac{616b^2c}{3a^3} - \frac{143c^2}{3a^2}\right) \int \frac{dx}{X^6} \end{aligned}$$

$$\int \frac{dx}{x^5X^6} = -\frac{1}{4ax^4X^5} - \frac{9b}{4a} \int \frac{dx}{x^4X^6} - \frac{7c}{2a} \int \frac{dx}{x^3X^6}$$

$$\int \frac{dx}{x^6X^6} = \left(-\frac{1}{5ax^5} + \frac{b}{2a^2x^4}\right) \frac{1}{X^5} + \left(\frac{9b^2}{2a^2} - \frac{3c}{a}\right) \int \frac{dx}{x^4X^6} + \frac{77bc}{a^2} \int \frac{dx}{x^3X^6}$$

$$\begin{aligned} \int \frac{dx}{x^7X^6} = & \left[-\frac{1}{6ax^6} + \frac{11b}{30a^2x^5} - \left(\frac{11b^2}{12a^3} - \frac{2c}{3a^2}\right) \frac{1}{x^4}\right] \frac{1}{X^5} \\ & - \left(\frac{33b^3}{4a^3} - \frac{23bc}{2a^2}\right) \int \frac{dx}{x^4X^6} - \left(\frac{77b^2c}{6a^3} - \frac{28c^2}{3a^2}\right) \int \frac{dx}{x^3X^6} \end{aligned}$$

$$\int \frac{dx}{x^8X^6} = -\frac{1}{7ax^7X^5} - \frac{12b}{7a} \int \frac{dx}{x^7X^6} - \frac{17c}{7a} \int \frac{dx}{x^6X^6}$$

Taf. XXXIX

$$\int \frac{x^m dx}{a + bx^3}$$

(a und b positiv oder negativ.)

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$$\text{VZ. } a + bx^3 = X, \quad \sqrt[3]{\frac{a}{b}} = k \quad \text{X: b} \left( x^3 + k^3 \right)$$


---

$$\int \frac{dx}{X} = \frac{1}{3bk^2} \left( \frac{1}{2} \log \frac{(x+k)^2}{x^2 - kx + k^2} + \sqrt{3} \cdot \text{Arc Tang} \frac{x\sqrt{3}}{2k - x} \right)$$

$$\int \frac{x dx}{X} = \frac{-1}{3bk} \left( \frac{1}{2} \log \frac{(x+k)^2}{x^2 - kx + k^2} - \sqrt{3} \cdot \text{Arc Tang} \frac{x\sqrt{3}}{2k - x} \right)$$

$$\int \frac{x^2 dx}{X} = \frac{1}{3b} \log X$$

$$\int \frac{x^3 dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^4 dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{x^3}{3b} - \frac{a}{3b^2} \log X$$

$$\int \frac{x^6 dx}{X} = \frac{x^4}{4b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^5}{5b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2} \int \frac{x dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^6}{6b} - \frac{ax^3}{3b^2} + \frac{a^2}{3b^3} \log X$$

$$\int \frac{x^9 dx}{X} = \frac{x^7}{7b} - \frac{ax^4}{4b^2} + \frac{a^2 x}{b^3} - \frac{a^3}{b^3} \int \frac{dx}{X}$$

---

Die Integrale  $\int \frac{dx}{X}$ ,  $\int \frac{x dx}{X}$  sind hier für  $x = 0$  verschwindend genommen worden. Uebrigens ist  $\log \frac{(x+k)^2}{x^2 - kx + k^2} = \log \frac{b(x+k)^2}{X}$ , oder mit Zuziehung der Constante,  $= \log \frac{(x+k)^2}{X} = 3 \log \frac{x+k}{\sqrt[3]{X}}$ .

see  
p 96  
b.

$$\int \frac{x^m dx}{(a+bx^3)^2}, \quad \int \frac{x^m dx}{(a+bx^3)^3} \quad \text{Taf. XL.}$$

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$$\text{VZ. } a + bx^3 = X$$


---

$$\int \frac{\partial x}{X^2} = \frac{x}{3aX} + \frac{2}{3a} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^2} = \frac{x^2}{3aX} + \frac{1}{3a} \int \frac{x \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^2} = -\frac{1}{3bX}$$

$$\int \frac{x^3 \partial x}{X^2} = -\frac{x}{3bX} + \frac{1}{3b} \int \frac{\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X^2} = -\frac{x^2}{3bX} + \frac{2}{3b} \int \frac{x \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^2} = \frac{a}{3b^2 X} + \frac{1}{3b^2} \log X$$

$$\int \frac{x^6 \partial x}{X^2} = \left( \frac{x^4}{b} + \frac{4ax}{3b^2} \right) \frac{1}{X} - \frac{4a}{3b^2} \int \frac{\partial x}{X}$$


---

$$\int \frac{\partial x}{X^3} = \left( \frac{5bx^4}{18a^2} + \frac{4x}{9a} \right) \frac{1}{X^2} + \frac{5}{9a^2} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^3} = \left( \frac{2bx^5}{9a^2} + \frac{7x^2}{18a} \right) \frac{1}{X^2} + \frac{2}{9a^2} \int \frac{x \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^3} = -\frac{1}{6bX^2}$$

$$\int \frac{x^3 \partial x}{X^3} = \left( \frac{x^4}{18a} - \frac{x}{9b} \right) \frac{1}{X^2} + \frac{1}{9ab} \int \frac{\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X^3} = \left( \frac{x^5}{9a} - \frac{x^2}{18b} \right) \frac{1}{X^2} + \frac{1}{9ab} \int \frac{x \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^3} = \frac{x^6}{6aX^2}$$

$$\int \frac{x^6 \partial x}{X^3} = \left( -\frac{7x^4}{18b} - \frac{2ax}{9b^2} \right) \frac{1}{X^2} + \frac{2}{9b^2} \int \frac{\partial x}{X}$$

Taf. XLI.

$$\int \frac{dx}{x^n(a+bx^3)}, \int \frac{dx}{x^n(a+bx^3)^2}$$

---


$$\text{VZ. } a+bx^3 = X$$


---

$$\int \frac{dx}{xX} = \frac{\log x}{a} - \frac{\log X}{3a} = \frac{1}{3a} \log \frac{x^3}{X} = -\frac{1}{3a} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{3a^2} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{a^2x} + \frac{b^2}{a^2} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{3a^2x^3} - \frac{b^2}{3a^2} \log \frac{X}{x^3}$$

---


$$\int \frac{dx}{xX^2} = \frac{1}{3aX} - \frac{1}{3a^2} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{4bx^2}{3a^2}\right) \frac{1}{X} - \frac{4b}{3a^2} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{5bx}{6a^2}\right) \frac{1}{X} - \frac{5b}{3a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4X^2} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2}\right) \frac{1}{X} + \frac{2b}{3a^2} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{7b}{4a^2x} + \frac{7b^2x^2}{3a^3}\right) \frac{1}{X} + \frac{7b^2}{3a^3} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2x^2} + \frac{4b^2x}{3a^3}\right) \frac{1}{X} + \frac{8b^2}{3a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2x^3} + \frac{b^2}{a^2}\right) \frac{1}{X} - \frac{b^2}{a^2} \log \frac{X}{x^3}$$

$$\int \frac{x^m dx}{a + bx^4}$$

Taf. XLII. a.

(a und b dieselben Zeichen)

$$\text{VZ. } a + bx^4 = X, \quad \sqrt[4]{\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = \frac{1}{4bk^3\sqrt{2}} \left( \log \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2 \text{Arc Tang} \frac{kx\sqrt{2}}{k^2 - x^2} \right)$$

$$\int \frac{x dx}{X} = -\frac{1}{2\sqrt{ab}} \text{Arc Tang} \frac{\sqrt{a}}{x^2\sqrt{b}}$$

$$\int \frac{x^2 dx}{X} = \frac{1}{4bk\sqrt{2}} \left( -\log \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2 \text{Arc Tang} \frac{kx\sqrt{2}}{k^2 - x^2} \right)$$

$$\int \frac{x^3 dx}{X} = \frac{1}{4b} \log X$$

$$\int \frac{x^4 dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X}$$

$$\int \frac{x^6 dx}{X} = \frac{x^3}{3b} - \frac{a}{b} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^4}{4b} - \frac{a}{b} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^5}{5b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{dx}{X}$$

$$\int \frac{x^9 dx}{X} = \frac{x^6}{6b} - \frac{ax^2}{2b} + \frac{a^2}{b^2} \int \frac{x dx}{X}$$

$$\text{Es ist } \log \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + \text{Const.} = 2 \log \frac{x^2 + kx\sqrt{2} + k^2}{\sqrt{X}} + \text{Const.}$$

$$\text{und } \text{Arc Tang} \frac{kx\sqrt{2}}{k^2 - x^2} = \text{Arc Sec} \frac{\sqrt{X}}{\sqrt{a - x^2\sqrt{b}}} = \text{Arc Cos} \frac{\sqrt{a - x^2\sqrt{b}}}{\sqrt{X}}.$$



Taf. XLII. b.

$$\int \frac{x^2 dx}{a + bx^2}$$

(a und b verschiedene Zeichen)

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$$\text{VL. } a + bx^2 = X, \quad \sqrt[4]{\frac{a}{b}} = k$$


---

$$\int \frac{dx}{X} = -\frac{1}{4bk^3} \left( \log \frac{x+k}{x-k} + 2 \text{ Arc Tang } \frac{x}{k} \right)$$

$$\int \frac{x dx}{X} = -\frac{1}{4bk^2} \log \frac{x^2 + k^2}{x^2 - k^2} \quad *)$$

$$\int \frac{x^2 dx}{X} = -\frac{1}{4bk} \left( \log \frac{x+k}{x-k} - 2 \text{ Arc Tang } \frac{x}{k} \right)$$

Die übrigen Integrale wie in Tafel XLII. a.

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$$*) \log \frac{x^2 + k^2}{x^2 - k^2} + \text{Const.} = \log \frac{k^2 + x^2}{k^2 - x^2} + \text{Const.},$$

und eben so

$$\log \frac{x+k}{x-k} + \text{Const.} = \log \frac{k+x}{k-x} + \text{Const.}$$

$$\int \frac{x^m dx}{(a+bx^4)^2}, \quad \int \frac{x^m dx}{(a+bx^4)^3} \quad \text{Taf. XLIII.}$$

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$$\text{VZ. } a + bx^4 = X$$


---

$$\int \frac{\partial x}{X^2} = \frac{x}{4aX} + \frac{3}{4a} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^2} = \frac{x^2}{4aX} + \frac{1}{2a} \int \frac{x \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^2} = \frac{x^3}{4aX} + \frac{1}{4a} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^3 \partial x}{X^2} = -\frac{1}{4bX}$$

$$\int \frac{x^4 \partial x}{X^2} = -\frac{x}{4bX} + \frac{1}{4b} \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^2} = -\frac{x^2}{4bX} + \frac{1}{2b} \int \frac{x \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X^2} = -\frac{x^3}{4bX} + \frac{3}{4b} \int \frac{x^2 \partial x}{X}$$

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$$\int \frac{\partial x}{X^3} = \left( \frac{7bx^5}{32a^2} + \frac{11x}{32a} \right) \frac{1}{X^2} + \frac{21}{32a^2} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^3} = \left( \frac{3bx^6}{16a^2} + \frac{5x^2}{16a} \right) \frac{1}{X^2} + \frac{3}{8a^2} \int \frac{x \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^3} = \left( \frac{5bx^7}{32a^2} + \frac{9x^3}{32a} \right) \frac{1}{X^2} + \frac{5}{32a^2} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^3 \partial x}{X^3} = -\frac{1}{8bX^2}$$

$$\int \frac{x^4 \partial x}{X^3} = \left( \frac{x^5}{32a} - \frac{3x}{32b} \right) \frac{1}{X^2} + \frac{3}{32ab} \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^3} = \left( \frac{x^6}{16a} - \frac{x^2}{16b} \right) \frac{1}{X^2} + \frac{1}{8ab} \int \frac{x \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X^3} = \left( \frac{3x^7}{32a} - \frac{x^3}{32ab} \right) \frac{1}{X^2} + \frac{3}{32ab} \int \frac{x^2 \partial x}{X}$$

Taf. XXXVII.

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^5}$$

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$$\text{VZ. } a + bx + cx^2 = X$$


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$$\int \frac{\partial x}{xX^5} = \frac{1}{8aX^4} + \frac{1}{6a^2X^3} + \frac{1}{4a^3X^2} + \frac{1}{2a^4X} + \frac{1}{2a^5} \log \frac{x^2}{X} - \frac{b}{2a} \int \frac{\partial x}{X^5}$$

$$- \frac{b}{2a^2} \int \frac{\partial x}{X^4} - \frac{b}{2a^3} \int \frac{\partial x}{X^3} - \frac{b}{2a^4} \int \frac{\partial x}{X^2} - \frac{b}{2a^5} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^2X^5} = -\frac{1}{axX^4} - \frac{5b}{a} \int \frac{\partial x}{xX^5} - \frac{9c}{a} \int \frac{\partial x}{X^5}$$

$$\int \frac{\partial x}{x^3X^5} = \left(-\frac{1}{2ax^2} + \frac{3b}{a^2x}\right) \frac{1}{X^4} + \left(\frac{15b^2}{a^3} - \frac{5c}{a}\right) \int \frac{\partial x}{xX^5} + \frac{27bc}{a^2} \int \frac{\partial x}{X^5}$$

$$\int \frac{\partial x}{x^4X^5} = \left[-\frac{1}{3ax^3} + \frac{7b}{6a^2x^2} - \left(\frac{7b^2}{a^3} - \frac{11c}{3a^2}\right) \frac{1}{x}\right] \frac{1}{X^4}$$

$$- \left(\frac{35b^3}{a^3} - \frac{30bc}{a^2}\right) \int \frac{\partial x}{xX^5} - \left(\frac{65b^2c}{a^3} - \frac{33c^2}{a^2}\right) \int \frac{\partial x}{X^5}$$

$$\int \frac{\partial x}{x^5X^5} = -\frac{1}{4ax^4X^4} - \frac{2b}{a} \int \frac{\partial x}{x^4X^5} - \frac{3c}{a} \int \frac{\partial x}{x^3X^5}$$

$$\int \frac{\partial x}{x^6X^5} = \left(-\frac{1}{5ax^5} + \frac{9b}{20a^2x^4}\right) \frac{1}{X^4} + \left(\frac{18b^2}{5a^3} - \frac{13c}{5a}\right) \int \frac{\partial x}{x^4X^5}$$

$$+ \frac{39c^2}{5a^2} \int \frac{\partial x}{x^3X^5}$$

$$\int \frac{\partial x}{x^7X^5} = \left[-\frac{1}{6ax^6} + \frac{b}{3a^2x^5} - \left(\frac{3b^2}{4a^3} - \frac{7c}{12a^2}\right) \frac{1}{x^4}\right] \frac{1}{X^4}$$

$$- \left(\frac{6b^3}{a^3} - \frac{9bc}{a^2}\right) \int \frac{\partial x}{x^4X^5} - \left(\frac{13c^3}{a^3} - \frac{7c^2}{a^2}\right) \int \frac{\partial x}{x^3X^5}$$

$$\int \frac{\partial x}{x^8X^5} = -\frac{1}{7ax^7} - \frac{11b}{7a} \int \frac{\partial x}{x^7X^5} - \frac{15c}{7a} \int \frac{\partial x}{x^6X^5}$$

$$\int \frac{dx}{x^m(a+bx+cx^2)^6} \quad \text{Taf. XXXVIII.}$$

$$\text{VZ. } a+bx+cx^2=X$$

$$\begin{aligned} \int \frac{dx}{xX^6} = & \frac{1}{10aX^5} + \frac{1}{8a^2X^4} + \frac{1}{6a^3X^3} + \frac{1}{4a^4X^2} + \frac{1}{2a^5X} + \frac{1}{2a^6} \log \frac{x^2}{X} \\ & - \frac{b}{2a} \int \frac{dx}{X^6} - \frac{b}{2a^2} \int \frac{dx}{X^5} - \frac{b}{2a^3} \int \frac{dx}{X^4} - \frac{b}{2a^4} \int \frac{dx}{X^3} \\ & - \frac{b}{2a^5} \int \frac{dx}{X^2} - \frac{b}{2a^6} \int \frac{dx}{X} \end{aligned}$$

$$\int \frac{dx}{x^2X^6} = -\frac{1}{axX^5} - \frac{6b}{a} \int \frac{dx}{xX^6} - \frac{11c}{a} \int \frac{dx}{X^6}$$

$$\int \frac{dx}{x^3X^6} = \left(-\frac{1}{2ax^2} + \frac{7b}{2a^2x}\right) \frac{1}{X^5} + \left(\frac{21b^2}{a^2} - \frac{6c}{a}\right) \int \frac{dx}{xX^6} + \frac{77bc}{a^2} \int \frac{dx}{X^6}$$

$$\begin{aligned} \int \frac{dx}{x^4X^6} = & \left[-\frac{1}{3ax^3} + \frac{4b}{3a^2x^2} - \left(\frac{28b^2}{3a^3} - \frac{13c}{3a^2}\right) \frac{1}{x}\right] \frac{1}{X^5} \\ & - \left(\frac{56b^3}{a^3} - \frac{42bc}{a^2}\right) \int \frac{dx}{xX^6} - \left(\frac{616b^2c}{3a^3} - \frac{143c^2}{3a^2}\right) \int \frac{dx}{X^6} \end{aligned}$$

$$\int \frac{dx}{x^5X^6} = -\frac{1}{4ax^4X^5} - \frac{9b}{4a} \int \frac{dx}{x^4X^6} - \frac{7c}{2a} \int \frac{dx}{x^3X^6}$$

$$\int \frac{dx}{x^6X^6} = \left(-\frac{1}{5ax^5} + \frac{b}{2a^2x^4}\right) \frac{1}{X^5} + \left(\frac{9b^2}{2a^2} - \frac{3c}{a}\right) \int \frac{dx}{x^4X^6} + \frac{77bc}{a^2} \int \frac{dx}{x^3X^6}$$

$$\begin{aligned} \int \frac{dx}{x^7X^6} = & \left[-\frac{1}{6ax^6} + \frac{11b}{30a^2x^5} - \left(\frac{11b^2}{12a^3} - \frac{2c}{3a^2}\right) \frac{1}{x^4}\right] \frac{1}{X^5} \\ & - \left(\frac{33b^3}{4a^3} - \frac{23bc}{2a^2}\right) \int \frac{dx}{x^4X^6} - \left(\frac{77b^2c}{6a^3} - \frac{28c^2}{3a^2}\right) \int \frac{dx}{x^3X^6} \end{aligned}$$

$$\int \frac{dx}{x^8X^6} = -\frac{1}{7ax^7X^5} - \frac{12b}{7a} \int \frac{dx}{x^7X^6} - \frac{17c}{7a} \int \frac{dx}{x^6X^6}$$

Taf. XXXIX

$$\int \frac{x^m dx}{a + bx^3}$$

(a und b positiv oder negativ.)

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$$\text{VZ. } a + bx^3 = X, \quad \sqrt[3]{\frac{a}{b}} = k, \quad X = b(x^3 + k^3)$$


---

$$\int \frac{dx}{X} = \frac{1}{3bk^2} \left( \frac{1}{2} \log \frac{(x+k)^2}{x^2 - kx + k^2} + \sqrt{3} \cdot \text{Arc Tang} \frac{x\sqrt{3}}{2k - x} \right)$$

$$\int \frac{x dx}{X} = \frac{-1}{3bk} \left( \frac{1}{2} \log \frac{(x+k)^2}{x^2 - kx + k^2} - \sqrt{3} \cdot \text{Arc Tang} \frac{x\sqrt{3}}{2k - x} \right)$$

$$\int \frac{x^2 dx}{X} = \frac{1}{3b} \log X$$

$$\int \frac{x^3 dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^4 dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{x^3}{3b} - \frac{a}{3b^2} \log X$$

$$\int \frac{x^6 dx}{X} = \frac{x^4}{4b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^5}{5b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2} \int \frac{x dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^6}{6b} - \frac{ax^3}{3b^2} + \frac{a^2}{3b^3} \log X$$

$$\int \frac{x^9 dx}{X} = \frac{x^7}{7b} - \frac{ax^4}{4b^2} + \frac{a^2 x}{b^3} - \frac{a^3}{b^3} \int \frac{dx}{X}$$

---

Die Integrale  $\int \frac{dx}{X}$ ,  $\int \frac{x dx}{X}$  sind hier für  $x = 0$  verschwindend genommen worden. Uebrigens ist  $\log \frac{(x+k)^2}{x^2 - kx + k^2} = \log \frac{b(x+k)^2}{X}$ , oder mit Zuziehung der Constante,  $= \log \frac{(x+k)^2}{X} = 3 \log \frac{x+k}{\sqrt[3]{X}}$ .

see  
p 96  
b7

$$\int \frac{x^m dx}{(a+bx^3)^2}, \quad \int \frac{x^m dx}{(a+bx^3)^3} \quad \text{Taf. XL.}$$

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$$\text{VZ. } a+bx^3 = X$$


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$$\int \frac{\partial x}{X^2} = \frac{x}{3aX} + \frac{2}{3a} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^2} = \frac{x^2}{3aX} + \frac{1}{3a} \int \frac{x \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^2} = -\frac{1}{3bX}$$

$$\int \frac{x^3 \partial x}{X^2} = -\frac{x}{3bX} + \frac{1}{3b} \int \frac{\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X^2} = -\frac{x^2}{3bX} + \frac{2}{3b} \int \frac{x \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^2} = \frac{a}{3b^2X} + \frac{1}{3b^2} \log X$$

$$\int \frac{x^6 \partial x}{X^2} = \left(\frac{x^4}{b} + \frac{4ax}{3b^2}\right) \frac{1}{X} - \frac{4a}{3b^2} \int \frac{\partial x}{X}$$


---

$$\int \frac{\partial x}{X^3} = \left(\frac{5bx^4}{18a^2} + \frac{4x}{9a}\right) \frac{1}{X^2} + \frac{5}{9a^2} \int \frac{\partial x}{X}$$

$$\int \frac{x \partial x}{X^3} = \left(\frac{2bx^5}{9a^2} + \frac{7x^2}{18a}\right) \frac{1}{X^2} + \frac{2}{9a^2} \int \frac{x \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^3} = -\frac{1}{6bX^2}$$

$$\int \frac{x^3 \partial x}{X^3} = \left(\frac{x^4}{18a} - \frac{x}{9b}\right) \frac{1}{X^2} + \frac{1}{9ab} \int \frac{\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X^3} = \left(\frac{x^5}{9a} - \frac{x^2}{18b}\right) \frac{1}{X^2} + \frac{1}{9ab} \int \frac{x \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X^3} = \frac{x^6}{6aX^2}$$

$$\int \frac{x^6 \partial x}{X^3} = \left(-\frac{7x^4}{18b} - \frac{2ax}{9b^2}\right) \frac{1}{X^2} + \frac{2}{9b^2} \int \frac{\partial x}{X}$$

Taf. XLI.

$$\int \frac{dx}{x^m(a+bx^3)}, \quad \int \frac{dx}{x^m(a+bx^3)^2}$$

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$$\text{VL. } a+bx^3 = X$$


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$$\int \frac{dx}{xX} = \frac{\log x}{a} - \frac{\log X}{3a} = \frac{1}{3a} \log \frac{x^3}{X} = -\frac{1}{3a} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4 X} = -\frac{1}{3ax^3} + \frac{b}{3a^2} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^5 X} = -\frac{1}{4ax^4} + \frac{b}{a^2 x} + \frac{b^2}{a^3} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^6 X} = -\frac{1}{5ax^5} + \frac{b}{2a^2 x^2} + \frac{b^2}{a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7 X} = -\frac{1}{6ax^6} + \frac{b}{3a^2 x^3} - \frac{b^2}{3a^3} \log \frac{X}{x^3}$$

---


$$\int \frac{dx}{xX^2} = \frac{1}{3aX} - \frac{1}{3a^2} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^2 X^2} = \left(-\frac{1}{ax} - \frac{4bx^2}{3a^2}\right) \frac{1}{X} - \frac{4b}{3a^2} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^3 X^2} = \left(-\frac{1}{2ax^2} - \frac{5bx}{6a^2}\right) \frac{1}{X} - \frac{5b}{3a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4 X^2} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2}\right) \frac{1}{X} + \frac{2b}{3a^3} \log \frac{X}{x^3}$$

$$\int \frac{dx}{x^5 X^2} = \left(-\frac{1}{4ax^4} + \frac{7b}{4a^2 x} + \frac{7b^2 x^2}{3a^3}\right) \frac{1}{X} + \frac{7b^2}{3a^3} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^6 X^2} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2 x^2} + \frac{4b^2 x}{3a^3}\right) \frac{1}{X} + \frac{8b^2}{3a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7 X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2 x^3} + \frac{b^2}{a^3}\right) \frac{1}{X} - \frac{b^2}{a^4} \log \frac{X}{x^3}$$

$$\int \frac{x^m dx}{a + bx^4}$$

Taf. XLII. a.

(a und b dieselben Zeichen)

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$$\text{VZ. } a + bx^4 = X, \quad \sqrt[4]{\frac{a}{b}} = k$$


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$$\int \frac{dx}{X} = \frac{1}{4bk^3\sqrt{2}} \left( \log \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2 \operatorname{ArcTang} \frac{kx\sqrt{2}}{k^2 - x^2} \right)$$

$$\int \frac{x dx}{X} = -\frac{1}{2\sqrt{ab}} \operatorname{ArcTang} \frac{\sqrt{a}}{x^2\sqrt{b}}$$

$$\int \frac{x^2 dx}{X} = \frac{1}{4bk\sqrt{2}} \left( -\log \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2 \operatorname{ArcTang} \frac{kx\sqrt{2}}{k^2 - x^2} \right)$$

$$\int \frac{x^3 dx}{X} = \frac{1}{4b} \log X$$

$$\int \frac{x^4 dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X}$$

$$\int \frac{x^6 dx}{X} = \frac{x^3}{3b} - \frac{a}{b} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^4}{4b} - \frac{a}{b} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^5}{5b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{dx}{X}$$

$$\int \frac{x^9 dx}{X} = \frac{x^6}{6b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2} \int \frac{x dx}{X}$$

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$$\text{Es ist } \log \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + \text{Const.} = 2 \log \frac{x^2 + kx\sqrt{2} + k^2}{\sqrt{X}} + \text{Const.}$$

$$\text{und } \operatorname{ArcTang} \frac{kx\sqrt{2}}{k^2 - x^2} = \operatorname{ArcSec} \frac{\sqrt{X}}{\sqrt{a - x^2}\sqrt{b}} = \operatorname{ArcCos} \frac{\sqrt{a - x^2}\sqrt{b}}{\sqrt{X}}.$$



Taf. XLII. b.

$$\int \frac{x^2 dx}{a + bx^4}$$

(a und b verschiedene Zeichen)

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$$\text{VZ. } a + bx^4 = X, \sqrt[4]{X} - \frac{a}{b} = k$$


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$$\int \frac{dx}{X} = -\frac{1}{4bk^3} \left( \log \frac{x+k}{x-k} + 2 \text{ Arc Tang } \frac{x}{k} \right)$$

$$\int \frac{x dx}{X} = -\frac{1}{4bk^2} \log \frac{x^2 + k^2}{x^2 - k^2} \quad *)$$

$$\int \frac{x^2 dx}{X} = -\frac{1}{4bk} \left( \log \frac{x+k}{x-k} - 2 \text{ Arc Tang } \frac{x}{k} \right)$$

Die übrigen Integrale wie in Tafel XLII. a.

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$$*) \log \frac{x^2 + k^2}{x^2 - k^2} + \text{Const.} = \log \frac{k^2 + x^2}{k^2 - x^2} + \text{Const.},$$

und eben so

$$\log \frac{x+k}{x-k} + \text{Const.} = \log \frac{k+x}{k-x} + \text{Const.}$$

$$\int \frac{x^n dx}{(a+bx^4)^2}, \quad \int \frac{x^n dx}{(a+bx^4)^3} \quad \text{Taf. XLIII.}$$

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$$\text{VZ. } a + bx^4 = X$$


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$$\int \frac{dx}{X^2} = \frac{x}{4aX} + \frac{3}{4a} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^2} = \frac{x^2}{4aX} + \frac{1}{2a} \int \frac{x dx}{X}$$

$$\int \frac{x^2 dx}{X^2} = \frac{x^3}{4aX} + \frac{1}{4a} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^3 dx}{X^2} = -\frac{1}{4bX}$$

$$\int \frac{x^4 dx}{X^2} = -\frac{x}{4bX} + \frac{1}{4b} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X^2} = -\frac{x^2}{4bX} + \frac{1}{2b} \int \frac{x dx}{X}$$

$$\int \frac{x^6 dx}{X^2} = -\frac{x^3}{4bX} + \frac{3}{4b} \int \frac{x^2 dx}{X}$$

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$$\int \frac{dx}{X^3} = \left( \frac{7bx^5}{32a^2} + \frac{11x}{32a} \right) \frac{1}{X^2} + \frac{21}{32a^2} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^3} = \left( \frac{3bx^6}{16a^2} + \frac{5x^3}{16a} \right) \frac{1}{X^2} + \frac{3}{8a^2} \int \frac{x dx}{X}$$

$$\int \frac{x^2 dx}{X^3} = \left( \frac{5bx^7}{32a^2} + \frac{9x^3}{32a} \right) \frac{1}{X^2} + \frac{5}{32a^2} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^3 dx}{X^3} = -\frac{1}{8bX^2}$$

$$\int \frac{x^4 dx}{X^3} = \left( \frac{x^5}{32a} - \frac{3x}{32b} \right) \frac{1}{X^2} + \frac{3}{32ab} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X^3} = \left( \frac{x^6}{16a} - \frac{x^3}{16b} \right) \frac{1}{X^2} + \frac{1}{8ab} \int \frac{x dx}{X}$$

$$\int \frac{x^6 dx}{X^3} = \left( \frac{3x^7}{32a} - \frac{x^3}{32ab} \right) \frac{1}{X^2} + \frac{3}{32ab} \int \frac{x^2 dx}{X}$$

Taf. XLIV.

$$\int \frac{\partial x}{x^m(a+bx^4)}, \quad \int \frac{\partial x}{x^m(a+bx^4)^2}$$

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$$\text{VL. } a + bx^4 = X$$


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$$\int \frac{\partial x}{xX} = \frac{\log x}{a} - \frac{\log X}{4a} = \frac{1}{4a} \log \frac{x^4}{X} = -\frac{1}{4a} \log \frac{X}{x^4}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{x \partial x}{X}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} - \frac{b}{a} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} - \frac{b}{a} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{a^2x} + \frac{b^2}{a^2} \int \frac{x^5 \partial x}{X}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2} \int \frac{x \partial x}{X}$$

---


$$\int \frac{\partial x}{xX^2} = \frac{1}{4aX} + \frac{1}{a} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{5bx^3}{4a^2}\right) \frac{1}{X} - \frac{5b}{4a^2} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{3bx^2}{4a^2}\right) \frac{1}{X} - \frac{3b}{2a^2} \int \frac{x \partial x}{X}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} - \frac{7bx}{12a^2}\right) \frac{1}{X} - \frac{7b}{4a^2} \int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} - \frac{b}{2a^2}\right) \frac{1}{X} - \frac{2b}{a^2} \int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{9b}{5a^2x} + \frac{9b^2x^3}{4a^3}\right) \frac{1}{X} + \frac{9b^2}{4a^3} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{5b}{6a^2x^2} + \frac{5b^2x^2}{4a^3}\right) \frac{1}{X} + \frac{5b^2}{2a^3} \int \frac{x \partial x}{X}$$

$$\int \frac{x^m dx}{a + bx^5}$$

Taf. XLV.

(a und b positiv oder negativ)

$$\text{VZ. } a + bx^5 = X, \quad \sqrt[5]{\frac{a}{b}} = k$$

$$x^2 - 2kx \cos 36^\circ + k^2 = Y$$

$$x^2 + 2kx \cos 72^\circ + k^2 = Y'$$

$$\frac{x \sin 36^\circ}{k - x \cos 36^\circ} = Z, \quad \frac{x \sin 72^\circ}{k + x \cos 72^\circ} = Z'$$

$$\int \frac{dx}{X} = \frac{1}{5bk^4} \left\{ \begin{array}{l} -\cos 36^\circ \log Y + 2 \sin 36^\circ \text{Arc Tang } Z \\ + \cos 72^\circ \log Y' + 2 \sin 72^\circ \text{Arc Tang } Z' \\ + \log(x + k) \end{array} \right\}$$

$$\int \frac{x dx}{X} = \frac{1}{5bk^3} \left\{ \begin{array}{l} -\cos 72^\circ \log Y + 2 \sin 72^\circ \text{Arc Tang } Z \\ + \cos 36^\circ \log Y' - 2 \sin 36^\circ \text{Arc Tang } Z' \\ - \log(x + k) \end{array} \right\}$$

$$\int \frac{x^2 dx}{X} = \frac{1}{5bk^2} \left\{ \begin{array}{l} \cos 72^\circ \log Y + 2 \sin 72^\circ \text{Arc Tang } Z \\ - \cos 36^\circ \log Y' - 2 \sin 36^\circ \text{Arc Tang } Z' \\ + \log(x + k) \end{array} \right\}$$

$$\int \frac{x^3 dx}{X} = \frac{1}{5bk} \left\{ \begin{array}{l} \cos 36^\circ \log Y + 2 \sin 36^\circ \text{Arc Tang } Z \\ - \cos 72^\circ \log Y' + 2 \sin 72^\circ \text{Arc Tang } Z' \\ - \log(x + k) \end{array} \right\}$$

$$\int \frac{x^4 dx}{X} = \frac{1}{5b} \log X$$

$$\int \frac{x^5 dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^6 dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^3}{3b} - \frac{a}{b} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^4}{4b} - \frac{a}{b} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^9 dx}{X} = \frac{x^5}{5b} - \frac{a}{b} \int \frac{x^4 dx}{X}$$

Taf. XLVI.

$$\int \frac{x^n dx}{(a+bx^5)^2}, \quad \int \frac{dx}{x^m(a+bx^5)}$$

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$$\text{VZ. } a+bx^5=X$$


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$$\int \frac{dx}{X^2} = \frac{x}{5aX} + \frac{4}{5a} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^2} = \frac{x^2}{5aX} + \frac{3}{5a} \int \frac{x dx}{X}$$

$$\int \frac{x^2 dx}{X^2} = \frac{x^3}{5aX} + \frac{2}{5a} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^3 dx}{X^2} = \frac{x^4}{5aX} + \frac{1}{5a} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^4 dx}{X^2} = -\frac{1}{5bX}$$

$$\int \frac{x^5 dx}{X^2} = -\frac{x}{5bX} + \frac{1}{5b} \int \frac{dx}{X}$$

$$\int \frac{x^6 dx}{X^2} = -\frac{x^2}{5bX} + \frac{2}{5b} \int \frac{x dx}{X}$$

$$\int \frac{x^7 dx}{X^2} = -\frac{x^3}{5bX} + \frac{3}{5b} \int \frac{x^2 dx}{X}$$

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$$\int \frac{dx}{xX} = \frac{\log x}{a} - \frac{\log X}{5a} = \frac{1}{5a} \log \frac{x^5}{X} = -\frac{1}{5a} \log \frac{X}{x^5}$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x^3 dx}{X}$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{x^2 dx}{X}$$

$$\int \frac{dx}{x^4 X} = -\frac{1}{3ax^3} - \frac{b}{a} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^5 X} = -\frac{1}{4ax^4} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^6 X} = -\frac{1}{5ax^5} - \frac{b}{a} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^7 X} = -\frac{1}{6ax^6} + \frac{b}{a^2 x} + \frac{b^2}{a^2} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^m dx}{a + bx^6}$$

Taf. XLVII. a.

(a und b dieselben Zeichen)

$$\text{VL. } a + bx^6 = X, \quad \sqrt[6]{\frac{a}{b}} = k$$

$$\int \frac{\partial x}{X} = \frac{1}{6bk^5} \left( \frac{\sqrt{3}}{2} \log \frac{x^2 + kx\sqrt{3} + k^2}{x^2 - kx\sqrt{3} + k^2} + \text{Arc Tang} \frac{3kx(k^2 - x^2)}{x^4 - 4k^2x^2 + k^4} \right)$$

$$\int \frac{x \partial x}{X} = \frac{1}{6bk^4} \left( \frac{1}{2} \log \frac{(x^2 + k^2)^2}{x^4 - k^2x^2 + k^4} + \sqrt{3} \cdot \text{Arc Tang} \frac{x^2\sqrt{3}}{2k^2 - x^2} \right)$$

$$\int \frac{x^2 \partial x}{X} = \frac{1}{3\sqrt{ab}} \text{Arc Tang } x^3 \sqrt{\frac{b}{a}}$$

$$\int \frac{x^3 \partial x}{X} = \frac{1}{6bk^3} \left( \frac{1}{2} \log \frac{x^4 - k^2x^2 + k^4}{(x^2 + k^2)^2} + \sqrt{3} \cdot \text{Arc Tang} \frac{x^3\sqrt{3}}{2k^2 - x^2} \right)$$

$$\int \frac{x^4 \partial x}{X} = \frac{1}{6bk^2} \left( \frac{\sqrt{3}}{2} \log \frac{x^2 - kx\sqrt{3} + k^2}{x^2 + kx\sqrt{3} + k^2} + \text{Arc Tang} \frac{3kx(k^2 - x^2)}{x^4 - 4k^2x^2 + k^4} \right)$$

$$\int \frac{x^5 \partial x}{X} = \frac{1}{6b} \log X$$

$$\int \frac{x^6 \partial x}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{\partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^3}{3b} - \frac{a}{b} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^4}{4b} - \frac{a}{b} \int \frac{x^3 \partial x}{X}$$

Diese Integrale verschwinden sämmtlich für  $x = 0$ . Auch ist

$$\log \frac{x^4 - k^2x^2 + k^4}{(x^2 + k^2)^2} + \text{Const.} = \log \frac{X}{(x^2 + k^2)^2} = 3 \log \frac{\sqrt[3]{X}}{x^2 + k^2}.$$

Taf. XLVII. b.

$$\int \frac{x^m dx}{a + bx^6}$$

( $a$  und  $b$  verschiedene Zeichen)

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$$\text{VZ. } a + bx^6 = X, \quad \sqrt[6]{-\frac{a}{b}} = k$$


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$$\int \frac{dx}{X} = \frac{-1}{6bk^5} \left( \frac{1}{2} \log \frac{(x+k)^2(x^2+kx+k^2)}{(x-k)^2(x^2-kx+k^2)} + \sqrt{3} \cdot \text{Arc Tang} \frac{kx\sqrt{3}}{k^2-x^2} \right)$$

$$\int \frac{x dx}{X} = \frac{-1}{6bk^4} \left( \frac{1}{2} \log \frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} + \sqrt{3} \cdot \text{Arc Tang} \frac{x^2\sqrt{3}}{2k^2+x^2} \right)$$

$$\int \frac{x^2 dx}{X} = \frac{-1}{6bk^3} \log \frac{x^3+k^3}{x^3-k^3}$$

$$\int \frac{x^3 dx}{X} = \frac{-1}{6bk^2} \left( \frac{1}{2} \log \frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} - \sqrt{3} \cdot \text{Arc Tang} \frac{x^2\sqrt{3}}{2k^2+x^2} \right)$$

$$\int \frac{x^4 dx}{X} = \frac{-1}{6bk} \left( \frac{1}{2} \log \frac{(x+k)^2(x^2+kx+k^2)}{(x-k)^2(x^2-kx+k^2)} - \sqrt{3} \cdot \text{Arc Tang} \frac{kx\sqrt{3}}{k^2-x^2} \right)$$

Die übrigen Integrale wie in Taf. XLVII. a.

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Diese Integrale verschwinden sämmtlich für  $x = 0$ . Auch ist

$$\log \frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} + \text{Const.} = \log \frac{X}{(x^2-k^2)^2} = 3 \log \frac{\sqrt[3]{X}}{x^2-k^2}.$$

$$\int \frac{x^m dx}{(a + bx^6)^2}, \int \frac{dx}{x^m(a + bx^6)} \quad \text{Taf. XLVIII.}$$

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$$\text{VZ. } a + bx^6 = X$$


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$$\int \frac{dx}{X^2} = \frac{x}{6aX} + \frac{5}{6a} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^2} = \frac{x^2}{6aX} + \frac{2}{3a} \int \frac{x dx}{X}$$

$$\int \frac{x^2 dx}{X^2} = \frac{x^3}{6aX} + \frac{1}{2a} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^3 dx}{X^2} = \frac{x^4}{6aX} + \frac{1}{3a} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^4 dx}{X^2} = \frac{x^5}{6aX} + \frac{1}{6a} \int \frac{x^4 dx}{X}$$

$$\int \frac{x^5 dx}{X^2} = -\frac{1}{6bX}$$

$$\int \frac{x^6 dx}{X^2} = -\frac{x}{6bX} + \frac{1}{6b} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X^2} = -\frac{x^2}{6bX} + \frac{1}{3b} \int \frac{x dx}{X}$$

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$$\int \frac{dx}{xX} = \frac{\log x}{a} - \frac{\log X}{6a} = \frac{1}{6a} \log \frac{x^6}{X} = -\frac{1}{6a} \log \frac{X}{x^6}$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x^4 dx}{X}$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{x^3 dx}{X}$$

$$\int \frac{dx}{x^4 X} = -\frac{1}{3ax^3} - \frac{b}{a} \int \frac{x^2 dx}{X}$$

$$\int \frac{dx}{x^5 X} = -\frac{1}{4ax^4} - \frac{b}{a} \int \frac{x dx}{X}$$

$$\int \frac{dx}{x^6 X} = -\frac{1}{5ax^5} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7 X} = -\frac{1}{6ax^6} - \frac{b}{a} \int \frac{dx}{xX}$$



Taf. XLIX. a.

$$\int \frac{x^m dx}{a + bx^2 + cx^4}$$

( $b^2 - 4ac$  eine positive GröÙe)

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$$a + bx^2 + cx^4 = X$$

$$\frac{1}{2}b - \frac{1}{2}\sqrt{b^2 - 4ac} = f, \quad \frac{1}{2}b + \frac{1}{2}\sqrt{b^2 - 4ac} = g$$

$$\sqrt{b^2 - 4ac} = g - f = h$$


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$$\int \frac{\partial x}{X} = \frac{c}{h} \left[ \int \frac{\partial x}{cx^2 + f} - \int \frac{\partial x}{cx^2 + g} \right]$$

$$\int \frac{x \partial x}{X} = \frac{1}{2h} \log \frac{cx^2 + f}{cx^2 + g}$$

$$\int \frac{x^2 \partial x}{X} = \frac{g}{h} \int \frac{\partial x}{cx^2 + g} - \frac{f}{h} \int \frac{\partial x}{cx^2 + f}$$

$$\int \frac{x^3 \partial x}{X} = \frac{1}{2ch} \left[ g \log (cx^2 + g) - f \log (cx^2 + f) \right]$$

$$\int \frac{x^4 \partial x}{X} = \frac{x}{c} - \frac{a}{c} \int \frac{\partial x}{X} - \frac{b}{c} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^2}{2c} - \frac{a}{c} \int \frac{x \partial x}{X} - \frac{b}{c} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^3}{3c} - \frac{bx}{c^2} + \frac{ab}{c^2} \int \frac{\partial x}{X} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^4}{4c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2} \int \frac{x \partial x}{X} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^5}{5c} - \frac{bx^3}{3c^2} + \left( \frac{b^2}{c^2} - \frac{a}{c^2} \right) x - \left( \frac{ab^2}{c^3} - \frac{a^2}{c^2} \right) \int \frac{\partial x}{X}$$

$$- \left( \frac{b^3}{c^3} - \frac{2ab}{c^2} \right) \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^6}{6c} - \frac{bx^4}{4c^2} + \left( \frac{b^2}{2c^3} - \frac{a}{2c^2} \right) x^2 - \left( \frac{ab^2}{c^3} - \frac{a^2}{c^2} \right) \int \frac{x \partial x}{X}$$

$$- \left( \frac{b^3}{c^3} - \frac{2ab}{c^2} \right) \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^{10} \partial x}{X} = \frac{x^7}{7c} - \frac{a}{c} \int \frac{x^6 \partial x}{X} - \frac{b}{c} \int \frac{x^8 \partial x}{X}$$

$$\int \frac{x^3 dx}{a + bx^2 + cx^4}$$

Taf. XLIX. b.

 ( $b^2 - 4ac$  eine negative GröÙe)

$$\text{VZ. } a + bx^2 + cx^4 = X, \sqrt[4]{\frac{a}{c}} = f$$

 $\alpha$  ein Winkel, dessen Cosinus  $= -\frac{b}{2\sqrt{ac}}$ 

$$\int \frac{dx}{X} = \frac{1}{4cf^3 \sin \alpha} \left\{ \begin{aligned} &\sin \frac{\alpha}{2} \log \frac{x^2 + 2fx \cos \frac{\alpha}{2} + f^2}{x^2 - 2fx \cos \frac{\alpha}{2} + f^2} \\ &+ 2 \cos \frac{\alpha}{2} \text{Arc Tang} \frac{2fx \sin \frac{\alpha}{2}}{f^2 - x^2} \end{aligned} \right\}$$

$$\int \frac{x dx}{X} = \frac{1}{2cf^2 \sin \alpha} \text{Arc Tang} \frac{f^2 \sin \alpha}{f^2 \cos \alpha - x^2}$$

$$\int \frac{x^2 dx}{X} = \frac{1}{4cf \sin \alpha} \left\{ \begin{aligned} &\sin \frac{\alpha}{2} \log \frac{x^2 - 2fx \cos \frac{\alpha}{2} + f^2}{x^2 + 2fx \cos \frac{\alpha}{2} + f^2} \\ &+ 2 \cos \frac{\alpha}{2} \text{Arc Tang} \frac{2fx \sin \frac{\alpha}{2}}{f^2 - x^2} \end{aligned} \right\}$$

$$\int \frac{x^3 dx}{X} = \frac{1}{4c \sin \alpha} \left\{ \begin{aligned} &\sin \alpha \log (x^4 - 2f^2 x^2 \sin \alpha + f^4) \\ &+ 2 \cos \alpha \text{Arc Tang} \frac{2fx \sin \frac{\alpha}{2}}{f^2 - x^2} \end{aligned} \right\}$$

Die übrigen Integrale wie in Tafel XLIX. a.

Ein Winkel, dessen Cosinus  $= -\frac{b}{2\sqrt{ac}}$ , laßt sich immer finden; denn ist diese GröÙe positiv, so ist der Winkel spitz, ist sie negativ, so ist der Winkel stumpf. GröÙer als 1 kann sie nicht werden, weil sonst  $b^2 - 4ac$  nicht negativ seyn könnte.

Taf. L

$$\int \frac{x^n dx}{(a + bx^2 + cx^4)^2}$$

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$$\text{VZ. } a + bx^2 + cx^4 = X, \quad 2a(b^2 - 4ac) = k$$


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$$\int \frac{dx}{X^2} = [bcx^3 + (b^2 - 2ac)x] \frac{1}{kX} + \frac{b^2 - 6ac}{k} \int \frac{dx}{X} + \frac{bc}{k} \int \frac{x^2 dx}{X}$$

$$\int \frac{x dx}{X^2} = [bcx^4 + (b^2 - 2ac)x^2] \frac{1}{kX} - \frac{4ac}{k} \int \frac{x dx}{X}$$

$$\int \frac{x^2 dx}{X^2} = [bcx^5 + (b^2 - 2ac)x^3] \frac{1}{kX} - \frac{bx}{k} + \frac{ab}{k} \int \frac{dx}{X} - \frac{2ac}{k} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^3 dx}{X^2} = [bcx^6 + (b^2 - 2ac)x^4] \frac{1}{kX} - \frac{bx^2}{k} + \frac{2ab}{k} \int \frac{x dx}{X}$$

$$\int \frac{x^4 dx}{X^2} = -\frac{x}{3cX} + \frac{a}{3c} \int \frac{dx}{X^2} - \frac{b}{3c} \int \frac{x^2 dx}{X^2}$$

$$\int \frac{x^5 dx}{X^2} = -\frac{x^2}{2cX} + \frac{a}{c} \int \frac{x dx}{X^2}$$

$$\int \frac{x^6 dx}{X^2} = \left(-\frac{x^3}{c} - \frac{bx}{3c^2}\right) \frac{1}{X} + \frac{ab}{3c^2} \int \frac{dx}{X^2} - \left(\frac{b^2}{3c^2} - \frac{3a}{c}\right) \int \frac{x^2 dx}{X^2}$$

$$\int \frac{x^7 dx}{X^2} = [bcx^{10} + (b^2 - 2ac)x^8] \frac{1}{kX} + \frac{8ac - 6b^2}{k} \int \frac{x^7 dx}{X} - \frac{6bc}{k} \int \frac{x^9 dx}{X}$$

$$\int \frac{x^8 dx}{X^2} = \frac{x^5}{cX} - \frac{5a}{c} \int \frac{x^4 dx}{X^2} - \frac{3b}{c} \int \frac{x^6 dx}{X^2}$$

$$\int \frac{x^9 dx}{X^2} = \frac{x^6}{2cX} - \frac{3a}{c} \int \frac{x^5 dx}{X^2} - \frac{2b}{c} \int \frac{x^7 dx}{X^2}$$

$$\int \frac{\partial x}{x^n(a + bx^2 + cx^4)}$$

Taf. LI.

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$$\text{VZ. } a + bx^2 + cx^4 = X$$


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$$\int \frac{\partial x}{xX} = \frac{\log x}{a} - \frac{b}{a} \int \frac{x \partial x}{X} - \frac{c}{a} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{\partial x}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{\partial x}{X} - \frac{c}{a} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{\partial x}{xX} - \frac{c}{a} \int \frac{x \partial x}{X}$$

$$\int \frac{\partial x}{x^4 X} = -\frac{1}{3ax^3} + \frac{b}{a^2 x} + \left(\frac{b^2}{a^3} - \frac{c}{a}\right) \int \frac{\partial x}{X} + \frac{bc}{a^2} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^5 X} = -\frac{1}{4ax^4} + \frac{b}{2a^2 x^2} + \left(\frac{b^2}{a^3} - \frac{c}{a}\right) \int \frac{\partial x}{xX} + \frac{bc}{a^2} \int \frac{x \partial x}{X}$$

$$\int \frac{\partial x}{x^6 X} = -\frac{1}{5ax^5} + \frac{b}{3a^2 x^3} - \left(\frac{b^2}{a^3} - \frac{c}{a}\right) \frac{1}{x} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right) \int \frac{\partial x}{X} - \left(\frac{b^2 c}{a^3} - \frac{c^2}{a^2}\right) \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^7 X} = -\frac{1}{6ax^6} - \frac{b}{a} \int \frac{\partial x}{x^3 X} - \frac{c}{a} \int \frac{\partial x}{x^5 X}$$

$$\int \frac{\partial x}{x^8 X} = -\frac{1}{7ax^7} - \frac{b}{a} \int \frac{\partial x}{x^6 X} - \frac{c}{a} \int \frac{\partial x}{x^4 X}$$

$$\int \frac{\partial x}{x^9 X} = -\frac{1}{8ax^8} + \frac{b}{6a^2 x^6} + \left(\frac{b^2}{a^3} - \frac{c}{a}\right) \int \frac{\partial x}{x^3 X} + \frac{bc}{a^2} \int \frac{\partial x}{x^5 X}$$

$$\int \frac{\partial x}{x^{10} X} = -\frac{1}{9ax^9} + \frac{b}{7a^2 x^7} + \left(\frac{b^2}{a^3} - \frac{c}{a}\right) \int \frac{\partial x}{x^6 X} + \frac{bc}{a^2} \int \frac{\partial x}{x^4 X}$$

$$\int \frac{\partial x}{x^{11} X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2 x^8} - \left(\frac{b^2}{6a^3} - \frac{c}{6a^2}\right) \frac{1}{x^6} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right) \int \frac{\partial x}{x^3 X} - \left(\frac{b^2 c}{a^3} - \frac{c^2}{a^2}\right) \int \frac{\partial x}{x^5 X}$$

Taf. LII.

$$\int \frac{\partial x}{x^n(a + bx^2 + cx^4)^2}$$

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$$\text{VZ. } a + bx^2 + cx^4 = X, \quad 2a(b^2 - 4ac) = k$$


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$$\int \frac{\partial x}{xX^2} = \frac{bcx^2 + b^2 - 2ac}{kX} + \frac{\log x}{a^2} + \left(\frac{2bc}{k} - \frac{b}{a^2}\right) \int \frac{x\partial x}{X} - \frac{bc}{a^2} \int \frac{x^3\partial x}{X}$$

$$\int \frac{\partial x}{x^2X^2} = -\frac{1}{axX} - \frac{3b}{a} \int \frac{\partial x}{X^2} - \frac{5c}{a} \int \frac{x^2\partial x}{X^2}$$

$$\int \frac{\partial x}{x^3X^2} = -\frac{1}{2ax^2X} - \frac{2b}{a} \int \frac{\partial x}{xX^2} - \frac{3c}{a} \int \frac{x\partial x}{X^2}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x}\right) \frac{1}{X} + \left(\frac{5b^2}{a^2} - \frac{7c}{3a}\right) \int \frac{\partial x}{X^2} + \frac{25c^2}{3a^2} \int \frac{x^2\partial x}{X^2}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2}\right) \frac{1}{X} + \left(\frac{3b^2}{a^2} - \frac{2c}{a}\right) \int \frac{\partial x}{xX^2} + \frac{9bc}{2a^2} \int \frac{x\partial x}{X^2}$$

$$\int \frac{\partial x}{x^6X^2} = -\frac{1}{5ax^5X} - \frac{7b}{5a} \int \frac{\partial x}{x^4X^2} - \frac{9c}{5a} \int \frac{\partial x}{x^2X^2}$$

$$\int \frac{\partial x}{x^7X^2} = -\frac{1}{6ax^6X} - \frac{4b}{3a} \int \frac{\partial x}{x^5X^2} - \frac{5c}{3a} \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5}\right) \frac{1}{X} + \left(\frac{9b^2}{5a^2} - \frac{11c}{7a}\right) \int \frac{\partial x}{x^4X^2} + \frac{81bc}{35a^2} \int \frac{\partial x}{x^2X^2}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{5b}{24a^2x^6}\right) \frac{1}{X} + \left(\frac{5b^2}{3a^2} - \frac{3c}{2a}\right) \int \frac{\partial x}{x^5X^2} + \frac{25bc}{12a^2} \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{x^n dx}{a + bx^3 + cx^6}$$

( $b^2 - 4ac$  eine positive GröÙe)

Taf. LIII. a.

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$$a + bx^3 + cx^6 = X$$

$$\frac{1}{2}b - \frac{1}{2}V(b^2 - 4ac) = f, \quad \frac{1}{2}b + \frac{1}{2}V(b^2 - 4ac) = g$$

$$V(b^2 - 4ac) = g - f = h$$


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$$\int \frac{\partial x}{X} = \frac{c}{h} \left[ \int \frac{\partial x}{cx^3 + f} - \int \frac{\partial x}{cx^3 + g} \right]$$

$$\int \frac{x \partial x}{X} = \frac{c}{h} \left[ \int \frac{x \partial x}{cx^3 + f} - \int \frac{x \partial x}{cx^3 + g} \right]$$

$$\int \frac{x^2 \partial x}{X} = \frac{1}{3h} \log \frac{cx^3 + f}{cx^3 + g}$$

$$\int \frac{x^3 \partial x}{X} = \frac{g}{h} \int \frac{\partial x}{cx^3 + g} - \frac{f}{h} \int \frac{\partial x}{cx^3 + f}$$

$$\int \frac{x^4 \partial x}{X} = \frac{g}{h} \int \frac{x \partial x}{cx^3 + g} - \frac{f}{h} \int \frac{x \partial x}{cx^3 + f}$$

$$\int \frac{x^5 \partial x}{X} = \frac{g}{3h} \log (cx^3 + g) - \frac{f}{3h} \log (cx^3 + f)$$

$$\int \frac{x^6 \partial x}{X} = \frac{x}{c} - \frac{a}{c} \int \frac{\partial x}{X} - \frac{b}{c} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^2}{2c} - \frac{a}{c} \int \frac{x \partial x}{X} - \frac{b}{c} \int \frac{x^4 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^3}{3c} - \frac{a}{c} \int \frac{x^2 \partial x}{X} - \frac{b}{c} \int \frac{x^5 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^4}{4c} - \frac{bx}{c^2} + \frac{ab}{c^2} \int \frac{\partial x}{X} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^{10} \partial x}{X} = \frac{x^5}{5c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2} \int \frac{x \partial x}{X} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^4 \partial x}{X}$$

Taf. LIII. b.

$$\int \frac{x^m dx}{a + bx^3 + cx^6}$$

(b<sup>2</sup> - 4ac eine negative GröÙe)

$$\text{VZ. } a + bx^3 + cx^6 = X, \quad \sqrt[6]{\frac{a}{c}} = f$$

$$\alpha \text{ ein Winkel, dessen Cosinus} = -\frac{b}{2\sqrt{ac}}$$

$$\frac{\alpha}{3} = \phi', \quad 120^\circ + \frac{\alpha}{3} = \phi'', \quad 240^\circ + \frac{\alpha}{3} = \phi'''$$

$$x^2 - 2fx \cos \phi' + f^2 = Y'$$

$$x^2 - 2fx \cos \phi'' + f^2 = Y''$$

$$x^2 - 2fx \cos \phi''' + f^2 = Y'''$$

$$\frac{x \sin \phi'}{f - x \cos \phi'} = Z', \quad \frac{x \sin \phi''}{f - x \cos \phi''} = Z'',$$

$$\frac{x \sin \phi'''}{f - x \cos \phi'''} = Z'''$$

$$\int \frac{dx}{X} = \frac{1}{6cf^3 \sin \alpha} \left\{ \begin{array}{l} - \sin 2\phi' \log Y' + 2 \cos 2\phi' \text{Arc Tang } Z' \\ - \sin 2\phi'' \log Y'' + 2 \cos 2\phi'' \text{Arc Tang } Z'' \\ - \sin 2\phi''' \log Y''' + 2 \cos 2\phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x dx}{X} = \frac{1}{6cf^2 \sin \alpha} \left\{ \begin{array}{l} - \sin \phi' \log Y' + 2 \cos \phi' \text{Arc Tang } Z' \\ - \sin \phi'' \log Y'' + 2 \cos \phi'' \text{Arc Tang } Z'' \\ - \sin \phi''' \log Y''' + 2 \cos \phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^2 dx}{X} = \frac{1}{3cf \sin \alpha} \text{Arc Tang } \frac{x^3 \sin \alpha}{f^3 - x^3 \cos \alpha}$$

$$\int \frac{x^3 dx}{X} = \frac{1}{6cf^2 \sin \alpha} \left\{ \begin{array}{l} \sin \phi' \log Y' + 2 \cos \phi' \text{Arc Tang } Z' \\ + \sin \phi'' \log Y'' + 2 \cos \phi'' \text{Arc Tang } Z'' \\ + \sin \phi''' \log Y''' + 2 \cos \phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^4 dx}{X} = \frac{1}{6cf \sin \alpha} \left\{ \begin{array}{l} \sin 2\phi' \log Y' + 2 \cos 2\phi' \text{Arc Tang } Z' \\ + \sin 2\phi'' \log Y'' + 2 \cos 2\phi'' \text{Arc Tang } Z'' \\ + \sin 2\phi''' \log Y''' + 2 \cos 2\phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^5 dx}{X} = \frac{1}{6c \sin \alpha} \left\{ \begin{array}{l} \sin 3\phi' \log Y' + 2 \cos 3\phi' \text{Arc Tang } Z' \\ + \sin 3\phi'' \log Y'' + 2 \cos 3\phi'' \text{Arc Tang } Z'' \\ + \sin 3\phi''' \log Y''' + 2 \cos 3\phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^m dx}{(a+bx^3+cx^6)^2}, \int \frac{dx}{x^m(a+bx^3+cx^6)} \quad \text{Taf. LIV.}$$

$$\text{VL. } a+bx^3+cx^6=X, \quad 3a(b^2-4ac)=k$$

$$\int \frac{dx}{X^2} = [bcx^4 + (b^2 - 2ac)x] \frac{1}{kX} + \frac{2b^2 - 10ac}{k} \int \frac{dx}{X} + \frac{2bc}{k} \int \frac{x^3 dx}{X}$$

$$\int \frac{x dx}{X^2} = [bcx^5 + (b^2 - 2ac)x^2] \frac{1}{kX} + \frac{b^2 - 8ac}{k} \int \frac{x dx}{X} + \frac{bc}{k} \int \frac{x^4 dx}{X}$$

$$\int \frac{x^2 dx}{X^2} = [bcx^6 + (b^2 - 2ac)x^3] \frac{1}{kX} - \frac{6ac}{k} \int \frac{x^2 dx}{X}$$

$$\int \frac{x^3 dx}{X^2} = [bcx^7 + (b^2 - 2ac)x^4] \frac{1}{kX} - \frac{b^2 + 4ac}{k} \int \frac{x^3 dx}{X} - \frac{bc}{k} \int \frac{x^6 dx}{X}$$

$$\int \frac{dx}{xX} = \frac{\log x}{a} - \frac{b}{a} \int \frac{x^2 dx}{X} - \frac{c}{a} \int \frac{x^5 dx}{X}$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x dx}{X} - \frac{c}{a} \int \frac{x^4 dx}{X}$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{X} - \frac{c}{a} \int \frac{x^2 dx}{X}$$

$$\int \frac{dx}{x^4 X} = -\frac{1}{3ax^3} - \frac{b}{a} \int \frac{dx}{xX} - \frac{c}{a} \int \frac{x^2 dx}{X}$$

$$\int \frac{dx}{x^5 X} = -\frac{1}{4ax^4} + \frac{b}{a^2 x} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{x dx}{X} + \frac{bc}{a^2} \int \frac{x^4 dx}{X}$$

$$\int \frac{dx}{x^6 X} = -\frac{1}{5ax^5} + \frac{b}{2a^2 x^2} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{dx}{X} + \frac{bc}{a^2} \int \frac{x^3 dx}{X}$$

$$\int \frac{dx}{x^7 X} = -\frac{1}{6ax^6} + \frac{b}{3a^2 x^3} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{dx}{xX} + \frac{bc}{a^2} \int \frac{x^2 dx}{X}$$

$$\int \frac{dx}{x^8 X} = -\frac{1}{7ax^7} + \frac{b}{4a^2 x^4} - \left(\frac{b^2}{a^2} - \frac{c}{a^2}\right) \frac{1}{x} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right) \int \frac{x dx}{X} - \left(\frac{b^2 c}{a^3} - \frac{c^2}{a^2}\right) \int \frac{x^4 dx}{X}$$



Taf. LV.

$$\int \frac{x^n dx}{X}$$

(X ein Product von binomischen und trinomischen Factoren.)

$$\int \frac{dx}{(x+f)(x+g)} = \frac{1}{g-f} \log \frac{x+f}{x+g}$$

$$\int \frac{x dx}{(x+f)(x+g)} = \frac{1}{g-f} [g \log(x+g) - f \log(x+f)]$$

$$\int \frac{dx}{(x+f)(x+g)^2} = \frac{1}{(g-f)(x+g)} + \frac{1}{(g-f)^2} \log \frac{x+f}{x+g}$$

$$\int \frac{x dx}{(x+f)(x+g)^2} = \frac{-g}{(g-f)(x+g)} - \frac{f}{(g-f)^2} \log \frac{x+f}{x+g}$$

$$\int \frac{x^2 dx}{(x+f)(x+g)^2} = \frac{g^2}{(g-f)(x+g)} + \frac{f^2}{(g-f)^2} \log(x+f) + \frac{g^2 - 2fg}{(g-f)^2} \log(x+g)$$

$$\int \frac{dx}{(x+f)^2(x+g)^2} = \frac{-1}{(g-f)^2} \left( \frac{1}{x+f} + \frac{1}{x+g} \right) - \frac{2}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x dx}{(x+f)^2(x+g)^2} = \frac{1}{(g-f)^2} \left( \frac{f}{x+f} + \frac{g}{x+g} \right) + \frac{f+g}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x^2 dx}{(x+f)^2(x+g)^2} = \frac{-1}{(g-f)^2} \left( \frac{f^2}{x+f} + \frac{g^2}{x+g} \right) - \frac{2fg}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x^3 dx}{(x+f)^2(x+g)^2} = \frac{1}{(g-f)^2} \left( \frac{f^3}{x+f} + \frac{g^3}{x+g} \right) + \frac{f^2(3g-f)}{(g-f)^3} \log(x+f) + \frac{g^2(g-3f)}{(g-f)^3} \log(x+g)$$

$$\int \frac{dx}{(x+f)(x+g)(x+h)} = \frac{1}{(g-f)(h-f)} \log(x+f) + \frac{1}{(f-g)(h-g)} \log(x+g) + \frac{1}{(f-h)(g-h)} \log(x+h)$$

$$\int \frac{x dx}{(x+f)(x+g)(x+h)} = -\frac{f}{(g-f)(h-g)} \log(x+f) - \frac{g}{(f-g)(h-g)} \log(x+g) - \frac{h}{(f-h)(g-h)} \log(x+h)$$

$$\int \frac{x^2 dx}{X}$$

Taf. LV.

(X ein Product von binomischen und trinomischen Factoren.)

$$\int \frac{x^2 dx}{(x+f)(x+g)(x+h)} = \frac{f^2}{(g-f)(h-f)} \log(x+f) + \frac{g^2}{(f-g)(h-g)} \log(x+g) + \frac{h^2}{(f-h)(g-h)} \log(x+h)$$

$$\int \frac{dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a} \left[ \log \frac{x+f}{\sqrt{x^2+a}} + f \int \frac{dx}{x^2+a} \right]$$

$$\int \frac{x dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a} \left[ f \log \frac{\sqrt{x^2+a}}{x+f} + a \int \frac{dx}{x^2+a} \right]$$

$$\int \frac{x^2 dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a} \left[ f^2 \log(x+f) + \frac{1}{2} a \log(x^2+a) \right] - \frac{af}{f^2+a} \int \frac{dx}{x^2+a}$$

$$\int \frac{dx}{(x^2+a)(x^2+b)} = \frac{1}{b-a} \left[ \int \frac{dx}{x^2+a} - \int \frac{dx}{x^2+b} \right]$$

$$\int \frac{x dx}{(x^2+a)(x^2+b)} = \frac{1}{2(b-a)} \log \frac{x^2+a}{x^2+b}$$

$$\int \frac{x^2 dx}{(x^2+a)(x^2+b)} = \frac{1}{a-b} \left[ a \int \frac{dx}{x^2+a} - b \int \frac{dx}{x^2+b} \right]$$

$$\int \frac{dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2} \left[ f \log \frac{(x+f)^2}{x^2+a} + (f^2-a) \int \frac{dx}{x^2+a} \right] - \frac{1}{(f^2+a)(x+f)}$$

$$\int \frac{x dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2} \left[ \frac{a-f^2}{2} \log \frac{(x+f)^2}{x^2+a} + 2af \int \frac{dx}{x^2+a} \right] + \frac{f}{(f^2+a)(x+f)}$$

$$\int \frac{x^2 dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2} \left[ -af \log \frac{(x+f)^2}{x^2+a} - a(f^2-a) \int \frac{dx}{x^2+a} \right] - \frac{f^2}{(f^2+a)(x+f)}$$

Taf. LV.

$$\int \frac{x^m dx}{X}$$

(X ein Product von binomischen und trinomischen Factoren)

$$\int \frac{x^3 dx}{(x+f)^2(x^2+a)} = \frac{f^2(f^2+3a)}{(f^2+a)^2} \log(x+f) - \frac{a(f^2-a)}{2(f^2+a)^2} \log(x^2+a) \\ - \frac{2a^2 f}{(f^2+a)^2} \int \frac{dx}{x^2+a} + \frac{f^3}{(f^2+a)(x+f)}$$

$$\int \frac{dx}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times \\ \left[ \frac{1}{2} \log \frac{(x+f)^2}{x^2+ax+b} + (f-\frac{1}{2}a) \int \frac{dx}{x^2+ax+b} \right]$$

$$\int \frac{x dx}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times \\ \left[ -\frac{1}{2} f \log \frac{(x+f)^2}{x^2+ax+b} + (b-\frac{1}{2}af) \int \frac{dx}{x^2+ax+b} \right]$$

$$\int \frac{x^2 dx}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times \\ \left[ f^2 \log(x+f) + \frac{1}{2}(b-af) \log(x^2+ax+b) \right. \\ \left. + \frac{1}{2}(a^2 f - ab - 2bf) \int \frac{dx}{x^2+ax+b} \right]$$

T a f e l  
einiger allgemeineren Formeln.

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$$\text{VL: } a + bx = X$$


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$$\int \frac{\partial x}{X} = -\frac{1}{(n-1)bX^{n-1}}$$

$$\int \frac{x^n \partial x}{X} = \frac{x^n}{mb} - \frac{a}{b} \int \frac{x^{n-1} \partial x}{X}$$

$$\int \frac{x^n \partial x}{X} = \frac{x^n}{mb} - \frac{ax^{n-1}}{(m-1)b^2} + \frac{a^2 x^{n-2}}{(m-2)b^3} - \frac{a^3 x^{n-3}}{(m-3)b^4} + \text{etc.}$$

$$+ \frac{a^{i-1} x^{n-i+1}}{(m-i+1)b^i} + \frac{a^i}{b^i} \int \frac{x^{n-i} \partial x}{X}$$

$$\int \frac{x^n \partial x}{X} = \frac{x^n}{mb} - \frac{ax^{n-1}}{(m-1)b^2} + \frac{a^2 x^{n-2}}{(m-2)b^3} - \frac{a^3 x^{n-3}}{(m-3)b^4} + \text{etc.}$$

$$+ \frac{a^{n-1} x}{b^n} + \frac{a^n}{b^{n+1}} \log X$$

$$\int \frac{x^n \partial x}{X^2} = \frac{x^n}{(m-1)bX} - \frac{ma}{(m-1)b} \int \frac{x^{n-1} \partial x}{X^2}$$

$$\int \frac{x^n \partial x}{X^2} = (Ax^n - Bx^{n-1} + Cx^{n-2} - Dx^{n-3} + Ex^{n-4} - \text{etc.})$$

$$+ Kx^{n-i+2} + Lx^{n-i+1} \frac{1}{X} + L(m-i+1)a \int \frac{x^{n-i} \partial x}{X^2}$$

$$A = \frac{1}{(m-1)b}, B = \frac{ma}{(m-2)b}, C = \frac{(m-1)a}{(m-3)b},$$

$$D = \frac{(m-2)a}{(m-4)b}, E = \frac{(m-3)a}{(m-5)b}, \text{etc.}, L = \frac{(m-i+2)a}{(m-i)b} K.$$

$$\int \frac{x^n \partial x}{X^3} = \frac{x^n}{(m-2)bX^2} - \frac{ma}{(m-2)b} \int \frac{x^{n-1} \partial x}{X^3}$$

$$\int \frac{x^n \partial x}{X^3} = (Ax^n - Bx^{n-1} + Cx^{n-2} - Dx^{n-3} + Ex^{n-4} - \text{etc.})$$

$$+ Kx^{n-i+2} + Lx^{n-i+1} \frac{1}{X^2} + L(m-i+1)a \int \frac{x^{n-i} \partial x}{X^3}$$

Taf. LII.

$$\int \frac{\partial x}{x^m(a + bx^2 + cx^4)^2}$$

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$$\text{VZ. } a + bx^2 + cx^4 = X, \quad 2a(b^2 - 4ac) = k$$


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$$\int \frac{\partial x}{xX^2} = \frac{bcx^2 + b^2 - 2ac}{kX} + \frac{\log x}{a^2} + \left(\frac{2bc}{k} - \frac{b}{a^2}\right) \int \frac{x\partial x}{X} - \frac{bc}{a^2} \int \frac{x^3\partial x}{X}$$

$$\int \frac{\partial x}{x^2X^2} = -\frac{1}{axX} - \frac{3b}{a} \int \frac{\partial x}{X^2} - \frac{5c}{a} \int \frac{x^2\partial x}{X^2}$$

$$\int \frac{\partial x}{x^3X^2} = -\frac{1}{2ax^2X} - \frac{2b}{a} \int \frac{\partial x}{xX^2} - \frac{3c}{a} \int \frac{x\partial x}{X^2}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x}\right) \frac{1}{X} + \left(\frac{5b^2}{a^2} - \frac{7c}{3a}\right) \int \frac{\partial x}{X^2} + \frac{25c^2}{3a^2} \int \frac{x^3\partial x}{X^2}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2}\right) \frac{1}{X} + \left(\frac{3b^2}{a^2} - \frac{2c}{a}\right) \int \frac{\partial x}{xX^2} + \frac{9bc}{2a^2} \int \frac{x\partial x}{X^2}$$

$$\int \frac{\partial x}{x^6X^2} = -\frac{1}{5ax^5X} - \frac{7b}{5a} \int \frac{\partial x}{x^4X^2} - \frac{9c}{5a} \int \frac{\partial x}{x^2X^2}$$

$$\int \frac{\partial x}{x^7X^2} = -\frac{1}{6ax^6X} - \frac{4b}{3a} \int \frac{\partial x}{x^5X^2} - \frac{5c}{3a} \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5}\right) \frac{1}{X} + \left(\frac{9b^2}{5a^2} - \frac{11c}{7a}\right) \int \frac{\partial x}{x^4X^2} + \frac{81bc}{35a^2} \int \frac{\partial x}{x^2X^2}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{5b}{24a^2x^6}\right) \frac{1}{X} + \left(\frac{5b^2}{3a^2} - \frac{3c}{2a}\right) \int \frac{\partial x}{x^5X^2} + \frac{25bc}{12a^2} \int \frac{\partial x}{x^3X^2}$$

$$\int \frac{x^m \partial x}{a + bx^3 + cx^6}$$

Taf. LIII. a.

 ( $b^2 - 4ac$  eine positive GröÙe)

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$$\begin{aligned} a + bx^3 + cx^6 &= X \\ \frac{1}{2}b - \frac{1}{2}\sqrt{b^2 - 4ac} &= f, \quad \frac{1}{2}b + \frac{1}{2}\sqrt{b^2 - 4ac} = g \\ \sqrt{b^2 - 4ac} &= g - f = h \end{aligned}$$


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$$\int \frac{\partial x}{X} = \frac{c}{h} \left[ \int \frac{\partial x}{cx^3 + f} - \int \frac{\partial x}{cx^3 + g} \right]$$

$$\int \frac{x \partial x}{X} = \frac{c}{h} \left[ \int \frac{x \partial x}{cx^3 + f} - \int \frac{x \partial x}{cx^3 + g} \right]$$

$$\int \frac{x^2 \partial x}{X} = \frac{1}{3h} \log \frac{cx^3 + f}{cx^3 + g}$$

$$\int \frac{x^3 \partial x}{X} = \frac{g}{h} \int \frac{\partial x}{cx^3 + g} - \frac{f}{h} \int \frac{\partial x}{cx^3 + f}$$

$$\int \frac{x^4 \partial x}{X} = \frac{g}{h} \int \frac{x \partial x}{cx^3 + g} - \frac{f}{h} \int \frac{x \partial x}{cx^3 + f}$$

$$\int \frac{x^5 \partial x}{X} = \frac{g}{3h} \log (cx^3 + g) - \frac{f}{3h} \log (cx^3 + f)$$

$$\int \frac{x^6 \partial x}{X} = \frac{x}{c} - \frac{a}{c} \int \frac{\partial x}{X} - \frac{b}{c} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^2}{2c} - \frac{a}{c} \int \frac{x \partial x}{X} - \frac{b}{c} \int \frac{x^4 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^3}{3c} - \frac{a}{c} \int \frac{x^2 \partial x}{X} - \frac{b}{c} \int \frac{x^5 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^4}{4c} - \frac{bx}{c^2} + \frac{ab}{c^2} \int \frac{\partial x}{X} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^{10} \partial x}{X} = \frac{x^5}{5c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2} \int \frac{x \partial x}{X} + \left( \frac{b^2}{c^2} - \frac{a}{c} \right) \int \frac{x^4 \partial x}{X}$$

Taf. LIII. b.

$$\int \frac{x^m \partial x}{a + bx^3 + cx^6}$$

(b<sup>2</sup> - 4ac eine negative GröÙe)

$$\text{VZ. } a + bx^3 + cx^6 = X, \quad \sqrt[6]{\frac{a}{c}} = f$$

$$\alpha \text{ ein Winkel, dessen Cosinus} = -\frac{b}{2\sqrt{ac}}$$

$$\frac{\alpha}{3} = \phi', \quad 120^\circ + \frac{\alpha}{3} = \phi'', \quad 240^\circ + \frac{\alpha}{3} = \phi'''$$

$$x^2 - 2fx \cos \phi' + f^2 = Y'$$

$$x^2 - 2fx \cos \phi'' + f^2 = Y''$$

$$x^2 - 2fx \cos \phi''' + f^2 = Y'''$$

$$\frac{x \sin \phi'}{f - x \cos \phi'} = Z', \quad \frac{x \sin \phi''}{f - x \cos \phi''} = Z'',$$

$$\frac{x \sin \phi'''}{f - x \cos \phi'''} = Z'''$$

$$\int \frac{\partial x}{X} = \frac{1}{6cf^5 \sin \alpha} \left\{ \begin{array}{l} - \sin 2\phi' \log Y' + 2 \cos 2\phi' \text{Arc Tang } Z' \\ - \sin 2\phi'' \log Y'' + 2 \cos 2\phi'' \text{Arc Tang } Z'' \\ - \sin 2\phi''' \log Y''' + 2 \cos 2\phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x \partial x}{X} = \frac{1}{6cf^4 \sin \alpha} \left\{ \begin{array}{l} - \sin \phi' \log Y' + 2 \cos \phi' \text{Arc Tang } Z' \\ - \sin \phi'' \log Y'' + 2 \cos \phi'' \text{Arc Tang } Z'' \\ - \sin \phi''' \log Y''' + 2 \cos \phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^2 \partial x}{X} = \frac{1}{3cf^3 \sin \alpha} \text{Arc Tang } \frac{x^3 \sin \alpha}{f^3 - x^3 \cos \alpha}$$

$$\int \frac{x^3 \partial x}{X} = \frac{1}{6cf^2 \sin \alpha} \left\{ \begin{array}{l} \sin \phi' \log Y' + 2 \cos \phi' \text{Arc Tang } Z' \\ + \sin \phi'' \log Y'' + 2 \cos \phi'' \text{Arc Tang } Z'' \\ + \sin \phi''' \log Y''' + 2 \cos \phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^4 \partial x}{X} = \frac{1}{6cf \sin \alpha} \left\{ \begin{array}{l} \sin 2\phi' \log Y' + 2 \cos 2\phi' \text{Arc Tang } Z' \\ + \sin 2\phi'' \log Y'' + 2 \cos 2\phi'' \text{Arc Tang } Z'' \\ + \sin 2\phi''' \log Y''' + 2 \cos 2\phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^5 \partial x}{X} = \frac{1}{6c \sin \alpha} \left\{ \begin{array}{l} \sin 3\phi' \log Y' + 2 \cos 3\phi' \text{Arc Tang } Z' \\ + \sin 3\phi'' \log Y'' + 2 \cos 3\phi'' \text{Arc Tang } Z'' \\ + \sin 3\phi''' \log Y''' + 2 \cos 3\phi''' \text{Arc Tang } Z''' \end{array} \right\}$$

$$\int \frac{x^m dx}{(a+bx^3+cx^6)^2}, \int \frac{\partial x}{x^m(a+bx^3+cx^6)} \quad \text{Taf. LIV.}$$

$$\text{VZ. } a+bx^3+cx^6=X, \quad 3a(b^2-4ac)=k$$

$$\int \frac{\partial x}{X^2} = [bcx^4 + (b^2 - 2ac)x] \frac{1}{kX} + \frac{2b^2 - 10ac}{k} \int \frac{\partial x}{X} + \frac{2bc}{k} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x \partial x}{X^2} = [bcx^5 + (b^2 - 2ac)x^2] \frac{1}{kX} + \frac{b^2 - 8ac}{k} \int \frac{x \partial x}{X} + \frac{bc}{k} \int \frac{x^4 \partial x}{X}$$

$$\int \frac{x^2 \partial x}{X^2} = [bcx^6 + (b^2 - 2ac)x^3] \frac{1}{kX} - \frac{6ac}{k} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^3 \partial x}{X^2} = [bcx^7 + (b^2 - 2ac)x^4] \frac{1}{kX} - \frac{b^2 + 4ac}{k} \int \frac{x^3 \partial x}{X} - \frac{bc}{k} \int \frac{x^6 \partial x}{X}$$

$$\int \frac{\partial x}{xX} = \frac{\log x}{a} - \frac{b}{a} \int \frac{x^2 \partial x}{X} - \frac{c}{a} \int \frac{x^5 \partial x}{X}$$

$$\int \frac{\partial x}{x^2 X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x \partial x}{X} - \frac{c}{a} \int \frac{x^4 \partial x}{X}$$

$$\int \frac{\partial x}{x^3 X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{\partial x}{X} - \frac{c}{a} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{\partial x}{x^4 X} = -\frac{1}{3ax^3} - \frac{b}{a} \int \frac{\partial x}{xX} - \frac{c}{a} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^5 X} = -\frac{1}{4ax^4} + \frac{b}{a^2 x} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{x \partial x}{X} + \frac{bc}{a^2} \int \frac{x^4 \partial x}{X}$$

$$\int \frac{\partial x}{x^6 X} = -\frac{1}{5ax^5} + \frac{b}{2a^2 x^2} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{\partial x}{X} + \frac{bc}{a^2} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{\partial x}{x^7 X} = -\frac{1}{6ax^6} + \frac{b}{3a^2 x^3} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{\partial x}{xX} + \frac{bc}{a^2} \int \frac{x^2 \partial x}{X}$$

$$\int \frac{\partial x}{x^8 X} = -\frac{1}{7ax^7} + \frac{b}{4a^2 x^4} - \left(\frac{b^2}{a^2} - \frac{c}{a^2}\right) \frac{1}{x} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right) \int \frac{x \partial x}{X} - \left(\frac{b^2 c}{a^3} - \frac{c^2}{a^2}\right) \int \frac{x^4 \partial x}{X}$$



Taf. LV.

$$\int \frac{x^n \partial x}{X}$$

(X ein Product von binomischen und trinomischen Factoren.)

$$\int \frac{\partial x}{(x+f)(x+g)} = \frac{1}{g-f} \log \frac{x+f}{x+g}$$

$$\int \frac{x \partial x}{(x+f)(x+g)} = \frac{1}{g-f} [g \log(x+g) - f \log(x+f)]$$

$$\int \frac{\partial x}{(x+f)(x+g)^2} = \frac{1}{(g-f)(x+g)} + \frac{1}{(g-f)^2} \log \frac{x+f}{x+g}$$

$$\int \frac{x \partial x}{(x+f)(x+g)^2} = \frac{-g}{(g-f)(x+g)} - \frac{f}{(g-f)^2} \log \frac{x+f}{x+g}$$

$$\int \frac{x^2 \partial x}{(x+f)(x+g)^2} = \frac{g^2}{(g-f)(x+g)} + \frac{f^2}{(g-f)^2} \log(x+f) \\ + \frac{g^2 - 2fg}{(g-f)^2} \log(x+g)$$

$$\int \frac{\partial x}{(x+f)^2(x+g)^2} = \frac{-1}{(g-f)^2} \left( \frac{1}{x+f} + \frac{1}{x+g} \right) - \frac{2}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x \partial x}{(x+f)^2(x+g)^2} = \frac{1}{(g-f)^2} \left( \frac{f}{x+f} + \frac{g}{x+g} \right) + \frac{f+g}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x^2 \partial x}{(x+f)^2(x+g)^2} = \frac{-1}{(g-f)^2} \left( \frac{f^2}{x+f} + \frac{g^2}{x+g} \right) - \frac{2fg}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x^3 \partial x}{(x+f)^2(x+g)^2} = \frac{1}{(g-f)^2} \left( \frac{f^3}{x+f} + \frac{g^3}{x+g} \right) + \frac{f^2(3g-f)}{(g-f)^3} \log(x+f) \\ + \frac{g^2(g-3f)}{(g-f)^3} \log(x+g)$$

$$\int \frac{\partial x}{(x+f)(x+g)(x+h)} = \frac{1}{(g-f)(h-f)} \log(x+f) \\ + \frac{1}{(f-g)(h-g)} \log(x+g) + \frac{1}{(f-h)(g-h)} \log(x+h)$$

$$\int \frac{x \partial x}{(x+f)(x+g)(x+h)} = -\frac{f}{(g-f)(h-g)} \log(x+f) \\ - \frac{g}{(f-g)(h-g)} \log(x+g) - \frac{h}{(f-h)(g-h)} \log(x+h)$$

$$\int \frac{x^m dx}{X}$$

Taf. LV.

(X ein Product von binomischen und trinomischen Factoren.)

$$\int \frac{x^2 dx}{(x+f)(x+g)(x+h)} = \frac{f^2}{(g-f)(h-f)} \log(x+f) + \frac{g^2}{(f-g)(h-g)} \log(x+g) + \frac{h^2}{(f-h)(g-h)} \log(x+h)$$

$$\int \frac{dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a} \left[ \log \frac{x+f}{\sqrt{x^2+a}} + f \int \frac{dx}{x^2+a} \right]$$

$$\int \frac{x dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a} \left[ f \log \frac{\sqrt{x^2+a}}{x+f} + a \int \frac{dx}{x^2+a} \right]$$

$$\int \frac{x^2 dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a} \left[ f^2 \log(x+f) + \frac{1}{2} a \log(x^2+a) \right] - \frac{af}{f^2+a} \int \frac{dx}{x^2+a}$$

$$\int \frac{dx}{(x^2+a)(x^2+b)} = \frac{1}{b-a} \left[ \int \frac{dx}{x^2+a} - \int \frac{dx}{x^2+b} \right]$$

$$\int \frac{x dx}{(x^2+a)(x^2+b)} = \frac{1}{2(b-a)} \log \frac{x^2+a}{x^2+b}$$

$$\int \frac{x^2 dx}{(x^2+a)(x^2+b)} = \frac{1}{a-b} \left[ a \int \frac{dx}{x^2+a} - b \int \frac{dx}{x^2+b} \right]$$

$$\int \frac{dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2} \left[ f \log \frac{(x+f)^2}{x^2+a} + (f^2-a) \int \frac{dx}{x^2+a} \right] - \frac{1}{(f^2+a)(x+f)}$$

$$\int \frac{x dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2} \left[ \frac{a-f^2}{2} \log \frac{(x+f)^2}{x^2+a} + 2af \int \frac{dx}{x^2+a} \right] + \frac{f}{(f^2+a)(x+f)}$$

$$\int \frac{x^2 dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2} \left[ -af \log \frac{(x+f)^2}{x^2+a} - a(f^2-a) \int \frac{dx}{x^2+a} \right] - \frac{f^2}{(f^2+a)(x+f)}$$

Taf. LV.

$$\int \frac{x^n dx}{X}$$

(X ein Product von binomischen und trinomischen Factoren)

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$$\int \frac{x^3 dx}{(x+f)^2(x^2+a)} = \frac{f^2(f^2+3a)}{(f^2+a)^2} \log(x+f) - \frac{a(f^2-a)}{2(f^2+a)^2} \log(x^2+a) \\ - \frac{2a^2f}{(f^2+a)^2} \int \frac{dx}{x^2+a} + \frac{f^3}{(f^2+a)(x+f)}$$

$$\int \frac{dx}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times \\ \left[ \frac{1}{2} \log \frac{(x+f)^2}{x^2+ax+b} + (f-\frac{1}{2}a) \int \frac{dx}{x^2+ax+b} \right]$$

$$\int \frac{x dx}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times \\ \left[ -\frac{1}{2}f \log \frac{(x+f)^2}{x^2+ax+b} + (b-\frac{1}{2}af) \int \frac{dx}{x^2+ax+b} \right]$$

$$\int \frac{x^2 dx}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times \\ \left[ f^2 \log(x+f) + \frac{1}{2}(b-af) \log(x^2+ax+b) \right. \\ \left. + \frac{1}{2}(a^2f-ab-2bf) \int \frac{dx}{x^2+ax+b} \right]$$


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## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a + bx = X$$

$$\int \frac{\partial x}{X^n} = -\frac{1}{(n-1)bX^{n-1}}$$

$$\int \frac{x^m \partial x}{X} = \frac{x^m}{mb} - \frac{a}{b} \int \frac{x^{m-1} \partial x}{X}$$

$$\begin{aligned} \int \frac{x^m \partial x}{X} &= \frac{x^m}{mb} - \frac{ax^{m-1}}{(m-1)b^2} + \frac{a^2 x^{m-2}}{(m-2)b^3} - \frac{a^3 x^{m-3}}{(m-3)b^4} + \text{etc.} \\ &\quad \pm \frac{a^{i-1} x^{m-i+1}}{(m-i+1)b^i} + \frac{a^i}{b^i} \int \frac{x^{m-i} \partial x}{X} \end{aligned}$$

$$\begin{aligned} \int \frac{x^m \partial x}{X} &= \frac{x^m}{mb} - \frac{ax^{m-1}}{(m-1)b^2} + \frac{a^2 x^{m-2}}{(m-2)b^3} - \frac{a^3 x^{m-3}}{(m-3)b^4} + \text{etc.} \\ &\quad \pm \frac{a^{m-1} x}{b^m} + \frac{a^m}{b^{m+1}} \log X \end{aligned}$$

$$\int \frac{x^m \partial x}{X^2} = \frac{x^m}{(m-1)bX} - \frac{ma}{(m-1)b} \int \frac{x^{m-1} \partial x}{X^2}$$

$$\begin{aligned} \int \frac{x^m \partial x}{X^2} &= (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.} \\ &\quad \pm Kx^{m-i+2} \mp Lx^{m-i+1}) \frac{1}{X^2} \pm L(m-i+1)a \int \frac{x^{m-i} \partial x}{X^2} \end{aligned}$$

$$A = \frac{1}{(m-1)b}, B = \frac{ma}{(m-2)b}, C = \frac{(m-1)a}{(m-3)b},$$

$$D = \frac{(m-2)a}{(m-4)b}, E = \frac{(m-3)a}{(m-5)b}, \text{ etc. }, L = \frac{(m-i+2)a}{(m-i)b}, K.$$

$$\int \frac{x^m \partial x}{X^3} = \frac{x^m}{(m-2)bX^2} - \frac{ma}{(m-2)b} \int \frac{x^{m-1} \partial x}{X^3}$$

$$\begin{aligned} \int \frac{x^m \partial x}{X^3} &= (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.} \\ &\quad \pm Kx^{m-i+2} \mp Lx^{m-i+1}) \frac{1}{X^3} \pm L(m-i+1)a \int \frac{x^{m-i} \partial x}{X^3} \end{aligned}$$

T a f e l  
einiger allgemeineren Formeln.

$$\text{VZ. } a + bx = X$$

$$A = \frac{1}{(m-2)b}, B = \frac{ma}{(m-3)b} A, C = \frac{(m-1)a}{(m-4)b} B, \\ D = \frac{(m-2)a}{(m-5)b} C, E = \frac{(m-3)a}{(m-6)b} D, \text{ etc.}, L = \frac{(m-i+2)a}{(m-i-1)b} K.$$

$$\int \frac{x^m \partial x}{X^4} = \frac{x^m}{(m-3)bX^3} - \frac{ma}{(m-3)b} \int \frac{x^{m-1} \partial x}{X^4}$$

$$\int \frac{x^m \partial x}{X^4} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.} \\ \pm Kx^{m-i+2} \mp Lx^{m-i+1}) \frac{1}{X^3} \pm L(m-i+1)a \int \frac{x^{m-1} \partial x}{X^4}$$

$$A = \frac{1}{(m-3)b}, B = \frac{ma}{(m-4)b} A, C = \frac{(m-1)a}{(m-5)b} B, \\ D = \frac{(m-2)a}{(m-6)b} C, E = \frac{(m-3)a}{(m-7)b} D, \text{ etc.}, L = \frac{(m-i+2)a}{(m-i-2)b} K.$$

$$\int \frac{x^m \partial x}{X^5} = \frac{x^m}{(m-4)bX^4} - \frac{ma}{(m-4)b} \int \frac{x^{m-1} \partial x}{X^5}$$

$$\int \frac{x^m \partial x}{X^5} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} + \text{etc.} \\ \pm Kx^{m-i+2} \mp Lx^{m-i+1}) \frac{1}{X^4} \pm L(m-i+1)a \int \frac{x^{m-1} \partial x}{X^5}$$

$$A = \frac{1}{(m-4)b}, B = \frac{ma}{(m-5)b} A, C = \frac{(m-1)a}{(m-6)b} B, \\ D = \frac{(m-2)a}{(m-7)b} C, E = \frac{(m-3)a}{(m-8)b} D, \text{ etc.}, L = \frac{(m-i+2)a}{(m-i-3)b} K.$$

$$\int \frac{x^m \partial x}{X^6} = \frac{x^m}{(m-5)bX^5} - \frac{ma}{(m-5)b} \int \frac{x^{m-1} \partial x}{X^6}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VL. } a + bx = X$$

$$\int \frac{x^m dx}{X^6} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.} \\ \pm Kx^{m-i+2} \mp Lx^{m-i+1}) \frac{1}{X^5} \pm L(m-i+1)a \int \frac{x^{m-i} dx}{X^5}$$

$$A = \frac{1}{(m-5)b}, B = \frac{ma}{(m-6)b}, C = \frac{(m-1)a}{(m-7)b},$$

$$D = \frac{(m-2)a}{(m-8)b}, E = \frac{(m-3)a}{(m-9)b}, \text{etc.}, L = \frac{(m-i+2)a}{(m-i-4)b} K.$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{\partial x}{x^{m-1} X}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} + \frac{b}{(m-2)a^2 x^{m-2}} - \frac{b^2}{(m-3)a^3 x^{m-3}} \\ + \frac{b^3}{(m-4)a^4 x^{m-4}} - \text{etc.} \pm \frac{b^{i-1}}{(m-i)a^i x^{m-i}} + \frac{b^i}{a^i} \int \frac{\partial x}{x^{m-i} X}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} + \frac{b}{(m-2)a^2 x^{m-2}} - \frac{b^2}{(m-3)a^3 x^{m-3}} \\ + \frac{b^3}{(m-4)a^4 x^{m-4}} - \text{etc.} \pm \frac{b^{m-2}}{a^{m-1}x} + \frac{b^{m-1}}{a^m} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{mb}{(m-1)a} \int \frac{\partial x}{x^{m-1} X^2}$$

$$\int \frac{\partial x}{x^m X^2} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.} \right. \\ \left. \pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}} \right) \frac{1}{X} \mp L(m-i+1)b \int \frac{\partial x}{x^{m-i} X^2}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{mb}{(m-2)a}, C = \frac{(m-1)b}{(m-3)a},$$

$$D = \frac{(m-2)b}{(m-4)a}, E = \frac{(m-3)b}{(m-5)a}, \text{etc.}, L = \frac{(m-i+2)b}{(m-i)a} K.$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+1)b}{(m-1)a} \int \frac{\partial x}{x^{m-1}X^3}$$

$$\int \frac{\partial x}{x^m X^3} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.} \right. \\ \left. \pm \frac{K}{x^{m-i+1}} + \frac{L}{x^{m-i}} \right) \frac{1}{X^2} + L(m-i+2)b \int \frac{\partial x}{x^{m-i}X^3}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+1)b}{(m-2)a} A, \quad C = \frac{mb}{(m-3)a} B,$$

$$D = \frac{(m-1)b}{(m-4)a} C, \quad E = \frac{(m-2)b}{(m-5)a} D, \text{ etc., } L = \frac{(m-i+3)b}{(m-i)a} K.$$

$$\int \frac{\partial x}{x^m X^4} = -\frac{1}{(m-1)ax^{m-1}X^3} - \frac{(m+2)b}{(m-1)a} \int \frac{\partial x}{x^{m-1}X^4}$$

$$\int \frac{\partial x}{x^m X^4} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.} \right. \\ \left. \pm \frac{K}{x^{m-i+1}} + \frac{L}{x^{m-i}} \right) \frac{1}{X^3} + L(m-i+3)b \int \frac{\partial x}{x^{m-i}X^4}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+2)b}{(m-2)a} A, \quad C = \frac{(m+1)b}{(m-3)a} B,$$

$$D = \frac{mb}{(m-4)a} C, \quad E = \frac{(m-1)b}{(m-5)a} D, \text{ etc., } L = \frac{(m-i+4)b}{(m-i)a} K.$$

$$\int \frac{\partial x}{x^m X^5} = -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+3)b}{(m-1)a} \int \frac{\partial x}{x^{m-1}X^5}$$

$$\int \frac{\partial x}{x^m X^5} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.} \right. \\ \left. \pm \frac{K}{x^{m-i+1}} + \frac{L}{x^{m-i}} \right) \frac{1}{X^4} + L(m-i+4)b \int \frac{\partial x}{x^{m-i}X^5}$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } a + bx = X$$


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$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+3)b}{(m-2)a} A, \quad C = \frac{(m+2)b}{(m-3)a} B,$$

$$D = \frac{(m+1)b}{(m-4)a} C, \quad E = \frac{mb}{(m-5)a} D, \text{ etc.}, \quad L = \frac{(m-i+5)b}{(m-i)a} K.$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+4)b}{(m-1)a} \int \frac{\partial x}{x^{m-1}X^2}$$

$$\int \frac{\partial x}{x^m X^2} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.} \right. \\ \left. + \frac{K}{x^{m-i+1}} + \frac{L}{x^{m-i}} \right) \frac{1}{X^2} + L(m-i+5)b \int \frac{\partial x}{x^{m-i}X^2}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+4)b}{(m-2)a} A, \quad C = \frac{(m+3)b}{(m-3)a} B,$$

$$D = \frac{(m+2)b}{(m-4)a} C, \quad E = \frac{(m+1)b}{(m-5)a} D, \text{ etc.}, \quad L = \frac{(m-i+6)b}{(m-i)a} K.$$


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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{\partial x}{X^2} = \frac{x}{(p-1)2aX^{p-1}} + \frac{2p-3}{(p-1)2a} \int \frac{\partial x}{X^{p-1}}$$

$$\int \frac{\partial x}{X^2} = \left( \frac{A}{X^{p-1}} + \frac{B}{X^{p-2}} + \frac{C}{X^{p-3}} + \frac{D}{X^{p-4}} + \frac{E}{X^{p-5}} + \text{etc.} \right. \\ \left. + \frac{K}{X^{p-i+1}} + \frac{L}{X^{p-i}} \right) x + L(2p-2i-1) \int \frac{\partial x}{X^{p-i}}$$

$$A = \frac{1}{(p-1)2a}, \quad B = \frac{2p-3}{(p-2)2a} A, \quad C = \frac{2p-5}{(p-3)2a} B,$$

$$D = \frac{2p-7}{(p-4)2a} C, \quad E = \frac{2p-9}{(p-5)2a} D, \text{ etc.}, \quad L = \frac{2p-2i+1}{(p-i)2a} K.$$



## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{x^m dx}{X} = \frac{x^{m-1}}{(m-1)b} - \frac{a}{b} \int \frac{x^{m-3} dx}{X}$$

$$\int \frac{x^m dx}{X} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.} \\ + Kx^{m-2i+5} + Lx^{m-2i+1}) \frac{1}{X} \pm L(m-2i+1)a \int \frac{x^{m-2i} dx}{X}$$

$$A = \frac{1}{(m-1)b}, B = \frac{(m-1)a}{(m-3)b} A, C = \frac{(m-3)a}{(m-5)b} B,$$

$$D = \frac{(m-5)a}{(m-7)b} C, E = \frac{(m-7)a}{(m-9)b} D, \text{ etc., } L = \frac{(m-2i+3)a}{(m-2i+1)b} K.$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-1} dx}{(m-3)bX} - \frac{(m-1)a}{(m-3)b} \int \frac{x^{m-3} dx}{X^2}$$

$$\int \frac{x^m dx}{X^2} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.} \\ + Kx^{m-2i+5} + Lx^{m-2i+1}) \frac{1}{X^2} \pm L(m-2i+1)a \int \frac{x^{m-2i} dx}{X^2}$$

$$A = \frac{1}{(m-3)b}, B = \frac{(m-1)a}{(m-5)b} A, C = \frac{(m-3)a}{(m-7)b} B,$$

$$D = \frac{(m-5)a}{(m-9)b} C, E = \frac{(m-7)a}{(m-11)b} D, \text{ etc., } L = \frac{(m-2i+3)a}{(m-2i-1)b} K.$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-1}}{(m-5)b} - \frac{(m-1)a}{(m-5)b} \int \frac{x^{m-3} dx}{X^3}$$

$$\int \frac{x^m dx}{X^3} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.} \\ + Kx^{m-2i+5} + Lx^{m-2i+1}) \frac{1}{X^3} \pm L(m-2i+1)a \int \frac{x^{m-2i} dx}{X^3}$$

$$A = \frac{1}{(m-5)b}, B = \frac{(m-1)a}{(m-7)b} A, C = \frac{(m-3)a}{(m-9)b} B,$$

$$D = \frac{(m-5)a}{(m-11)b} C, E = \frac{(m-7)a}{(m-13)b} D, \text{ etc., } L = \frac{(m-2i+3)a}{(m-2i-3)b} K.$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VL. } a + bx^2 = X$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-1}}{(m-7)bX^2} - \frac{(m-1)a}{(m-7)b} \int \frac{x^{m-2} dx}{X^2}$$

$$\int \frac{x^m dx}{X^2} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.} \\ \pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^2} \pm L(m-2i+1)a \int \frac{x^{m-2i} dx}{X^2}$$

$$A = \frac{1}{(m-7)b}, B = \frac{(m-1)a}{(m-9)b} A, C = \frac{(m-3)a}{(m-11)b} B,$$

$$D = \frac{(m-5)a}{(m-13)b} C, E = \frac{(m-7)a}{(m-15)b} D, \text{etc.}, L = \frac{(m-2i+3)a}{(m-2i-5)b} K.$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-1}}{(m-9)bX^3} - \frac{(m-1)a}{(m-9)b} \int \frac{x^{m-2} dx}{X^3}$$

$$\int \frac{x^m dx}{X^3} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.} \\ \pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^3} \pm L(m-2i+1)a \int \frac{x^{m-2i} dx}{X^3}$$

$$A = \frac{1}{(m-9)b}, B = \frac{(m-1)a}{(m-11)b} A, C = \frac{(m-3)a}{(m-13)b} B,$$

$$D = \frac{(m-5)a}{(m-15)b} C, E = \frac{(m-7)a}{(m-17)b} D, \text{etc.}, L = \frac{(m-2i+3)a}{(m-2i-7)b} K.$$

$$\int \frac{x^m dx}{X^6} = \frac{x^{m-1}}{(m-11)bX^6} - \frac{(m-1)a}{(m-11)b} \int \frac{x^{m-2} dx}{X^6}$$

$$\int \frac{x^m dx}{X^6} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.} \\ \pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^6} \pm L(m-2i+1)a \int \frac{x^{m-2i} dx}{X^6}$$

$$A = \frac{1}{(m-11)b}, B = \frac{(m-1)a}{(m-13)b} A, C = \frac{(m-3)a}{(m-15)b} B,$$

$$D = \frac{(m-5)a}{(m-17)b} C, E = \frac{(m-7)a}{(m-19)b} D, \text{etc.}, L = \frac{(m-2i+3)a}{(m-2i-9)b} K.$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{\partial x}{x^{m-2} X}$$

$$\int \frac{\partial x}{x^m X} = \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}$$

$$+ \frac{K}{x^{m-2i+3}} + \frac{L}{x^{m-2i+1}} + L(m-2i+1)b \int \frac{\partial x}{x^{m-2i} X}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m-1)b}{(m-3)a} A, \quad C = \frac{(m-5)b}{(m-5)a} B,$$

$$D = \frac{(m-5)b}{(m-7)a} C, \quad E = \frac{(m-7)b}{(m-9)a} D, \text{ etc.}, \quad L = \frac{(m-2i+5)b}{(m-2i+1)a} K.$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1} X} - \frac{(m+1)b}{(m-1)a} \int \frac{\partial x}{x^{m-2} X^2}$$

$$\int \frac{\partial x}{x^m X^2} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.} \right.$$

$$\left. + \frac{K}{x^{m-2i+3}} + \frac{L}{x^{m-2i+1}} \right) \frac{1}{X} + L(m-2i+3)b \int \frac{\partial x}{x^{m-2i} X^2}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+1)b}{(m-3)a} A, \quad C = \frac{(m-1)b}{(m-5)a} B,$$

$$D = \frac{(m-3)b}{(m-7)a} C, \quad E = \frac{(m-5)b}{(m-9)a} D, \text{ etc.}, \quad L = \frac{(m-2i+5)b}{(m-2i+1)a} K.$$

$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1} X^2} - \frac{(m+3)b}{(m-1)a} \int \frac{\partial x}{x^{m-2} X^3}$$

$$\int \frac{\partial x}{x^m X^3} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.} \right.$$

$$\left. + \frac{K}{x^{m-2i+3}} + \frac{L}{x^{m-2i+1}} \right) \frac{1}{X^2} + L(m-2i+5)b \int \frac{\partial x}{x^{m-2i} X^3}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+3)b}{(m-3)a} A, \quad C = \frac{(m+1)b}{(m-5)a} B,$$

$$D = \frac{(m-1)b}{(m-7)a} C, \quad E = \frac{(m-5)b}{(m-9)a} D, \text{ etc.}, \quad L = \frac{(m-2i+7)b}{(m-2i+1)a} K.$$

T a f e l  
einiger allgemeineren Formeln.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{x^m X^4} = -\frac{1}{(m-1)ax^{m-1}X^3} - \frac{(m+5)b}{(m-1)a} \int \frac{\partial x}{x^{m-2}X^4}$$

$$\int \frac{\partial x}{x^m X^4} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-5}} + \frac{C}{x^{m-9}} - \frac{D}{x^{m-13}} + \frac{E}{x^{m-17}} - \text{etc.} \right.$$

$$\left. + \frac{K}{x^{m-2i+3}} + \frac{L}{x^{m-2i+1}} \right) \frac{1}{X^3} + L(m-2i+7)b \int \frac{\partial x}{x^{m-2i}X^4}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+5)b}{(m-3)a} A, \quad C = \frac{(m+3)b}{(m-5)a} B,$$

$$D = \frac{(m+1)b}{(m-7)a} C, \quad E = \frac{(m-1)b}{(m-9)a} D, \quad \text{etc.}, \quad L = \frac{(m-2i+9)b}{(m-2i+1)a} K.$$

$$\int \frac{\partial x}{x^m X^5} = -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+7)b}{(m-1)a} \int \frac{\partial x}{x^{m-2}X^5}$$

$$\int \frac{\partial x}{x^m X^5} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-5}} + \frac{C}{x^{m-9}} - \frac{D}{x^{m-13}} + \frac{E}{x^{m-17}} - \text{etc.} \right.$$

$$\left. + \frac{K}{x^{m-2i+3}} + \frac{L}{x^{m-2i+1}} \right) \frac{1}{X^4} + L(m-2i+9)b \int \frac{\partial x}{x^{m-2i}X^5}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+7)b}{(m-3)a} A, \quad C = \frac{(m+5)b}{(m-5)a} B,$$

$$D = \frac{(m+3)b}{(m-7)a} C, \quad E = \frac{(m+1)b}{(m-9)a} D, \quad \text{etc.}, \quad L = \frac{(m-2i+11)b}{(m-2i+1)a} K.$$

$$\int \frac{\partial x}{x^m X^6} = -\frac{1}{(m-1)ax^{m-1}X^5} - \frac{(m+9)b}{(m-1)a} \int \frac{\partial x}{x^{m-2}X^6}$$

$$\int \frac{\partial x}{x^m X^6} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-5}} + \frac{C}{x^{m-9}} - \frac{D}{x^{m-13}} + \frac{E}{x^{m-17}} - \text{etc.} \right.$$

$$\left. + \frac{K}{x^{m-2i+3}} + \frac{L}{x^{m-2i+1}} \right) \frac{1}{X^5} + L(m-2i+11)b \int \frac{\partial x}{x^{m-2i}X^6}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+9)b}{(m-3)a} A, \quad C = \frac{(m+7)b}{(m-5)a} B,$$

$$D = \frac{(m+5)b}{(m-7)a} C, \quad E = \frac{(m+3)b}{(m-9)a} D, \quad \text{etc.}, \quad L = \frac{(m-2i+13)b}{(m-2i+1)a} K.$$

## T a f e l

einiger allgemeineren Formeln.

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$$V. \quad a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


---

$$\int \frac{dx}{X} = \frac{2cx + b}{(p-1)kX^{p-1}} + \frac{(2p-3)2c}{(p-1)k} \int \frac{dx}{X^{p-1}}$$

$$\int \frac{dx}{X} = \left( \frac{A}{X^{p-1}} + \frac{B}{X^{p-2}} + \frac{C}{X^{p-3}} + \text{etc.} + \frac{K}{X^{p-i+1}} + \frac{L}{X^{p-i}} \right) (2cx + b) \\ + L(4p-2i-2)c \int \frac{dx}{X^{p-i}}$$

$$A = \frac{1}{(p-1)k}, \quad B = \frac{(4p-6)c}{(p-2)k} A, \quad C = \frac{(4p-10)c}{(p-3)k} B,$$

$$D = \frac{(4p-14)c}{(p-4)k} C, \quad E = \frac{(4p-18)c}{(p-5)k} D, \text{ etc.}, \quad L = \frac{(4p-4i+2)c}{(p-i)k} K.$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-1}}{(m-1)c} - \frac{a}{c} \int \frac{x^{m-2} dx}{X} - \frac{b}{c} \int \frac{x^{m-1} dx}{X}$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-1}}{(m-3)cX} - \frac{(m-1)a}{(m-3)c} \int \frac{x^{m-2} dx}{X^2} - \frac{(m-2)b}{(m-3)c} \int \frac{x^{m-1} dx}{X^2}$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-1}}{(m-5)cX^2} - \frac{(m-1)a}{(m-5)c} \int \frac{x^{m-2} dx}{X^3} - \frac{(m-3)b}{(m-5)c} \int \frac{x^{m-1} dx}{X^3}$$

$$\int \frac{x^m dx}{X^4} = \frac{x^{m-1}}{(m-7)cX^3} - \frac{(m-1)a}{(m-7)c} \int \frac{x^{m-2} dx}{X^4} - \frac{(m-4)b}{(m-7)c} \int \frac{x^{m-1} dx}{X^4}$$

$$\int \frac{x^m dx}{X^5} = \frac{x^{m-1}}{(m-9)cX^4} - \frac{(m-1)a}{(m-9)c} \int \frac{x^{m-2} dx}{X^5} - \frac{(m-5)b}{(m-9)c} \int \frac{x^{m-1} dx}{X^5}$$

$$\int \frac{x^m dx}{X^6} = \frac{x^{m-1}}{(m-11)cX^5} - \frac{(m-1)a}{(m-11)c} \int \frac{x^{m-2} dx}{X^6} - \frac{(m-6)b}{(m-11)c} \int \frac{x^{m-1} dx}{X^6}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-1} X} - \frac{c}{a} \int \frac{dx}{x^{m-2} X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1} X} - \frac{mb}{(m-1)a} \int \frac{dx}{x^{m-1} X^2} - \frac{(m+1)c}{(m-1)a} \int \frac{dx}{x^{m-2} X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1} X^2} - \frac{(m+1)b}{(m-1)a} \int \frac{dx}{x^{m-1} X^3} - \frac{(m+3)c}{(m-1)a} \int \frac{dx}{x^{m-2} X^3}$$

## T a f e l

einiger allgemeineren Formeln.

---


$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


---

$$\begin{aligned} \int \frac{dx}{x^m X^4} &= -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+2)b}{(m-1)a} \int \frac{dx}{x^{m-1}X^4} - \frac{(m+5)c}{(m-1)a} \int \frac{dx}{x^{m-2}X^4} \\ \int \frac{dx}{x^m X^5} &= -\frac{1}{(m-1)ax^{m-1}X^5} - \frac{(m+3)b}{(m-1)a} \int \frac{dx}{x^{m-1}X^5} - \frac{(m+7)c}{(m-1)a} \int \frac{dx}{x^{m-2}X^5} \\ \int \frac{dx}{x^m X^6} &= -\frac{1}{(m-1)ax^{m-1}X^6} - \frac{(m+4)b}{(m-1)a} \int \frac{dx}{x^{m-1}X^6} - \frac{(m+9)c}{(m-1)a} \int \frac{dx}{x^{m-2}X^6} \end{aligned}$$


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$$\text{VZ. } a + bx^2 = X$$


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$$\begin{aligned} \int \frac{x^m dx}{X} &= \frac{x^{m-2}}{(m-2)b} - \frac{a}{b} \int \frac{x^{m-5} dx}{X} \\ \int \frac{x^m dx}{X^2} &= \frac{x^{m-2}}{(m-5)bX} - \frac{(m-2)a}{(m-5)b} \int \frac{x^{m-5} dx}{X^2} \\ \int \frac{x^m dx}{X^3} &= \frac{x^{m-2}}{(m-8)bX^2} - \frac{(m-2)a}{(m-8)b} \int \frac{x^{m-5} dx}{X^3} \\ \int \frac{dx}{x^m X} &= -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-3}X} \\ \int \frac{dx}{x^m X^2} &= -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+2)b}{(m-1)a} \int \frac{dx}{x^{m-3}X^2} \\ \int \frac{dx}{x^m X^3} &= -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+5)b}{(m-1)a} \int \frac{dx}{x^{m-3}X^3} \end{aligned}$$


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$$\text{VZ. } a + bx^4 = X$$


---

$$\begin{aligned} \int \frac{x^m dx}{X} &= \frac{x^{m-3}}{(m-3)b} - \frac{a}{b} \int \frac{x^{m-7} dx}{X} \\ \int \frac{x^m dx}{X^2} &= \frac{x^{m-3}}{(m-7)bX} - \frac{(m-3)a}{(m-7)b} \int \frac{x^{m-7} dx}{X^2} \end{aligned}$$

**T a f e l**  
 einiger allgemeineren Formeln.

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VZ.  $a + bx^4 = X$

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$$\int \frac{x^m dx}{X^3} = \frac{x^{m-3}}{(m-11)bX^2} - \frac{(m-3)a}{(m-11)b} \int \frac{x^{m-4} dx}{X^3}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-1} X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1} X} - \frac{(m+5)b}{(m-1)a} \int \frac{dx}{x^{m-1} X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1} X^2} - \frac{(m+7)b}{(m-1)a} \int \frac{dx}{x^{m-1} X^3}$$


---

VZ.  $a + bx^5 = X$

---

$$\int \frac{x^m dx}{X} = \frac{x^{m-4}}{(m-4)b} - \frac{a}{b} \int \frac{x^{m-5} dx}{X}$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-4}}{(m-9)b} - \frac{(m-4)a}{(m-9)b} \int \frac{x^{m-5} dx}{X^2}$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-4}}{(m-14)b} - \frac{(m-4)a}{(m-14)b} \int \frac{x^{m-5} dx}{X^3}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-5} X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1} X} - \frac{(m+4)b}{(m-1)a} \int \frac{dx}{x^{m-5} X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1} X^2} - \frac{(m+9)b}{(m-1)a} \int \frac{dx}{x^{m-5} X^3}$$


---

VZ.  $a + bx^6 = X$

---

$$\int \frac{x^m dx}{X} = \frac{x^{m-5}}{(m-5)b} - \frac{a}{b} \int \frac{x^{m-6} dx}{X}$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-5}}{(m-11)bX} - \frac{(m-5)a}{(m-11)b} \int \frac{x^{m-6} dx}{X^2}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a + bx^6 = X$$

$$\begin{aligned} \int \frac{x^m dx}{X^3} &= \frac{x^{m-5}}{(m-17)bX^2} - \frac{(m-5)a}{(m-17)b} \int \frac{x^{m-6} dx}{X^3} \\ \int \frac{\partial x}{x^m X} &= -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{\partial x}{x^{m-6} X} \\ \int \frac{\partial x}{x^m X^2} &= -\frac{1}{(m-1)ax^{m-1} X} - \frac{(m+5)b}{(m-1)a} \int \frac{\partial x}{x^{m-6} X^2} \\ \int \frac{\partial x}{x^m X^3} &= -\frac{1}{(m-1)ax^{m-1} X^2} - \frac{(m+11)b}{(m-1)a} \int \frac{\partial x}{x^{m-6} X^3} \end{aligned}$$

$$\text{VZ. } a + bx^2 + cx^4 = X, (p-1)(b^2 - 4ac)2a = k$$

$$\begin{aligned} \int \frac{\partial x}{X^2} &= \frac{bcx^3 + (b^2 - 2ac)x}{kX^{2-1}} + \frac{(4p-7)bc}{k} \int \frac{x^2 \partial x}{X^{2-1}} \\ &\quad + \frac{2(p-1)(b^2 - 4ac) + 2ac - b^2}{k} \int \frac{\partial x}{X^{2-1}} \\ \int \frac{x^m \partial x}{X^2} &= \frac{bcx^{m+3} + (b^2 - 2ac)x^{m+1}}{kX^{2-1}} + \frac{(4p-m-5)bc}{k} \int \frac{x^{m+2} \partial x}{X^{2-1}} \\ &\quad + \frac{2(p-1)(b^2 - 4ac) + (m+1)(2ac - b^2)}{k} \int \frac{x^m \partial x}{X^{2-1}} \\ \int \frac{x^m \partial x}{X^2} &= \frac{x^{m-3}}{(m-4p+1)cX^{2-1}} - \frac{(m-3)a}{(m-4p+1)c} \int \frac{x^{m-4} \partial x}{X^2} \\ &\quad - \frac{(m-2p-1)b}{(m-4p+1)c} \int \frac{x^{m-2} \partial x}{X^2} \\ \int \frac{\partial x}{x^m X^2} &= -\frac{1}{(m-1)ax^{m-1} X^{2-1}} - \frac{(m+2p-3)b}{(m-1)a} \int \frac{\partial x}{x^{m-2} X^2} \\ &\quad - \frac{m+4p-5}{(m-1)a} \int \frac{\partial x}{x^{m-4} X^2} \end{aligned}$$



## Table I

Integration of Rational Functions

$$\text{Let } x^2 - 2x + 1 = (x-1)^2 \quad \text{and} \quad x^2 - 2x + 1 = (x-1)^2$$

$$\int \frac{x}{x^2 - 2x + 1} = \frac{x^2 - (x^2 - 2x + 1)}{(x-1)^2} = \frac{2x - 1}{(x-1)^2} = \frac{2(x-1) + 1}{(x-1)^2} = \frac{2}{x-1} + \frac{1}{(x-1)^2}$$

$$\int \frac{x^2}{x^2 - 2x + 1} = \frac{x^2 - (x^2 - 2x + 1)}{(x-1)^2} = \frac{2x - 1}{(x-1)^2} = \frac{2(x-1) + 1}{(x-1)^2} = \frac{2}{x-1} + \frac{1}{(x-1)^2}$$

$$\int \frac{x^3}{x^2 - 2x + 1} = \frac{x^3 - (x^2 - 2x + 1)x}{(x-1)^2} = \frac{x^3 - x^3 + 2x^2 - x}{(x-1)^2} = \frac{2x^2 - x}{(x-1)^2} = \frac{2(x-1) + 1}{(x-1)^2} = \frac{2}{x-1} + \frac{1}{(x-1)^2}$$

$$\int \frac{x^4}{x^2 - 2x + 1} = \frac{x^4 - (x^2 - 2x + 1)x^2}{(x-1)^2} = \frac{x^4 - x^4 + 2x^3 - x^2}{(x-1)^2} = \frac{2x^3 - x^2}{(x-1)^2} = \frac{2(x-1) + 1}{(x-1)^2} = \frac{2}{x-1} + \frac{1}{(x-1)^2}$$

# T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a + bx^2 = X, \quad \pi = 180^\circ$$

Es ist im Allgemeinen, wenn  $m < n$ , oder  $= 0$ ,

$$\int \frac{x^m dx}{X} = U + V,$$

wo  $U$  eine bloße logarithmische Function oder  $= 0$  ist, und  $V$  ein Aggregat von Gliedern der folgenden Form:

$$\frac{1}{nbk^{n-m-1}} \left\{ \begin{aligned} &\cos(n-m-1)\varphi \log(x^2 - 2kx \cos \varphi + k^2) \\ &+ 2 \sin(n-m-1)\varphi \text{Arc Tang} \frac{x \sin \varphi}{k - x \cos \varphi} \end{aligned} \right\}$$

## N ä h e r e B e s t i m m u n g.

1)  $n$  eine ungerade Zahl;  $a$  und  $b$  beliebige Vorzeichen.

Man setze  $k = \sqrt[n]{\frac{a}{b}}$ ; alsdann ist

$$U = \frac{1}{nb(-k)^{n-m-1}} \log(x+k)$$

und  $V$  ein Aggregat von Gliedern der obigen Form, welche sämmtlich erhalten werden, wenn man in diesem Ausdrücke für  $\varphi$  successive die  $\frac{n-1}{2}$  Werthe  $\frac{\pi}{n}, \frac{3\pi}{n}, \frac{5\pi}{n}, \frac{7\pi}{n}, \dots, \frac{(n-2)\pi}{n}$  setzt.

2)  $n$  eine gerade Zahl;  $a$  und  $b$  verschiedene Vorzeichen.

Man setze  $k = \sqrt[n]{-\frac{a}{b}}$ ; alsdann ist

$$U = \frac{1}{nbk^{n-m-1}} \log(x-k) + \frac{1}{nb(-k)^{n-m-1}} \log(x+k)$$

und  $V$  ein Aggregat von Gliedern der obigen Form, welche sämmtlich erhalten werden, wenn man in diesem Ausdrücke für  $\varphi$  successive die  $\frac{n-2}{2}$  Werthe  $\frac{2\pi}{n}, \frac{4\pi}{n}, \frac{6\pi}{n}, \dots, \frac{(n-2)\pi}{n}$  setzt.

3)  $n$  eine gerade Zahl;  $a$  und  $b$  dieselben Vorzeichen.

Man setze  $k = \sqrt[n]{\frac{a}{b}}$ ; alsdann ist  $U = 0$ , und  $V$  ein Aggregat von Gliedern, welche aus dem obigen Ausdrücke entstehen, wenn man darin für  $\varphi$  die  $\frac{n}{2}$  Werthe  $\frac{\pi}{n}, \frac{3\pi}{n}, \frac{5\pi}{n}, \frac{7\pi}{n}, \dots, \frac{(n-1)\pi}{n}$  setzt.

## T a f e l

einiger allgemeineren Formeln.

---


$$\text{VZ. } a + bx^n + cx^{2n} = X, \quad \pi = 180^\circ$$


---

Das Integral  $\int \frac{x^n \partial x}{X}$  erhält, wenn reelle Ausdrücke gefordert werden, zwey verschiedene Formen, je nachdem  $4ac - b^2$  eine positive oder eine negative Gröfse ist.

I.  $4ac - b^2$  eine positive Gröfse;  $m < 2n$ .

Es sey  $k = \sqrt[n]{\frac{a}{c}}$ , und  $\alpha$  ein Winkel, dessen Cosinus  $= -\frac{b}{2\sqrt[n]{ac}}$ ,

so ist  $\int \frac{x^n \partial x}{X}$  ein Aggregat von  $n$  Gliedern der folgenden Form:

$$\frac{1}{2nck^{2n-m-1}\sin\alpha} \left\{ -\sin(n-m-1)\varphi \log(x^2 - 2kx \cos\varphi + k^2) \right. \\ \left. + 2\cos(n-m-1)\varphi \operatorname{ArcTang} \frac{x \sin\varphi}{k - x \cos\varphi} \right\}$$

welche sämmtlich erhalten werden, wenn man in diesem Ausdrucke successive  $\frac{\alpha}{n}, \frac{2\pi + \alpha}{n}, \frac{4\pi + \alpha}{n}, \frac{6\pi + \alpha}{n}, \frac{8\pi + \alpha}{n}, \dots$   
 $\frac{(2n-2)\pi + \alpha}{n}$  für  $\varphi$  setzt.

Wenn  $m > 2n$ , wird man das Integral  $\int \frac{x^n \partial x}{X}$  auf andere reduciren, für welche  $m < 2n$ .

II.  $4ac - b^2$  eine negative Gröfse.

Es sey

$$\frac{1}{2}b - \frac{1}{2}\sqrt{b^2 - 4ac} = f$$

$$\frac{1}{2}b + \frac{1}{2}\sqrt{b^2 - 4ac} = g$$

$$\sqrt{b^2 - 4ac} = g - f = h$$

so ist

$$\int \frac{\partial x}{X} = \frac{c}{h} \left[ \int \frac{\partial x}{cx^2 + f} - \int \frac{\partial x}{cx^2 + g} \right]$$

$$\int \frac{x^n \partial x}{X} = \frac{c}{h} \left[ \int \frac{x^n \partial x}{cx^2 + f} - \int \frac{x^n \partial x}{cx^2 + g} \right]$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } \begin{cases} Ax^4 + Bx^{4-1} + Cx^{4-2} + Dx^{4-3} + \text{etc.} + Kx + L = U \\ ax^4 + bx^{4-1} + cx^{4-2} + dx^{4-3} + \text{etc.} + kx + l = V \end{cases}$$

Es sey

$$\frac{\partial V}{\partial x} = nax^{n-1} + (n-1)bx^{n-2} + (n-2)cx^{n-3} + \dots + k = Z.$$

Es seyen ferner

 $r', r'', r''', r''', \dots, r^{(n)}$  die  $n$  Wurzeln der Gleichung  $V=0$ ,

 $U', U'', U''', U''', \dots, U^{(n)}$  die Werthe der Function  $U$ , wenn diese Wurzeln für  $x$  substituirt werden,

 $Z', Z'', Z''', Z''', \dots, Z^{(n)}$  die Werthe der Function  $Z$ , wenn diese Wurzeln für  $x$  substituirt werden;

 so ist im Allgemeinen, jedoch nur unter der Voraussetzung, daß die Wurzeln  $r', r'', r''', \text{etc.}$ , sämmtlich von einander verschieden seyen, und  $h < n$ :

$$\int \frac{U dx}{V} = \frac{U'}{Z'} \log(x-r') + \frac{U''}{Z''} \log(x-r'') + \frac{U'''}{Z'''} \log(x-r''') + \dots + \frac{U^{(n)}}{Z^{(n)}} \log(x-r^{(n)})$$

$$\int \frac{x^h U dx}{V} = \frac{U'}{Z'} \int \frac{x^h dx}{x-r'} + \frac{U''}{Z''} \int \frac{x^h dx}{x-r''} + \frac{U'''}{Z'''} \int \frac{x^h dx}{x-r'''} + \dots + \frac{U^{(n)}}{Z^{(n)}} \int \frac{x^h dx}{x-r^{(n)}}$$

$$\int \frac{U dx}{x^h V} = \frac{U'}{Z'} \int \frac{dx}{x^h(x-r')} + \frac{U''}{Z''} \int \frac{dx}{x^h(x-r'')} + \frac{U'''}{Z'''} \int \frac{dx}{x^h(x-r''')} + \dots + \frac{U^{(n)}}{Z^{(n)}} \int \frac{dx}{x^h(x-r^{(n)})}$$

 und diese Formeln werden reell, wenn die Wurzeln  $r', r'', r''', \text{etc.}$  reell sind.

*Erinnerungen zu der vorhergehenden Tafel.*

1) Die Formeln Seite 97 — 110 sind sämmtlich aus den allgemeinen Reductionsformeln Seite 19 — 23 abgeleitet. Bey der ersten Formel Seite 106 ist jedoch zu bemerken, daß sie nicht unmittelbar in dieser Gestalt gefunden wird; denn wenn man in der Formel V, Seite 23,  $-p$  für  $p$  und  $m=1$ ,  $n=1$  setzt, so erhält man eigentlich

$$\int \frac{\partial x}{X^2} = \frac{Ax + Bx^2}{KX^{p-1}} + \frac{C}{K} \int \frac{\partial x}{X^{p-1}} + \frac{D}{K} \int \frac{x \partial x}{X^{p-1}};$$

wo  $A=2ac-b^2$ ,  $B=-bc$ ,  $C=(p-1)(4ac-b^2)+b^2-2ac$ ,  $D=(-2p+4)bc$ ,  $K=(p-1)(4ac-b^2)a$ . Es ist aber

$$\int \frac{x \partial x}{X^{p-1}} = \frac{-1}{2c(p-2)X^{p-2}} - \frac{b}{2c} \int \frac{\partial x}{X^{p-1}}.$$

Substituirt man diesen Werth, so erhält man nach der gehörigen Reduction die Formel, welche Seite 106 angegeben worden.

2) Die Zerlegung der Brüche  $\frac{x^m}{a+bx^2}$ ,  $\frac{x^m}{a+bx^2+cx^2}$ , welche zur Integrirung der Differentiale  $\frac{x^m \partial x}{a+bx^2}$ ,  $\frac{x^m \partial x}{a+bx^2+cx^2}$  erfordert wird, geschieht am leichtesten nach der zweiten Methode des dritten Falles Seite 10. Es ist bekannt, daß jeder trinomische Factor von  $x^2 + \frac{a}{b}$  und  $x^2 + \frac{b}{c}x^2 + \frac{a}{c}$  (wenn  $b^2 - 4ac$  negativ) die Form  $x^2 - 2kx \cos \phi + k^2$  hat. Man darf also nur  $\cos \phi + \sin \phi \sqrt{-1}$  für  $x$  setzen, und sich dabey erinnern, daß im Allgemeinen  $(\cos \phi + \sin \phi \sqrt{-1})^n = \cos n\phi + \sin n\phi \sqrt{-1}$ . Man erhält auf diese Weise für den ersten Bruch partielle Brüche von der Form

$$\frac{2}{nbk^{2n-2}} \cdot \frac{-k \cos(n-m)\phi + \cos(n-m-1)\phi \cdot x}{x^2 - 2kx \cos \phi + k^2}$$

und für den zweiten Bruch

$$\frac{1}{nck^{2n-2} \sin \phi} \cdot \frac{k \sin(n-m)\phi - \sin(n-m-1)\phi \cdot x}{x^2 - 2kx \cos \phi + k^2}$$

**Integraltafeln**  
für  
**irrationale Differentiale.**

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(Es werden in diesem Abschnitte die Integrale der am häufigsten in der Ausübung vorkommenden irrationalen Differentiale gegeben. Man hat sich dabey größtentheils auf solche beschränkt, worin nur quadratische Wurzelgrößen vorkommen, theils aus dem Grunde, weil die vollständige Integration bey den andern Wurzelgrößen nur in wenigen Fällen ausführbar ist, theils auch deshalb, weil sie gegenwärtig nur selten oder fast gar nicht gebraucht werden. Die allgemeineren Methoden und Formeln sind jedoch am Ende angeführt.)

$$\int \frac{x^n dx}{V(a+bx)}$$

Taf. I

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$$\text{VZ. } a + bx = X$$


---

$$\int \frac{dx}{VX} = \frac{a}{b} V X$$

$$\int \frac{x dx}{VX} = \left( \frac{1}{2} X - a \right) \frac{2VX}{b^2}$$

$$\int \frac{x^2 dx}{VX} = \left( \frac{1}{3} X^2 - \frac{2}{5} a X + a^2 \right) \frac{2VX}{b^3}$$

$$\int \frac{x^3 dx}{VX} = \left( \frac{1}{4} X^3 - \frac{3}{5} a X^2 + a^2 X - a^3 \right) \frac{2VX}{b^4}$$

$$\int \frac{x^4 dx}{VX} = \left( \frac{1}{5} X^4 - \frac{4}{7} a X^3 + \frac{6}{5} a^2 X^2 - \frac{4}{5} a^3 X + a^4 \right) \frac{2VX}{b^5}$$

$$\int \frac{x^5 dx}{VX} = \left( \frac{1}{11} X^5 - \frac{5}{9} a X^4 + \frac{10}{7} a^2 X^3 - 2a^3 X^2 + \frac{5}{5} a^4 X - a^5 \right) \frac{2VX}{b^6}$$

$$\int \frac{x^6 dx}{VX} = \left( \frac{1}{15} X^6 - \frac{6}{21} a X^5 + \frac{5}{8} a^2 X^4 - \frac{20}{7} a^3 X^3 + 3a^4 X^2 - 2a^5 X + a^6 \right) \frac{2VX}{b^7}$$

$$\int \frac{x^7 dx}{VX} = \left( \frac{1}{15} X^7 - \frac{7}{25} a X^6 + \frac{21}{21} a^2 X^5 - \frac{55}{9} a^3 X^4 + 5a^4 X^3 - \frac{21}{5} a^5 X^2 + \frac{7}{8} a^6 X - a^7 \right) \frac{2VX}{b^8}$$

$$\int \frac{x^8 dx}{VX} = \left( \frac{1}{17} X^8 - \frac{8}{25} a X^7 + \frac{28}{25} a^2 X^6 - \frac{56}{11} a^3 X^5 + \frac{70}{9} a^4 X^4 - 8a^5 X^3 + \frac{28}{5} a^6 X^2 - \frac{8}{5} a^7 X + a^8 \right) \frac{2VX}{b^9}$$

$$\int \frac{x^9 dx}{VX} = \left( \frac{1}{19} X^9 - \frac{9}{17} a X^8 + \frac{12}{5} a^2 X^7 - \frac{84}{15} a^3 X^6 + \frac{126}{11} a^4 X^5 - 14a^5 X^4 + 12a^6 X^3 - \frac{56}{5} a^7 X^2 + 3a^8 X - a^9 \right) \frac{2VX}{b^{10}}$$

$$\int \frac{x^{10} dx}{VX} = \left( \frac{1}{21} X^{10} - \frac{10}{19} a X^9 + \frac{45}{17} a^2 X^8 - 8a^3 X^7 + \frac{210}{15} a^4 X^6 - \frac{252}{11} a^5 X^5 + \frac{70}{5} a^6 X^4 - \frac{120}{7} a^7 X^3 + 9a^8 X^2 - \frac{10}{5} a^9 X + a^{10} \right) \frac{2VX}{b^{11}}$$



Taf. H.

$$\int \frac{dx}{x^2 \sqrt{a+bx}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{x \sqrt{X}} = \int \frac{dx}{x \sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{x^2 \sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^3 \sqrt{X}} = \left( -\frac{1}{2ax^2} + \frac{3b}{4a^2x} \right) \sqrt{X} + \frac{3b^2}{8a^2} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^4 \sqrt{X}} = \left( -\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \frac{5b^2}{8a^3x} \right) \sqrt{X} - \frac{5b^3}{16a^3} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^5 \sqrt{X}} = \left( -\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \frac{35b^2}{96a^3x^2} + \frac{35b^3}{64a^4x} \right) \sqrt{X} + \frac{35b^4}{128a^4} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^6 \sqrt{X}} = \left( -\frac{1}{5ax^5} + \frac{9b}{40a^2x^4} - \frac{21b^2}{80a^3x^3} + \frac{21b^3}{64a^4x^2} - \frac{63b^4}{128a^5x} \right) \sqrt{X} - \frac{63b^5}{256a^5} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^7 \sqrt{X}} = \left( -\frac{1}{6ax^6} + \frac{11b}{60a^2x^5} - \frac{33b^2}{160a^3x^4} + \frac{77b^3}{320a^4x^3} - \frac{77b^4}{256a^5x^2} + \frac{231b^5}{512a^6x} \right) \sqrt{X} + \frac{231b^6}{1024a^6} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^8 \sqrt{X}} = \left( -\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \frac{143b^2}{840a^3x^5} + \frac{429b^3}{2240a^4x^4} - \frac{145b^4}{640a^5x^3} + \frac{143b^5}{512a^6x^2} - \frac{429b^6}{1024a^7x} \right) \sqrt{X} - \frac{429b^7}{2048a^7} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^9 \sqrt{X}} = \left( -\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \frac{65b^2}{448a^3x^6} + \frac{143b^3}{896a^4x^5} - \frac{1287b^4}{7168a^5x^4} + \frac{429b^5}{2048a^6x^3} - \frac{2145b^6}{8192a^7x^2} + \frac{6435b^7}{16384a^8x} \right) \sqrt{X} + \frac{6435b^8}{32768a^8} \int \frac{dx}{x \sqrt{X}}$$

*Anmerkung zu der vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} + \text{Const.}$$

$$\text{oder } \int \frac{dx}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-a}} \text{ArcTang} \frac{\sqrt{a+bx}}{\sqrt{-a}} + \text{Const.}$$

Der erste Ausdruck wird reell, wenn  $a$  positiv, der zweite, wenn  $a$  negativ ist.

*I. a positiv.*

$$\begin{aligned} \int \frac{dx}{x\sqrt{a+bx}} + \text{Const.} &= \frac{2}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \\ &= \frac{2}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{x}} = -\frac{2}{\sqrt{a}} \log \frac{\sqrt{x}}{\sqrt{a+bx} - \sqrt{a}}, \end{aligned}$$

und in diesen Ausdrücken kann  $\sqrt{a}$  sowohl positiv als negativ genommen werden. Der Integralausdruck  $\int \frac{dx}{x\sqrt{a+bx}}$  kann, wenn er endlich seyn soll, nicht von  $x=0$  anfangen, und daher auch nicht für diesen Werth verschwinden.

*II. a negativ.*

$$\begin{aligned} \int \frac{dx}{x\sqrt{bx-a}} &= \frac{2}{\sqrt{a}} \text{ArcTang} \sqrt{\frac{bx-a}{a}} = \frac{2}{\sqrt{a}} \text{ArcCot} \sqrt{\frac{a}{bx-a}} \\ &= \frac{2}{\sqrt{a}} \text{ArcSec} \sqrt{\frac{bx}{a}} = \frac{2}{\sqrt{a}} \text{ArcCosec} \sqrt{\frac{bx}{bx-a}} = \frac{2}{\sqrt{a}} \text{ArcCos} \sqrt{\frac{a}{bx}} \\ &= \frac{2}{\sqrt{a}} \text{ArcSin} \sqrt{\frac{bx-a}{bx}} = \frac{1}{\sqrt{a}} \text{ArcCos} \frac{2a-bx}{bx} \\ &= \frac{1}{\sqrt{a}} \text{ArcSin vers} \frac{2a-bx}{bx}. \end{aligned}$$

In dem Integralausdruck  $\int \frac{dx}{x\sqrt{bx-a}}$  kann  $b$  nicht negativ seyn; auch kann sich derselbe erst von  $x=\frac{a}{b}$  anfangen, und daher für keinen kleinern Werth von  $x$  verschwinden. Für diesen Werth des  $x$  verschwinden nun auch die hier angegebenen Integrale.

Taf. III.

$$\int \frac{x^n dx}{(a+bx)^{\frac{1}{2}}}$$

---


$$\text{VL. } a + bx = X$$


---

$$\int \frac{dx}{X^{\frac{1}{2}}} = -\frac{2}{b\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = (X+a) \frac{2}{b^2 \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{3} X^3 - 2aX - a^2 \right) \frac{2}{b^3 \sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{5} X^5 - aX^3 + 3a^2 X + a^3 \right) \frac{2}{b^4 \sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{7} X^7 - \frac{4}{5} aX^5 + 2a^2 X^3 - 4a^3 X - a^4 \right) \frac{2}{b^5 \sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{9} X^9 - \frac{5}{7} aX^7 + 2a^2 X^5 - \frac{10}{3} a^3 X^3 + 5a^4 X + a^5 \right) \frac{2}{b^6 \sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{11} X^{11} - \frac{3}{5} aX^9 + \frac{15}{7} a^2 X^7 - 4a^3 X^5 + 5a^4 X^3 - 6a^5 X - a^6 \right) \frac{2}{b^7 \sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{13} X^{13} - \frac{7}{11} aX^{11} + \frac{7}{3} a^2 X^9 - 5a^3 X^7 + 7a^4 X^5 - 7a^5 X^3 + 7a^6 X + a^7 \right) \frac{2}{b^8 \sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{15} X^{15} - \frac{8}{13} aX^{13} + \frac{28}{11} a^2 X^{11} - \frac{56}{9} a^3 X^9 + 10a^4 X^7 - \frac{56}{5} a^5 X^5 + \frac{28}{3} a^6 X^3 - 8a^7 X - a^8 \right) \frac{2}{b^9 \sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left( \frac{1}{17} X^{17} - \frac{5}{13} aX^{15} + \frac{36}{13} a^2 X^{13} - \frac{84}{11} a^3 X^{11} + 14a^4 X^9 - 18a^5 X^7 + \frac{84}{5} a^6 X^5 - 12a^7 X^3 + 9a^8 X + a^9 \right) \frac{2}{b^{10} \sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{1}{2}}} = \left( \frac{1}{19} X^{19} - \frac{10}{17} aX^{17} + 3a^2 X^{15} - \frac{120}{13} a^3 X^{13} + \frac{210}{11} a^4 X^{11} - 28a^5 X^9 + 30a^6 X^7 - 24a^7 X^5 + 15a^8 X^3 - 10a^9 X - a^{10} \right) \frac{2}{b^{11} \sqrt{X}}$$

$$\int \frac{dx}{x^m(a+bx)^{\frac{1}{2}}}$$

Taf. IV.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = \frac{2}{a\sqrt{X}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{a^2}\right) \frac{1}{\sqrt{X}} - \frac{3b}{2a^2} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{4a^3}\right) \frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \frac{35b^2}{24a^3x} - \frac{35b^3}{8a^4}\right) \frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^3} - \frac{21b^2}{32a^3x^2} + \frac{105b^3}{64a^4x} + \frac{315b^4}{64a^5}\right) \frac{1}{\sqrt{X}} + \frac{315b^4}{128a^5} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4} - \frac{33b^2}{80a^3x^3} + \frac{231b^3}{320a^4x^2} - \frac{231b^4}{128a^5x} - \frac{693b^5}{128a^6}\right) \frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \frac{143b^2}{480a^3x^4} + \frac{143b^3}{320a^4x^3} - \frac{1001b^4}{1280a^5x^2} + \frac{1001b^5}{512a^6x} + \frac{3003b^6}{512a^7}\right) \frac{1}{\sqrt{X}} + \frac{3003b^6}{1024a^7} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^7} + \frac{5b}{28a^2x^6} - \frac{13b^2}{56a^3x^5} + \frac{143b^3}{448a^4x^4} - \frac{429b^4}{896a^5x^3} + \frac{429b^5}{512a^6x^2} - \frac{2145b^6}{1024a^7x} - \frac{6435b^7}{1024a^8}\right) \frac{1}{\sqrt{X}} - \frac{6435b^7}{2048a^8} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7} - \frac{85b^2}{448a^3x^6} + \frac{221b^3}{896a^4x^5} - \frac{2431b^4}{7168a^5x^4} + \frac{7293b^5}{14336a^6x^3} - \frac{7293b^6}{8192a^7x^2} + \frac{36465b^7}{16384a^8x} + \frac{109395b^8}{16384a^9}\right) \frac{1}{\sqrt{X}} + \frac{109395b^8}{32768a^{10}} \int \frac{dx}{x\sqrt{X}}$$

Taf. V.

$$\int \frac{x^m dx}{(a+bx)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = -\frac{2}{3b\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = \left(-X + \frac{1}{3}a\right) \frac{2}{b^2\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left(X^2 + 2aX - \frac{1}{3}a^2\right) \frac{2}{b^3\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{3}X^3 - 3aX^2 - 3a^2X + \frac{1}{5}a^3\right) \frac{2}{b^4\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{5}X^4 - \frac{4}{3}aX^3 + 6a^2X^2 + 4a^3X - \frac{1}{3}a^4\right) \frac{2}{b^5\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{7}X^5 - aX^4 + \frac{10}{3}a^2X^3 - 10a^3X^2 - 5a^4X + \frac{1}{5}a^5\right) \frac{2}{b^6\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{9}X^6 - \frac{6}{7}aX^5 + 3a^2X^4 - \frac{20}{3}a^3X^3 + 15a^4X^2 + 6a^5X - \frac{1}{3}a^6\right) \times \frac{2}{b^7\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{11}X^7 - \frac{7}{9}aX^6 + 3a^2X^5 - 7a^3X^4 + \frac{55}{3}a^4X^3 - 21a^5X^2 - 7a^6X + \frac{1}{3}a^7\right) \frac{2}{b^8\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{13}X^8 - \frac{8}{11}aX^7 + \frac{28}{9}a^2X^6 - 8a^3X^5 + 14a^4X^4 - \frac{56}{3}a^5X^3 + 28a^6X^2 + 8a^7X - \frac{1}{3}a^8\right) \frac{2}{b^9\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{15}X^9 - \frac{9}{13}aX^8 + \frac{56}{11}a^2X^7 - \frac{28}{3}a^3X^6 + 18a^4X^5 - \frac{126}{5}a^5X^4 + 28a^6X^3 - 36a^7X^2 - 9a^8X + \frac{1}{3}a^9\right) \frac{2}{b^{10}\sqrt{X}}$$

$$\int \frac{dx}{x^m(a+bx)^{\frac{1}{2}}}$$

Taf. VI.

$$\text{VZ. } a + bx = X$$

$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left( \frac{8}{5a} + \frac{2bx}{a^2} \right) \frac{1}{XVX} + \frac{1}{a^2} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left( -\frac{1}{ax} - \frac{20b}{3a^2} - \frac{5b^2x}{a^3} \right) \frac{1}{XVX} - \frac{5b}{2a^3} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left( -\frac{1}{2ax^2} + \frac{7b}{4a^2x} + \frac{35b^2}{3a^3} + \frac{35b^2x}{4a^4} \right) \frac{1}{XVX} + \frac{35b^2}{8a^4} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left( -\frac{1}{3ax^3} + \frac{3b}{4a^2x^2} - \frac{21b^2}{8a^3x} - \frac{35b^3}{2a^4} - \frac{105b^4x}{8a^5} \right) \frac{1}{XVX} - \frac{105b^3}{16a^5} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left( -\frac{1}{4ax^4} + \frac{11b}{24a^2x^3} - \frac{33b^2}{32a^3x^2} + \frac{231b^3}{64a^4x} + \frac{385b^4}{16a^5} + \frac{1155b^5x}{64a^6} \right) \frac{1}{XVX} + \frac{1155b^4}{128a^6} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left( -\frac{1}{5ax^5} + \frac{13b}{40a^2x^4} - \frac{143b^2}{240a^3x^3} + \frac{429b^3}{320a^4x^2} - \frac{3003b^4}{640a^5x} - \frac{1001b^5}{32a^6} - \frac{3003b^6x}{128a^7} \right) \frac{1}{XVX} - \frac{3003b^5}{256a^7} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left( -\frac{1}{6ax^6} + \frac{b}{4a^2x^5} - \frac{13b^2}{32a^3x^4} + \frac{143b^3}{192a^4x^3} - \frac{429b^4}{256a^5x^2} + \frac{3003b^5}{512a^6x} + \frac{5005b^6}{128a^7} + \frac{15015b^7x}{512a^8} \right) \frac{1}{XVX} + \frac{15015b^6}{1024a^8} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left( -\frac{1}{7ax^7} + \frac{17b}{84a^2x^6} - \frac{17b^2}{56a^3x^5} + \frac{221b^3}{448a^4x^4} - \frac{2431b^4}{2688a^5x^3} + \frac{7293b^5}{3584a^6x^2} - \frac{7293b^6}{1024a^7x} - \frac{12155b^7}{256a^8} - \frac{36465b^8x}{1024a^9} \right) \frac{1}{XVX} - \frac{36465b^7}{2048a^9} \int \frac{dx}{xVX}$$

Taf. VII

$$\int \frac{x^m \partial x}{(a+bx)^{\frac{1}{2}}}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x}{X^{\frac{1}{2}}} = -\frac{2}{5bX^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x \partial x}{X^{\frac{1}{2}}} = \left(-\frac{1}{3}X + \frac{1}{3}a\right) \frac{2}{b^2 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^2 \partial x}{X^{\frac{1}{2}}} = \left(-X^2 + \frac{2}{3}aX - \frac{1}{3}a^2\right) \frac{2}{b^3 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^3 \partial x}{X^{\frac{1}{2}}} = \left(X^3 + 3aX^2 - a^2X + \frac{1}{5}a^3\right) \frac{2}{b^4 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^4 \partial x}{X^{\frac{1}{2}}} = \left(\frac{1}{3}X^4 - 4aX^3 - 6a^2X^2 + \frac{4}{5}a^3X - \frac{1}{5}a^4\right) \frac{2}{b^5 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^5 \partial x}{X^{\frac{1}{2}}} = \left(\frac{1}{5}X^5 - \frac{5}{5}aX^4 + 10a^2X^3 + 10a^3X^2 - \frac{6}{5}a^4X + \frac{1}{5}a^5\right) \frac{2}{b^6 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^6 \partial x}{X^{\frac{1}{2}}} = \left(\frac{1}{7}X^6 - \frac{6}{5}aX^5 + 5a^2X^4 - 20a^3X^3 - 15a^4X^2 + 2a^5X - \frac{1}{5}a^6\right) \frac{2}{b^7 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^7 \partial x}{X^{\frac{1}{2}}} = \left(\frac{1}{9}X^7 - aX^6 + \frac{21}{5}a^2X^5 - \frac{55}{5}a^3X^4 + 35a^4X^3 + 21a^5X^2 - \frac{7}{5}a^6X + \frac{1}{5}a^7\right) \frac{2}{b^8 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^8 \partial x}{X^{\frac{1}{2}}} = \left(\frac{1}{11}X^8 - \frac{8}{9}aX^7 + 4a^2X^6 - \frac{56}{5}a^3X^5 + \frac{70}{3}a^4X^4 - 56a^5X^3 - 28a^6X^2 + \frac{8}{5}a^7X - \frac{1}{5}a^8\right) \frac{2}{b^9 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^9 \partial x}{X^{\frac{1}{2}}} = \left(\frac{1}{13}X^9 - \frac{9}{11}aX^8 + 4a^2X^7 - 12a^3X^6 + \frac{126}{5}a^4X^5 - 42a^5X^4 + 84a^6X^3 + 36a^7X^2 - 3a^8X + \frac{1}{5}a^9\right) \frac{2}{b^{10} X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{dx}{x^m(a+bx)^{\frac{1}{2}}}$$

Taf. VIII.

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$$\text{VL. } a+bx=X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left( \frac{46}{15a} + \frac{14bx}{3a^2} + \frac{2b^2x^2}{a^3} \right) \frac{1}{X^2\sqrt{X}} + \frac{1}{a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left( -\frac{1}{2ax^2} + \frac{9b}{4a^2x} \right) \frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left( -\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \frac{33b^2}{8a^3x} \right) \frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left( -\frac{1}{4ax^4} + \frac{13b}{24a^2x^3} - \frac{143b^2}{96a^3x^2} + \frac{429b^3}{64a^4x} \right) \frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left( -\frac{1}{5ax^5} + \frac{3b}{8a^2x^4} - \frac{13b^2}{16a^3x^3} + \frac{143b^3}{64a^4x^2} - \frac{1287b^4}{128a^5x} \right) \frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left( -\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \frac{17b^2}{32a^3x^4} + \frac{221b^3}{192a^4x^3} - \frac{2431b^4}{768a^5x^2} + \frac{7293b^5}{512a^6x} \right) \frac{1}{X^2\sqrt{X}} + \frac{51051b^6}{1024a^6} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left( -\frac{1}{7ax^7} + \frac{19b}{84a^2x^6} - \frac{323b^2}{840a^3x^5} + \frac{323b^3}{448a^4x^4} - \frac{4199b^4}{2688a^5x^3} + \frac{46189b^5}{10752a^6x^2} - \frac{138567b^6}{7168a^7x} \right) \frac{1}{X^2\sqrt{X}} - \frac{138567b^7}{2048a^7} \int \frac{dx}{xX^{\frac{1}{2}}}$$



Taf. IX.

$$\int \frac{x^m dx}{(a+bx)^{\frac{3}{2}}}, \quad \int \frac{dx}{x^m(a+bx)^{\frac{3}{2}}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2}{7bX^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = \left(-\frac{1}{5}X + \frac{1}{7}a\right) \frac{2}{b^2 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \left(-\frac{1}{3}X^2 + \frac{2}{5}aX - \frac{1}{7}a^2\right) \frac{2}{b^3 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left(-X^3 + aX^2 - \frac{3}{5}a^2X + \frac{1}{7}a^3\right) \frac{2}{b^4 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left(X^4 + 4aX^3 - 2a^2X^2 + \frac{4}{5}a^3X - \frac{1}{7}a^4\right) \frac{2}{b^5 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{3}X^5 - 5aX^4 - 10a^2X^3 + \frac{10}{3}a^3X^2 - a^4X + \frac{1}{7}a^5\right) \frac{2}{b^6 X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{5}X^6 - 2aX^5 + 15a^2X^4 + 20a^3X^3 - 5a^4X^2 + \frac{6}{5}a^5X - \frac{1}{7}a^6\right) \frac{2}{b^7 X^{\frac{1}{2}}\sqrt{X}}$$

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$$\int \frac{dx}{xX^{\frac{3}{2}}} = \left(\frac{352}{105a} + \frac{116bx}{15a^2} + \frac{20b^2x^2}{3a^3} + \frac{2b^3x^3}{a^4}\right) \frac{1}{X^{\frac{1}{2}}\sqrt{X}} + \frac{1}{a^4} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = -\frac{1}{axX^{\frac{1}{2}}\sqrt{X}} - \frac{9b}{2a} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{11b}{4a^2x}\right) \frac{1}{X^{\frac{1}{2}}\sqrt{X}} + \frac{99b^2}{8a^2} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{13b}{12a^2x^2} - \frac{143b^2}{24a^3x}\right) \frac{1}{X^{\frac{1}{2}}\sqrt{X}} - \frac{429b^3}{16a^3} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^3} - \frac{65b^2}{32a^3x^2} + \frac{715b^3}{64a^4x}\right) \frac{1}{X^{\frac{1}{2}}\sqrt{X}} + \frac{6435b^4}{128a^4} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int x^m dx \sqrt{a+bx}$$

Taf. X.

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$$\text{VZ. } a + bx = X$$


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$$\int dx \sqrt{X} = \frac{2X\sqrt{X}}{3b}$$

$$\int x dx \sqrt{X} = \left( \frac{1}{5}X - \frac{1}{5}a \right) \frac{2X\sqrt{X}}{b^2}$$

$$\int x^2 dx \sqrt{X} = \left( \frac{1}{7}X^2 - \frac{2}{5}aX + \frac{1}{5}a^2 \right) \frac{2X\sqrt{X}}{b^3}$$

$$\int x^3 dx \sqrt{X} = \left( \frac{1}{9}X^3 - \frac{5}{7}aX^2 + \frac{6}{5}a^2X - \frac{2}{5}a^3 \right) \frac{2X\sqrt{X}}{b^4}$$

$$\int x^4 dx \sqrt{X} = \left( \frac{1}{11}X^4 - \frac{4}{9}aX^3 + \frac{6}{7}a^2X^2 - \frac{4}{5}a^3X + \frac{1}{5}a^4 \right) \frac{2X\sqrt{X}}{b^5}$$

$$\int x^5 dx \sqrt{X} = \left( \frac{1}{13}X^5 - \frac{5}{11}aX^4 + \frac{10}{9}a^2X^3 - \frac{10}{7}a^3X^2 + a^4X - \frac{1}{5}a^5 \right) \frac{2X\sqrt{X}}{b^6}$$

$$\int x^6 dx \sqrt{X} = \left( \frac{1}{15}X^6 - \frac{6}{13}aX^5 + \frac{15}{11}a^2X^4 - \frac{20}{9}a^3X^3 + \frac{15}{7}a^4X^2 - \frac{6}{5}a^5X + \frac{1}{5}a^6 \right) \frac{2X\sqrt{X}}{b^7}$$

$$\int x^7 dx \sqrt{X} = \left( \frac{1}{17}X^7 - \frac{7}{15}aX^6 + \frac{21}{13}a^2X^5 - \frac{55}{11}a^3X^4 + \frac{55}{9}a^4X^3 - 3a^5X^2 + \frac{7}{5}a^6X - \frac{1}{5}a^7 \right) \frac{2X\sqrt{X}}{b^8}$$

$$\int x^8 dx \sqrt{X} = \left( \frac{1}{19}X^8 - \frac{8}{17}aX^7 + \frac{28}{15}a^2X^6 - \frac{56}{13}a^3X^5 + \frac{70}{11}a^4X^4 - \frac{56}{9}a^5X^3 + 4a^6X^2 - \frac{8}{5}a^7X + \frac{1}{5}a^8 \right) \frac{2X\sqrt{X}}{b^9}$$

$$\int x^9 dx \sqrt{X} = \left( \frac{1}{21}X^9 - \frac{9}{19}aX^8 + \frac{36}{17}a^2X^7 - \frac{28}{5}a^3X^6 + \frac{126}{13}a^4X^5 - \frac{126}{11}a^5X^4 + \frac{28}{5}a^6X^3 - \frac{56}{7}a^7X^2 + \frac{9}{5}a^8X - \frac{1}{5}a^9 \right) \frac{2X\sqrt{X}}{b^{10}}$$

Taf. II

$$\int \frac{\partial x}{x^m \sqrt{a+bx}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{x \sqrt{X}} = \int \frac{\partial x}{x \sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{\partial x}{x^2 \sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^3 \sqrt{X}} = \left( -\frac{1}{2ax^2} + \frac{3b}{4a^2x} \right) \sqrt{X} + \frac{3b^2}{8a^2} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^4 \sqrt{X}} = \left( -\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \frac{5b^2}{8a^3x} \right) \sqrt{X} - \frac{5b^3}{16a^3} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^5 \sqrt{X}} = \left( -\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \frac{35b^2}{96a^3x^2} + \frac{35b^3}{64a^4x} \right) \sqrt{X} + \frac{35b^4}{128a^4} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^6 \sqrt{X}} = \left( -\frac{1}{5ax^5} + \frac{9b}{40a^2x^4} - \frac{21b^2}{80a^3x^3} + \frac{21b^3}{64a^4x^2} - \frac{63b^4}{128a^5x} \right) \sqrt{X} - \frac{63b^5}{256a^5} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^7 \sqrt{X}} = \left( -\frac{1}{6ax^6} + \frac{11b}{60a^2x^5} - \frac{33b^2}{160a^3x^4} + \frac{77b^3}{320a^4x^3} - \frac{77b^4}{256a^5x^2} + \frac{231b^5}{512a^6x} \right) \sqrt{X} + \frac{231b^6}{1024a^6} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^8 \sqrt{X}} = \left( -\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \frac{143b^2}{840a^3x^5} + \frac{429b^3}{2240a^4x^4} - \frac{143b^4}{640a^5x^3} + \frac{143b^5}{512a^6x^2} - \frac{429b^6}{1024a^7x} \right) \sqrt{X} - \frac{429b^7}{2048a^7} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^9 \sqrt{X}} = \left( -\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \frac{65b^2}{448a^3x^6} + \frac{143b^3}{896a^4x^5} - \frac{1287b^4}{7168a^5x^4} + \frac{429b^5}{2048a^6x^3} - \frac{2145b^6}{8192a^7x^2} + \frac{6435b^7}{16384a^8x} \right) \sqrt{X} + \frac{6435b^8}{32768a^8} \int \frac{\partial x}{x \sqrt{X}}$$

*Anmerkung zu der vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{\partial x}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} + \text{Const.}$$

$$\text{oder } \int \frac{\partial x}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-a}} \text{ArcTang} \frac{\sqrt{a+bx}}{\sqrt{-a}} + \text{Const.}$$

Der erste Ausdruck wird reell, wenn  $a$  positiv, der zweite, wenn  $a$  negativ ist.

*I. a positiv.*

$$\begin{aligned} \int \frac{\partial x}{x\sqrt{a+bx}} + \text{Const.} &= \frac{2}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \\ &= \frac{2}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{x}} = -\frac{2}{\sqrt{a}} \log \frac{\sqrt{x}}{\sqrt{a+bx} - \sqrt{a}}, \end{aligned}$$

und in diesen Ausdrücken kann  $\sqrt{a}$  sowohl positiv als negativ genommen werden. Der Integralausdruck  $\int \frac{\partial x}{x\sqrt{a+bx}}$  kann, wenn er endlich seyn soll, nicht von  $x=0$  anfangen, und daher auch nicht für diesen Werth verschwinden.

*II. a negativ.*

$$\begin{aligned} \int \frac{\partial x}{x\sqrt{bx-a}} &= \frac{2}{\sqrt{a}} \text{ArcTang} \sqrt{\frac{bx-a}{a}} = \frac{2}{\sqrt{a}} \text{ArcCot} \sqrt{\frac{a}{bx-a}} \\ &= \frac{2}{\sqrt{a}} \text{ArcSec} \sqrt{\frac{bx}{a}} = \frac{2}{\sqrt{a}} \text{ArcCosec} \sqrt{\frac{bx}{bx-a}} = \frac{2}{\sqrt{a}} \text{ArcCos} \sqrt{\frac{a}{bx}} \\ &= \frac{2}{\sqrt{a}} \text{ArcSin} \sqrt{\frac{bx-a}{bx}} = \frac{1}{\sqrt{a}} \text{ArcCos} \frac{2a-bx}{bx} \\ &= \frac{1}{\sqrt{a}} \text{ArcSin vers} \frac{x(bx-a)}{bx}. \end{aligned}$$

In dem Integralausdruck  $\int \frac{\partial x}{x\sqrt{bx-a}}$  kann  $b$  nicht negativ seyn; auch kann sich derselbe erst von  $x=\frac{a}{b}$  anfangen, und daher für keinen kleinern Werth von  $x$  verschwinden. Für diesen Werth des  $x$  verschwinden nun auch die hier angegebenen Integrale.

Taf. III.

$$\int \frac{x^m dx}{(a+bx)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{X^{\frac{1}{2}}} = -\frac{2}{b\sqrt{X}}$$

$$\int \frac{x \partial x}{X^{\frac{1}{2}}} = (X + a) \frac{2}{b^2 \sqrt{X}}$$

$$\int \frac{x^2 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{5} X^3 - 2aX - a^2 \right) \frac{2}{b^3 \sqrt{X}}$$

$$\int \frac{x^3 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{5} X^3 - aX^2 + 3a^2 X + a^3 \right) \frac{2}{b^4 \sqrt{X}}$$

$$\int \frac{x^4 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{7} X^4 - \frac{4}{5} aX^3 + 2a^2 X^2 - 4a^3 X - a^4 \right) \frac{2}{b^5 \sqrt{X}}$$

$$\int \frac{x^5 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{9} X^5 - \frac{5}{7} aX^4 + 2a^2 X^3 - \frac{10}{3} a^3 X^2 + 5a^4 X + a^5 \right) \frac{2}{b^6 \sqrt{X}}$$

$$\int \frac{x^6 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{11} X^6 - \frac{2}{5} aX^5 + \frac{15}{7} a^2 X^4 - 4a^3 X^3 + 5a^4 X^2 - 6a^5 X - a^6 \right) \frac{2}{b^7 \sqrt{X}}$$

$$\int \frac{x^7 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{13} X^7 - \frac{7}{11} aX^6 + \frac{7}{3} a^2 X^5 - 5a^3 X^4 + 7a^4 X^3 - 7a^5 X^2 + 7a^6 X + a^7 \right) \frac{2}{b^8 \sqrt{X}}$$

$$\int \frac{x^8 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{15} X^8 - \frac{8}{13} aX^7 + \frac{28}{11} a^2 X^6 - \frac{56}{9} a^3 X^5 + 10a^4 X^4 - \frac{56}{6} a^5 X^3 + \frac{28}{5} a^6 X^2 - 8a^7 X - a^8 \right) \frac{2}{b^9 \sqrt{X}}$$

$$\int \frac{x^9 \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{17} X^9 - \frac{3}{5} aX^8 + \frac{36}{13} a^2 X^7 - \frac{84}{11} a^3 X^6 + 14a^4 X^5 - 18a^5 X^4 + \frac{84}{5} a^6 X^3 - 12a^7 X^2 + 9a^8 X + a^9 \right) \frac{2}{b^{10} \sqrt{X}}$$

$$\int \frac{x^{10} \partial x}{X^{\frac{1}{2}}} = \left( \frac{1}{19} X^{10} - \frac{10}{17} aX^9 + 3a^2 X^8 - \frac{120}{13} a^3 X^7 + \frac{210}{11} a^4 X^6 - 28a^5 X^5 + 30a^6 X^4 - 24a^7 X^3 + 15a^8 X^2 - 10a^9 X - a^{10} \right) \frac{2}{b^{11} \sqrt{X}}$$

$$\int \frac{dx}{x^m(a+bx)^{\frac{1}{2}}}$$

Taf. IV.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = \frac{2}{a\sqrt{X}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{a^2}\right) \frac{1}{\sqrt{X}} - \frac{3b}{2a^2} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{4a^3}\right) \frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \frac{35b^2}{24a^3x} - \frac{35b^3}{8a^4}\right) \frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^3} - \frac{21b^2}{32a^3x^2} + \frac{105b^3}{64a^4x} + \frac{315b^4}{64a^5}\right) \frac{1}{\sqrt{X}} + \frac{315b^4}{128a^5} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4} - \frac{33b^2}{80a^3x^3} + \frac{231b^3}{320a^4x^2} - \frac{231b^4}{128a^5x} - \frac{693b^5}{128a^6}\right) \frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \frac{143b^2}{480a^3x^4} + \frac{143b^3}{320a^4x^3} - \frac{1001b^4}{1280a^5x^2} + \frac{1001b^5}{512a^6x} + \frac{3003b^6}{512a^7}\right) \frac{1}{\sqrt{X}} + \frac{3003b^6}{1024a^7} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^7} + \frac{5b}{28a^2x^6} - \frac{13b^2}{56a^3x^5} + \frac{143b^3}{448a^4x^4} - \frac{429b^4}{896a^5x^3} + \frac{429b^5}{512a^6x^2} - \frac{2145b^6}{1024a^7x} - \frac{6435b^7}{1024a^8}\right) \frac{1}{\sqrt{X}} - \frac{6435b^7}{2048a^8} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7} - \frac{85b^2}{448a^3x^6} + \frac{221b^3}{896a^4x^5} - \frac{2431b^4}{7168a^5x^4} + \frac{7293b^5}{14336a^6x^3} - \frac{7293b^6}{8192a^7x^2} + \frac{36465b^7}{16384a^8x} + \frac{109395b^8}{16384a^9}\right) \frac{1}{\sqrt{X}} + \frac{109395b^8}{32768a^{10}} \int \frac{dx}{x\sqrt{X}}$$

Taf. V.

$$\int \frac{x^n dx}{(a+bx)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = -\frac{2}{3bX\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = \left(-X + \frac{1}{3}a\right) \frac{2}{b^2 X\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left(X^2 + 2aX - \frac{1}{3}a^2\right) \frac{2}{b^3 X\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{3}X^3 - 3aX^2 - 3a^2X + \frac{1}{5}a^3\right) \frac{2}{b^4 X\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{5}X^4 - \frac{4}{3}aX^3 + 6a^2X^2 + 4a^3X - \frac{1}{5}a^4\right) \frac{2}{b^5 X\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{7}X^5 - aX^4 + \frac{10}{3}a^2X^3 - 10a^3X^2 - 5a^4X + \frac{1}{5}a^5\right) \frac{2}{b^6 X\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{9}X^6 - \frac{6}{7}aX^5 + 3a^2X^4 - \frac{20}{3}a^3X^3 + 15a^4X^2 + 6a^5X - \frac{1}{3}a^6\right) \times \frac{2}{b^7 X\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{11}X^7 - \frac{7}{9}aX^6 + 3a^2X^5 - 7a^3X^4 + \frac{55}{3}a^4X^3 - 21a^5X^2 - 7a^6X + \frac{1}{3}a^7\right) \frac{2}{b^8 X\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{13}X^8 - \frac{8}{11}aX^7 + \frac{28}{9}a^2X^6 - 8a^3X^5 + 14a^4X^4 - \frac{56}{3}a^5X^3 + 28a^6X^2 + 8a^7X - \frac{1}{3}a^8\right) \frac{2}{b^9 X\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{15}X^9 - \frac{9}{13}aX^8 + \frac{56}{11}a^2X^7 - \frac{28}{3}a^3X^6 + 18a^4X^5 - \frac{126}{5}a^5X^4 + 28a^6X^3 - 36a^7X^2 - 9a^8X + \frac{1}{5}a^9\right) \frac{2}{b^{10} X\sqrt{X}}$$

$$\int \frac{dx}{x^m(a+bx)^{\frac{1}{2}}}$$

Taf. VI.

$$\text{VZ. } a + bx = X$$

$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left( \frac{8}{5a} + \frac{2bx}{a^2} \right) \frac{1}{X\sqrt{X}} + \frac{1}{a^2} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left( -\frac{1}{ax} - \frac{20b}{3a^2} - \frac{5b^2x}{a^3} \right) \frac{1}{X\sqrt{X}} - \frac{5b}{2a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left( -\frac{1}{2ax^2} + \frac{7b}{4a^2x} + \frac{35b^2}{3a^3} + \frac{35b^3x}{4a^4} \right) \frac{1}{X\sqrt{X}} + \frac{35b^2}{8a^4} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left( -\frac{1}{3ax^3} + \frac{3b}{4a^2x^2} - \frac{21b^2}{8a^3x} - \frac{35b^3}{2a^4} - \frac{105b^4x}{8a^5} \right) \frac{1}{X\sqrt{X}} - \frac{105b^3}{16a^5} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left( -\frac{1}{4ax^4} + \frac{11b}{24a^2x^3} - \frac{33b^2}{32a^3x^2} + \frac{231b^3}{64a^4x} + \frac{385b^4}{16a^5} + \frac{1155b^5x}{64a^6} \right) \frac{1}{X\sqrt{X}} + \frac{1155b^4}{128a^6} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left( -\frac{1}{5ax^5} + \frac{13b}{40a^2x^4} - \frac{143b^2}{240a^3x^3} + \frac{429b^3}{320a^4x^2} - \frac{3003b^4}{640a^5x} - \frac{1001b^5}{32a^6} - \frac{3003b^6x}{128a^7} \right) \frac{1}{X\sqrt{X}} - \frac{3003b^5}{256a^7} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left( -\frac{1}{6ax^6} + \frac{b}{4a^2x^5} - \frac{13b^2}{32a^3x^4} + \frac{143b^3}{192a^4x^3} - \frac{429b^4}{256a^5x^2} + \frac{3003b^5}{512a^6x} + \frac{5005b^6}{128a^7} + \frac{15015b^7x}{512a^8} \right) \frac{1}{X\sqrt{X}} + \frac{15015b^6}{1024a^8} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left( -\frac{1}{7ax^7} + \frac{17b}{84a^2x^6} - \frac{17b^2}{56a^3x^5} + \frac{221b^3}{448a^4x^4} - \frac{2431b^4}{2688a^5x^3} + \frac{7293b^5}{3584a^6x^2} - \frac{7293b^6}{1024a^7x} - \frac{12155b^7}{256a^8} - \frac{36465b^8x}{1024a^9} \right) \frac{1}{X\sqrt{X}} - \frac{36465b^7}{2048a^9} \int \frac{dx}{x\sqrt{X}}$$



Taf. VII

$$\int \frac{x^n dx}{(a+bx)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = -\frac{2}{5bX^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = \left(-\frac{1}{5}X + \frac{1}{5}a\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left(-X^2 + \frac{2}{5}aX - \frac{1}{5}a^2\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left(X^3 + 3aX^2 - a^2X + \frac{1}{5}a^3\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{5}X^4 - 4aX^3 - 6a^2X^2 + \frac{4}{5}a^3X - \frac{1}{5}a^4\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{5}X^5 - \frac{5}{5}aX^4 + 10a^2X^3 + 10a^3X^2 - \frac{6}{5}a^4X + \frac{1}{5}a^5\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{7}X^6 - \frac{6}{5}aX^5 + 5a^2X^4 - 20a^3X^3 - 15a^4X^2 + 2a^5X - \frac{1}{5}a^6\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{9}X^7 - aX^6 + \frac{21}{5}a^2X^5 - \frac{85}{5}a^3X^4 + 35a^4X^3 + 21a^5X^2 - \frac{7}{5}a^6X + \frac{1}{5}a^7\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{11}X^8 - \frac{8}{9}aX^7 + 4a^2X^6 - \frac{56}{5}a^3X^5 + \frac{70}{5}a^4X^4 - 56a^5X^3 - 28a^6X^2 + \frac{8}{5}a^7X - \frac{1}{5}a^8\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left(\frac{1}{13}X^9 - \frac{9}{11}aX^8 + 4a^2X^7 - 12a^3X^6 + \frac{126}{5}a^4X^5 - 42a^5X^4 + 84a^6X^3 + 36a^7X^2 - 3a^8X + \frac{1}{5}a^9\right) \frac{2}{b^{\frac{1}{2}}X^{\frac{1}{2}}\sqrt{X}}$$

$$\int \frac{dx}{x^n(a+bx)^{\frac{1}{2}}}$$

Taf. VIII.

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$$\text{VZ. } a+bx=X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left( \frac{4b}{15a} + \frac{14b^2x}{3a^2} + \frac{2b^3x^2}{a^3} \right) \frac{1}{X^2\sqrt{X}} + \frac{1}{a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left( -\frac{1}{2ax^2} + \frac{9b}{4a^2x} \right) \frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left( -\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \frac{33b^2}{8a^3x} \right) \frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left( -\frac{1}{4ax^4} + \frac{13b}{24a^2x^3} - \frac{143b^2}{96a^3x^2} + \frac{429b^3}{64a^4x} \right) \frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left( -\frac{1}{5ax^5} + \frac{3b}{8a^2x^4} - \frac{13b^2}{16a^3x^3} + \frac{143b^3}{64a^4x^2} - \frac{1287b^4}{128a^5x} \right) \frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left( -\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \frac{17b^2}{32a^3x^4} + \frac{221b^3}{192a^4x^3} - \frac{2431b^4}{768a^5x^2} + \frac{7293b^5}{512a^6x} \right) \frac{1}{X^2\sqrt{X}} + \frac{51051b^6}{1024a^6} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left( -\frac{1}{7ax^7} + \frac{19b}{84a^2x^6} - \frac{323b^2}{840a^3x^5} + \frac{323b^3}{448a^4x^4} - \frac{4199b^4}{2688a^5x^3} + \frac{46189b^5}{10752a^6x^2} - \frac{138567b^6}{7168a^7x} \right) \frac{1}{X^2\sqrt{X}} - \frac{138567b^7}{2048a^7} \int \frac{dx}{xX^{\frac{1}{2}}}$$

Taf. IX.

$$\int \frac{x^m dx}{(a+bx)^{\frac{3}{2}}}, \quad \int \frac{dx}{x^m(a+bx)^{\frac{3}{2}}}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2}{7bX^{\frac{3}{2}}\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = \left(-\frac{1}{5}X + \frac{1}{7}a\right) \frac{2}{b^2 X^{\frac{3}{2}}\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \left(-\frac{1}{3}X^2 + \frac{2}{5}aX - \frac{1}{7}a^2\right) \frac{2}{b^3 X^{\frac{3}{2}}\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left(-X^3 + aX^2 - \frac{8}{5}a^2X + \frac{1}{7}a^3\right) \frac{2}{b^4 X^{\frac{3}{2}}\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left(X^4 + 4aX^3 - 2a^2X^2 + \frac{4}{5}a^3X - \frac{1}{7}a^4\right) \frac{2}{b^5 X^{\frac{3}{2}}\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{3}X^5 - 5aX^4 + 10a^2X^3 + \frac{10}{3}a^3X^2 - a^4X + \frac{1}{7}a^5\right) \frac{2}{b^6 X^{\frac{3}{2}}\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{5}X^6 - 2aX^5 + 15a^2X^4 + 20a^3X^3 - 5a^4X^2 + \frac{6}{5}a^5X - \frac{1}{7}a^6\right) \frac{2}{b^7 X^{\frac{3}{2}}\sqrt{X}}$$

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$$\int \frac{dx}{xX^{\frac{3}{2}}} = \left(\frac{352}{105a} + \frac{116bx}{15a^2} + \frac{20b^2x^2}{3a^3} + \frac{2b^3x^3}{a^4}\right) \frac{1}{X^3\sqrt{X}} + \frac{1}{a^4} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = -\frac{1}{axX^3\sqrt{X}} - \frac{9b}{2a} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{11b}{4a^2x}\right) \frac{1}{X^3\sqrt{X}} + \frac{99b^2}{8a^2} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{13b}{12a^2x^2} - \frac{143b^2}{24a^3x}\right) \frac{1}{X^3\sqrt{X}} - \frac{429b^3}{16a^3} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^3} - \frac{65b^2}{32a^3x^2} + \frac{715b^3}{64a^4x}\right) \frac{1}{X^3\sqrt{X}} + \frac{6435b^4}{128a^4} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int x^m dx V(a+bx)$$

Taf. X.

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$$\text{VZ. } a + bx = X$$


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$$\int dx V X = \frac{2XVX}{3b}$$

$$\int x dx V X = \left( \frac{1}{5}X - \frac{1}{5}a \right) \frac{2XVX}{b^2}$$

$$\int x^2 dx V X = \left( \frac{1}{7}X^2 - \frac{2}{5}aX + \frac{1}{5}a^2 \right) \frac{2XVX}{b^3}$$

$$\int x^3 dx V X = \left( \frac{1}{9}X^3 - \frac{3}{7}aX^2 + \frac{6}{5}a^2X - \frac{1}{5}a^3 \right) \frac{2XVX}{b^4}$$

$$\int x^4 dx V X = \left( \frac{1}{11}X^4 - \frac{4}{9}aX^3 + \frac{6}{7}a^2X^2 - \frac{4}{5}a^3X + \frac{1}{5}a^4 \right) \frac{2XVX}{b^5}$$

$$\int x^5 dx V X = \left( \frac{1}{13}X^5 - \frac{5}{11}aX^4 + \frac{10}{9}a^2X^3 - \frac{10}{7}a^3X^2 + a^4X - \frac{1}{5}a^5 \right) \frac{2XVX}{b^6}$$

$$\int x^6 dx V X = \left( \frac{1}{15}X^6 - \frac{6}{13}aX^5 + \frac{15}{11}a^2X^4 - \frac{20}{9}a^3X^3 + \frac{15}{7}a^4X^2 - \frac{6}{5}a^5X + \frac{1}{5}a^6 \right) \frac{2XVX}{b^7}$$

$$\int x^7 dx V X = \left( \frac{1}{17}X^7 - \frac{7}{15}aX^6 + \frac{21}{13}a^2X^5 - \frac{55}{11}a^3X^4 + \frac{55}{9}a^4X^3 - 3a^5X^2 + \frac{7}{5}a^6X - \frac{1}{5}a^7 \right) \frac{2XVX}{b^8}$$

$$\int x^8 dx V X = \left( \frac{1}{19}X^8 - \frac{8}{17}aX^7 + \frac{28}{15}a^2X^6 - \frac{56}{13}a^3X^5 + \frac{70}{11}a^4X^4 - \frac{56}{9}a^5X^3 + 4a^6X^2 - \frac{8}{5}a^7X + \frac{1}{5}a^8 \right) \frac{2XVX}{b^9}$$

$$\int x^9 dx V X = \left( \frac{1}{21}X^9 - \frac{9}{19}aX^8 + \frac{56}{17}a^2X^7 - \frac{28}{5}a^3X^6 + \frac{126}{15}a^4X^5 - \frac{126}{11}a^5X^4 + \frac{28}{5}a^6X^3 - \frac{56}{7}a^7X^2 + \frac{9}{5}a^8X - \frac{1}{5}a^9 \right) \frac{2XVX}{b^{10}}$$

Taf. XI.

$$\int \frac{\partial x V(a+bx)}{x^m}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x V X}{x} = 2 V X + a \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^2} = -\frac{V X}{x} + \frac{b}{2} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^3} = -\frac{X V X}{2 a x^2} + \frac{b V X}{4 a x} - \frac{b^2}{8 a} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^4} = \left(-\frac{1}{3 a x^3} + \frac{b}{4 a^2 x^2}\right) X V X - \frac{b^2 V X}{8 a^2 x} + \frac{b^3}{16 a^2} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^5} = \left(-\frac{1}{4 a x^4} + \frac{5 b}{24 a^2 x^3} - \frac{5 b^2}{32 a^3 x^2}\right) X V X + \frac{5 b^3 V X}{64 a^3 x} - \frac{5 b^4}{128 a^3} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^6} = \left(-\frac{1}{5 a x^5} + \frac{7 b}{40 a^2 x^4} - \frac{7 b^2}{48 a^3 x^3} + \frac{7 b^3}{64 a^4 x^2}\right) X V X - \frac{7 b^4 V X}{128 a^4 x} + \frac{7 b^5}{256 a^4} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^7} = \left(-\frac{1}{6 a x^6} + \frac{3 b}{20 a^2 x^5} - \frac{21 b^2}{160 a^3 x^4} + \frac{7 b^3}{64 a^4 x^3} - \frac{21 b^4}{256 a^5 x^2}\right) X V X + \frac{21 b^5 V X}{512 a^5 x} - \frac{21 b^6}{1024 a^5} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^8} = -\frac{X V X}{7 a x^7} - \frac{11 b}{14 a} \int \frac{\partial x V X}{x^7}$$

$$\int \frac{\partial x V X}{x^9} = \left(-\frac{1}{8 a x^8} + \frac{13 b}{112 a^2 x^7}\right) X V X + \frac{143 b^2}{224 a^2} \int \frac{\partial x V X}{x^7}$$

$$\int \frac{\partial x V X}{x^{10}} = \left(-\frac{1}{9 a x^9} + \frac{5 b}{48 a^2 x^8} - \frac{65 b^2}{672 a^3 x^7}\right) X V X - \frac{715 b^3}{1344 a^3} \int \frac{\partial x V X}{x^7}$$

$$\int x^m dx (a + bx)^{\frac{1}{2}}$$

Taf. XII.

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$$\text{VZ. } a + bx = X$$


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$$\int dx X^{\frac{1}{2}} = \frac{2X^{\frac{1}{2}} \sqrt{X}}{5b}$$

$$\int x dx X^{\frac{1}{2}} = \left( \frac{1}{7} X - \frac{1}{5} a \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{1}{9} X^2 - \frac{2}{7} a X + \frac{1}{5} a^2 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{1}{11} X^3 - \frac{1}{5} a X^2 + \frac{5}{7} a^2 X - \frac{1}{5} a^3 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{1}{13} X^4 - \frac{4}{11} a X^3 + \frac{2}{5} a^2 X^2 - \frac{4}{7} a^3 X + \frac{1}{5} a^4 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{1}{15} X^5 - \frac{5}{13} a X^4 + \frac{10}{11} a^2 X^3 - \frac{10}{9} a^3 X^2 + \frac{5}{7} a^4 X - \frac{1}{5} a^5 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{1}{17} X^6 - \frac{2}{5} a^2 X^5 + \frac{15}{13} a^3 X^4 - \frac{20}{11} a^4 X^3 + \frac{5}{5} a^5 X^2 - \frac{6}{7} a^6 X + \frac{1}{5} a^7 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^7}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{1}{19} X^7 - \frac{7}{17} a X^6 + \frac{7}{5} a^2 X^5 - \frac{55}{13} a^3 X^4 + \frac{55}{11} a^4 X^3 - \frac{7}{5} a^5 X^2 + a^6 X - \frac{1}{5} a^7 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^8}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{1}{21} X^8 - \frac{8}{19} a X^7 + \frac{28}{17} a^2 X^6 - \frac{56}{15} a^3 X^5 + \frac{70}{13} a^4 X^4 - \frac{56}{11} a^5 X^3 + \frac{28}{9} a^6 X^2 - \frac{8}{7} a^7 X + \frac{1}{5} a^8 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^9}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{1}{23} X^9 - \frac{3}{7} a X^8 + \frac{36}{19} a^2 X^7 - \frac{84}{17} a^3 X^6 + \frac{42}{5} a^4 X^5 - \frac{126}{13} a^5 X^4 + \frac{84}{11} a^6 X^3 - 4 a^7 X^2 + \frac{9}{7} a^8 X - \frac{1}{5} a^9 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^{10}}$$

Taf. XIII.

$$\int \frac{\partial x(a+bx)^{\frac{1}{2}}}{x^m}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left(\frac{1}{5}X + a\right) 2\sqrt{X} + a^2 \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X\sqrt{X}}{ax} + \frac{3b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{b}{4a^2x}\right) X^2\sqrt{X} + \frac{3b^2}{8a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left(-\frac{1}{3ax^3} + \frac{b}{12a^2x^2} + \frac{b^2}{24a^3x}\right) X^2\sqrt{X} - \frac{b^3}{16a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{b}{8a^2x^3} - \frac{b^2}{32a^3x^2} - \frac{b^3}{64a^4x}\right) X^2\sqrt{X} \\ + \frac{3b^4}{128a^4} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{b}{8a^2x^4} - \frac{b^2}{16a^3x^3} + \frac{b^3}{64a^4x^2} + \frac{b^4}{128a^5x}\right) X^2\sqrt{X} \\ - \frac{3b^5}{256a^5} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{7b}{60a^2x^5} - \frac{7b^2}{96a^3x^4} + \frac{7b^3}{192a^4x^3} - \frac{7b^4}{768a^5x^2} \\ - \frac{7b^5}{1536a^6x}\right) X^2\sqrt{X} + \frac{7b^6}{1024a^6} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = -\frac{X^2\sqrt{X}}{7ax^7} - \frac{9b}{14a} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{11b}{112a^2x^7}\right) X^2\sqrt{X} + \frac{99b^2}{224a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{13b}{144a^2x^8} - \frac{143b^2}{2016a^3x^7}\right) X^2\sqrt{X} - \frac{143b^3}{448a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int x^n dx (a + bx)^{\frac{1}{2}}$$

Taf. XIV.

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$$\text{VL. } a + bx = X$$


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$$\int dx X^{\frac{1}{2}} = \frac{2X^{\frac{3}{2}} \sqrt{X}}{7b}$$

$$\int x dx X^{\frac{1}{2}} = \left( \frac{1}{9} X - \frac{1}{7} a \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{1}{11} X^2 - \frac{2}{9} a X + \frac{1}{7} a^2 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{1}{13} X^3 - \frac{3}{11} a X^2 + \frac{1}{5} a^2 X - \frac{1}{7} a^3 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{1}{15} X^4 - \frac{4}{13} a X^3 + \frac{6}{11} a^2 X^2 - \frac{4}{9} a^3 X + \frac{1}{7} a^4 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{1}{17} X^5 - \frac{5}{13} a X^4 + \frac{10}{11} a^2 X^3 - \frac{10}{11} a^3 X^2 + \frac{5}{9} a^4 X - \frac{1}{7} a^5 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{1}{19} X^6 - \frac{6}{17} a X^5 + a^2 X^4 - \frac{20}{13} a^3 X^3 + \frac{15}{11} a^4 X^2 - \frac{3}{5} a^5 X + \frac{1}{7} a^6 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^7}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{1}{21} X^7 - \frac{7}{19} a X^6 + \frac{21}{17} a^2 X^5 - \frac{7}{5} a^3 X^4 + \frac{35}{13} a^4 X^3 - \frac{21}{11} a^5 X^2 + \frac{7}{9} a^6 X - \frac{1}{7} a^7 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^8}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{1}{23} X^8 - \frac{8}{21} a X^7 + \frac{28}{19} a^2 X^6 - \frac{66}{17} a^3 X^5 + \frac{14}{5} a^4 X^4 - \frac{56}{13} a^5 X^3 + \frac{28}{11} a^6 X^2 - \frac{8}{9} a^7 X + \frac{1}{7} a^8 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^9}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{1}{25} X^9 - \frac{9}{23} a X^8 + \frac{12}{7} a^2 X^7 - \frac{84}{19} a^3 X^6 + \frac{126}{17} a^4 X^5 - \frac{42}{5} a^5 X^4 + \frac{84}{13} a^6 X^3 - \frac{36}{11} a^7 X^2 + a^8 X - \frac{1}{7} a^9 \right) \frac{2X^{\frac{3}{2}} \sqrt{X}}{b^{10}}$$



Taf. XV.

$$\int \frac{\partial x(a+bx)^{\frac{1}{2}}}{x^n}$$

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$$\text{VZ. } a+bx=X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left(\frac{1}{5}X^2 + \frac{1}{3}aX + a^2\right)2\sqrt{X} + a^3 \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X^{\frac{3}{2}}\sqrt{X}}{ax} + \frac{5b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{3b}{4a^2x}\right)X^{\frac{3}{2}}\sqrt{X} + \frac{15b^2}{8a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{b}{12a^2x^2} - \frac{b^2}{8a^3x}\right)X^{\frac{3}{2}}\sqrt{X} + \frac{5b^3}{16a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{b}{24a^2x^3} + \frac{b^2}{96a^3x^2} + \frac{b^3}{64a^4x}\right)X^{\frac{3}{2}}\sqrt{X} - \frac{5b^4}{128a^4} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{3b}{40a^2x^4} - \frac{b^2}{80a^3x^3} - \frac{b^3}{320a^4x^2} - \frac{3b^4}{640a^5x}\right)X^{\frac{3}{2}}\sqrt{X} \\ + \frac{3b^5}{256a^5} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{1}{12a^2x^5} - \frac{b^2}{32a^3x^4} + \frac{b^3}{192a^4x^3} + \frac{b^4}{768a^5x^2} \\ + \frac{b^5}{512a^6x}\right)X^{\frac{3}{2}}\sqrt{X} - \frac{5b^6}{1024a^6} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = -\frac{X^{\frac{3}{2}}\sqrt{X}}{7ax^7} - \frac{b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{9b}{112a^2x^7}\right)X^{\frac{3}{2}}\sqrt{X} + \frac{9b^2}{32a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{11b}{144a^2x^8} - \frac{11b^2}{224a^3x^7}\right)X^{\frac{3}{2}}\sqrt{X} - \frac{11b^3}{64a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int x^m dx (a + bx)^{\frac{1}{2}}$$

Taf. XVI.

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$$\text{VL. } a + bx = X$$


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$$\int dx X^{\frac{1}{2}} = \frac{2X^{\frac{1}{2}} \sqrt{X}}{9b}$$

$$\int x dx X^{\frac{1}{2}} = \left( \frac{1}{11} X - \frac{1}{9} a \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{1}{15} X^2 - \frac{2}{11} a X + \frac{1}{9} a^2 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{1}{15} X^3 - \frac{3}{13} a X^2 + \frac{3}{11} a^2 X - \frac{1}{9} a^3 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{1}{17} X^4 - \frac{4}{15} a X^3 + \frac{6}{13} a^2 X^2 - \frac{4}{11} a^3 X + \frac{1}{9} a^4 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{1}{19} X^5 - \frac{5}{17} a X^4 + \frac{8}{13} a^2 X^3 - \frac{10}{15} a^3 X^2 + \frac{5}{11} a^4 X - \frac{1}{9} a^5 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{1}{21} X^6 - \frac{6}{19} a X^5 + \frac{15}{17} a^2 X^4 - \frac{4}{5} a^3 X^3 + \frac{15}{13} a^4 X^2 - \frac{6}{11} a^5 X + \frac{1}{9} a^6 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^7}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{1}{23} X^7 - \frac{7}{19} a X^6 + \frac{21}{15} a^2 X^5 - \frac{35}{17} a^3 X^4 + \frac{7}{5} a^4 X^3 - \frac{21}{13} a^5 X^2 + \frac{7}{11} a^6 X - \frac{1}{9} a^7 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^8}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{1}{25} X^8 - \frac{8}{23} a X^7 + \frac{4}{5} a^2 X^6 - \frac{56}{19} a^3 X^5 + \frac{70}{17} a^4 X^4 - \frac{56}{15} a^5 X^3 + \frac{28}{13} a^6 X^2 - \frac{8}{11} a^7 X + \frac{1}{9} a^8 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^9}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{1}{27} X^9 - \frac{9}{25} a X^8 + \frac{36}{23} a^2 X^7 - 4 a^3 X^6 + \frac{126}{19} a^4 X^5 - \frac{126}{17} a^5 X^4 + \frac{28}{5} a^6 X^3 - \frac{36}{13} a^7 X^2 + \frac{9}{11} a^8 X - \frac{1}{9} a^9 \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^{10}}$$

Taf. XVII

$$\int \frac{\partial x(a+bx)^{\frac{7}{2}}}{x^n}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x X^{\frac{7}{2}}}{x} = \left( \frac{1}{7} X^7 + \frac{1}{5} a X^5 + \frac{1}{3} a^2 X^3 + a^3 \right) \sqrt{X} + a^4 \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^2} = -\frac{X \sqrt{X}}{ax} + \frac{7b}{2a} \int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^3} = \left( -\frac{1}{2ax^2} - \frac{5b}{4a^2x} \right) X \sqrt{X} + \frac{55b^2}{8a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left( -\frac{1}{5ax^3} - \frac{b}{4a^2x^2} - \frac{5b^2}{8a^3x} \right) X \sqrt{X} + \frac{55b^3}{16a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{b}{24a^2x^3} - \frac{b^2}{32a^3x^2} - \frac{5b^3}{64a^4x} \right) X \sqrt{X} + \frac{35b^4}{128a^4} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = \left( -\frac{1}{5ax^5} + \frac{b}{40a^2x^4} + \frac{b^2}{240a^3x^3} + \frac{b^3}{320a^4x^2} + \frac{b^4}{128a^5x} \right) X \sqrt{X} - \frac{7b^5}{256a^5} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left( -\frac{1}{6ax^6} + \frac{b}{20a^2x^5} - \frac{b^2}{160a^3x^4} - \frac{b^3}{960a^4x^3} - \frac{b^4}{1280a^5x^2} - \frac{b^5}{5120a^6x} \right) X \sqrt{X} + \frac{7b^6}{1024a^6} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = -\frac{X \sqrt{X}}{7ax^7} - \frac{5b}{14a} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left( -\frac{1}{8ax^8} + \frac{b}{16a^2x^7} \right) X \sqrt{X} + \frac{5b^2}{32a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} = \left( -\frac{1}{9ax^9} + \frac{b}{16a^2x^8} - \frac{b^2}{32a^3x^7} \right) X \sqrt{X} - \frac{5b^3}{64a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x^7}$$

$$\int x^m \partial x (a + bx)^{\frac{1}{2}}, \quad \int \frac{\partial x (a + bx)^{\frac{1}{2}}}{x^m} \quad \text{Taf. XVIII.}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \partial x X^{\frac{1}{2}} = \frac{2X^{\frac{1}{2}} \vee X}{11b}$$

$$\int x \partial x X^{\frac{1}{2}} = \left( \frac{1}{13} X - \frac{2}{11} a \right) \frac{2X^{\frac{1}{2}} \vee X}{b^2}$$

$$\int x^2 \partial x X^{\frac{1}{2}} = \left( \frac{1}{15} X^2 - \frac{2}{13} a X + \frac{1}{11} a^2 \right) \frac{2X^{\frac{1}{2}} \vee X}{b^3}$$

$$\int x^3 \partial x X^{\frac{1}{2}} = \left( \frac{1}{17} X^3 - \frac{1}{5} a X^2 + \frac{2}{13} a^2 X - \frac{1}{11} a^3 \right) \frac{2X^{\frac{1}{2}} \vee X}{b^4}$$

$$\int x^4 \partial x X^{\frac{1}{2}} = \left( \frac{1}{19} X^4 - \frac{4}{17} a X^3 + \frac{2}{5} a^2 X^2 - \frac{4}{13} a^3 X + \frac{1}{11} a^4 \right) \frac{2X^{\frac{1}{2}} \vee X}{b^5}$$

$$\int x^5 \partial x X^{\frac{1}{2}} = \left( \frac{1}{21} X^5 - \frac{6}{19} a X^4 + \frac{10}{17} a^2 X^3 - \frac{2}{5} a^3 X^2 + \frac{6}{13} a^4 X - \frac{1}{11} a^5 \right) \frac{2X^{\frac{1}{2}} \vee X}{b^6}$$

$$\int x^6 \partial x X^{\frac{1}{2}} = \left( \frac{1}{23} X^6 - \frac{2}{7} a X^5 + \frac{15}{19} a^2 X^4 - \frac{20}{17} a^3 X^3 + a^4 X^2 - \frac{6}{13} a^5 X + \frac{1}{11} a^6 \right) \frac{2X^{\frac{1}{2}} \vee X}{b^7}$$

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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left( \frac{1}{9} X^4 + \frac{2}{7} a X^3 + \frac{1}{5} a^2 X^2 + \frac{1}{3} a^3 X + a^4 \right) 2 \vee X + a, \quad \int \frac{\partial x}{x \vee X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X^{\frac{1}{2}} \vee X}{ax} + \frac{9b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = \left( -\frac{1}{2ax^2} - \frac{7b}{4a^2 x} \right) X^{\frac{1}{2}} \vee X + \frac{63b^2}{8a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left( -\frac{1}{3ax^3} - \frac{5b}{12a^2 x^2} - \frac{35b^2}{24a^3 x} \right) X^{\frac{1}{2}} \vee X + \frac{105b^3}{16a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{b}{8a^2 x^3} - \frac{5b^2}{32a^3 x^2} - \frac{35b^3}{64a^4 x} \right) X^{\frac{1}{2}} \vee X + \frac{315b^4}{128a^4} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

Taf. XIX.

$$\int \frac{x^n dx}{\sqrt[3]{a+bx}}, \int \frac{x^n dx}{\sqrt[3]{(a+bx)^2}}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{dx}{\sqrt[3]{X}} = \frac{3\sqrt[3]{X^2}}{2b}$$

$$\int \frac{x dx}{\sqrt[3]{X}} = \left(\frac{1}{5}X - \frac{1}{5}a\right) \frac{3\sqrt[3]{X^2}}{b^2}$$

$$\int \frac{x^2 dx}{\sqrt[3]{X}} = \left(\frac{1}{8}X^2 - \frac{2}{5}aX + \frac{1}{5}a^2\right) \frac{3\sqrt[3]{X^2}}{b^3}$$

$$\int \frac{x^3 dx}{\sqrt[3]{X}} = \left(\frac{1}{11}X^3 - \frac{3}{8}aX^2 + \frac{3}{5}a^2X - \frac{1}{5}a^3\right) \frac{3\sqrt[3]{X^2}}{b^4}$$

$$\int \frac{x^4 dx}{\sqrt[3]{X}} = \left(\frac{1}{14}X^4 - \frac{4}{11}aX^3 + \frac{3}{4}a^2X^2 - \frac{4}{5}a^3X + \frac{1}{5}a^4\right) \frac{3\sqrt[3]{X^2}}{b^5}$$

$$\int \frac{x^5 dx}{\sqrt[3]{X}} = \left(\frac{1}{17}X^5 - \frac{5}{14}aX^4 + \frac{10}{11}a^2X^3 - \frac{5}{4}a^3X^2 + a^4X - \frac{1}{5}a^5\right) \frac{3\sqrt[3]{X^2}}{b^6}$$

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$$\int \frac{dx}{\sqrt[3]{X^2}} = \frac{3\sqrt[3]{X}}{b}$$

$$\int \frac{x dx}{\sqrt[3]{X^2}} = \left(\frac{1}{4}X - a\right) \frac{3\sqrt[3]{X}}{b^2}$$

$$\int \frac{x^2 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{7}X^2 - \frac{1}{2}aX + a^2\right) \frac{3\sqrt[3]{X}}{b^3}$$

$$\int \frac{x^3 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{10}X^3 - \frac{3}{7}aX^2 + \frac{3}{4}a^2X - a^3\right) \frac{3\sqrt[3]{X}}{b^4}$$

$$\int \frac{x^4 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{13}X^4 - \frac{2}{5}aX^3 + \frac{6}{7}a^2X^2 - a^3X + a^4\right) \frac{3\sqrt[3]{X}}{b^5}$$

$$\int \frac{x^5 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{16}X^5 - \frac{5}{13}aX^4 + a^2X^3 - \frac{10}{7}a^3X^2 + \frac{5}{4}a^4X - a^5\right) \frac{3\sqrt[3]{X}}{b^6}$$

$$\int \frac{dx}{x^m \sqrt[3]{a+bx}}, \int \frac{dx}{x^m \sqrt[3]{a+bx}^2} \quad \text{Taf. XX}$$

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$$\text{VZ. } a+bx = X$$


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$$\int \frac{dx}{x \sqrt[3]{X}} = \frac{1}{\sqrt[3]{a}} \left[ \frac{3}{2} \log \frac{\sqrt[3]{X} - \sqrt[3]{a}}{\sqrt[3]{x}} + \sqrt[3]{3} \cdot \text{Arc Tang} \frac{\sqrt[3]{3} \cdot \sqrt[3]{X}}{\sqrt[3]{X} + 2\sqrt[3]{a}} \right]$$

$$\int \frac{dx}{x^2 \sqrt[3]{X}} = -\frac{\sqrt[3]{X}^2}{ax} - \frac{b}{3a} \int \frac{dx}{x \sqrt[3]{X}}$$

$$\int \frac{dx}{x^3 \sqrt[3]{X}} = \left( -\frac{1}{2ax^2} + \frac{2b}{3a^2x} \right) \sqrt[3]{X}^2 + \frac{2b^2}{9a^3} \int \frac{dx}{x \sqrt[3]{X}}$$

$$\int \frac{dx}{x^4 \sqrt[3]{X}} = \left( -\frac{1}{3ax^3} + \frac{7b}{18a^2x^2} - \frac{14b^2}{27a^3x} \right) \sqrt[3]{X}^2 - \frac{14b^3}{81a^3} \int \frac{dx}{x \sqrt[3]{X}}$$

$$\int \frac{dx}{x^5 \sqrt[3]{X}} = \left( -\frac{1}{4ax^4} + \frac{5b}{18a^2x^3} - \frac{35b^2}{108a^3x^2} + \frac{35b^3}{81a^4x} \right) \sqrt[3]{X}^2 + \frac{35b^4}{243a^4} \int \frac{dx}{x \sqrt[3]{X}}$$


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$$\int \frac{dx}{x \sqrt[3]{X}^2} = \frac{1}{\sqrt[3]{a^2}} \left[ \frac{3}{2} \log \frac{\sqrt[3]{X} - \sqrt[3]{a}}{\sqrt[3]{x}} - \sqrt[3]{3} \cdot \text{Arc Tang} \frac{\sqrt[3]{3} \cdot \sqrt[3]{X}}{\sqrt[3]{X} + 2\sqrt[3]{a}} \right]$$

$$\int \frac{dx}{x^2 \sqrt[3]{X}^2} = -\frac{\sqrt[3]{X}}{ax} - \frac{2b}{3a} \int \frac{dx}{x \sqrt[3]{X}^2}$$

$$\int \frac{dx}{x^3 \sqrt[3]{X}^2} = \left( -\frac{1}{2ax^2} + \frac{5b}{6a^2x} \right) \sqrt[3]{X} + \frac{5b^2}{9a^3} \int \frac{dx}{x \sqrt[3]{X}^2}$$

$$\int \frac{dx}{x^4 \sqrt[3]{X}^2} = \left( -\frac{1}{3ax^3} + \frac{4b}{9a^2x^2} - \frac{20b^2}{27a^3x} \right) \sqrt[3]{X} - \frac{40b^3}{81a^3} \int \frac{dx}{x \sqrt[3]{X}^2}$$

$$\int \frac{dx}{x^5 \sqrt[3]{X}^2} = \left( -\frac{1}{4ax^4} + \frac{11b}{36a^2x^3} - \frac{11b^2}{27a^3x^2} + \frac{55b^3}{81a^4x} \right) \sqrt[3]{X} + \frac{110b^4}{243a^4} \int \frac{dx}{x \sqrt[3]{X}^2}$$

Taf. XXI.

$$\int x^n dx \sqrt[3]{a+bx}, \int x^n dx \sqrt[3]{(a+bx)^2}$$

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$$\text{VZ. } a + bx = X$$


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$$\int dx \sqrt[3]{X} = \frac{3X\sqrt[3]{X}}{4b}$$

$$\int x dx \sqrt[3]{X} = \left(\frac{1}{7}X - \frac{1}{4}a\right) \frac{3X\sqrt[3]{X}}{b^2}$$

$$\int x^2 dx \sqrt[3]{X} = \left(\frac{1}{10}X^2 - \frac{2}{7}aX + \frac{1}{4}a^2\right) \frac{3X\sqrt[3]{X}}{b^3}$$

$$\int x^3 dx \sqrt[3]{X} = \left(\frac{1}{15}X^3 - \frac{3}{10}aX^2 + \frac{3}{7}a^2X - \frac{1}{4}a^3\right) \frac{3X\sqrt[3]{X}}{b^4}$$

$$\int x^4 dx \sqrt[3]{X} = \left(\frac{1}{16}X^4 - \frac{4}{15}aX^3 + \frac{5}{5}a^2X^2 - \frac{4}{7}a^3X + \frac{1}{4}a^4\right) \frac{3X\sqrt[3]{X}}{b^5}$$

$$\int x^5 dx \sqrt[3]{X} = \left(\frac{1}{19}X^5 - \frac{5}{16}aX^4 + \frac{10}{15}a^2X^3 - a^3X^2 + \frac{5}{7}a^4X - \frac{1}{5}a^5\right) \frac{3X\sqrt[3]{X}}{b^6}$$


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$$\int dx \sqrt[3]{X^2} = \frac{3X\sqrt[3]{X^2}}{5b}$$

$$\int x dx \sqrt[3]{X^2} = \left(\frac{1}{8}X - \frac{1}{5}a\right) \frac{3X\sqrt[3]{X^2}}{b^2}$$

$$\int x^2 dx \sqrt[3]{X^2} = \left(\frac{1}{11}X^2 - \frac{1}{4}aX + \frac{1}{5}a^2\right) \frac{3X\sqrt[3]{X^2}}{b^3}$$

$$\int x^3 dx \sqrt[3]{X^2} = \left(\frac{1}{14}X^3 - \frac{3}{11}aX^2 + \frac{5}{8}a^2X - \frac{1}{5}a^3\right) \frac{3X\sqrt[3]{X^2}}{b^4}$$

$$\int x^4 dx \sqrt[3]{X^2} = \left(\frac{1}{17}X^4 - \frac{2}{7}aX^3 + \frac{6}{11}a^2X^2 - \frac{1}{2}a^3X + \frac{1}{5}a^4\right) \frac{3X\sqrt[3]{X^2}}{b^5}$$

$$\int x^5 dx \sqrt[3]{X^2} = \left(\frac{1}{20}X^5 - \frac{5}{17}aX^4 + \frac{5}{7}a^2X^3 - \frac{10}{11}a^3X^2 + \frac{5}{8}a^4X - \frac{1}{5}a^5\right) \frac{3X\sqrt[3]{X^2}}{b^6}$$

$$\int \frac{\partial x \sqrt[3]{a+bx}}{x^m}, \int \frac{\partial x \sqrt[3]{(a+bx)^2}}{x^m} \quad \text{Taf. XXII.}$$

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$$\forall Z. \quad a + bx = X$$


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$$\int \frac{\partial x \sqrt[3]{X}}{x} = 3 \sqrt[3]{X} + a \int \frac{\partial x}{x \sqrt[3]{X^2}}$$

$$\int \frac{\partial x \sqrt[3]{X}}{x^2} = -\frac{X \sqrt[3]{X}}{ax} + \frac{b}{3a} \int \frac{\partial x \sqrt[3]{X}}{x}$$

$$\int \frac{\partial x \sqrt[3]{X}}{x^3} = \left(-\frac{1}{2ax^2} + \frac{b}{3a^2x}\right) X \sqrt[3]{X} - \frac{b^2}{9a^2} \int \frac{\partial x \sqrt[3]{X}}{x}$$

$$\int \frac{\partial x \sqrt[3]{X}}{x^4} = \left(-\frac{1}{3ax^3} + \frac{5b}{18a^2x^2} - \frac{5b^2}{27a^3x}\right) X \sqrt[3]{X} + \frac{5b^3}{81a^3} \int \frac{\partial x \sqrt[3]{X}}{x}$$

$$\int \frac{\partial x \sqrt[3]{X}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{2b}{9a^2x^3} - \frac{5b^2}{27a^3x^2} + \frac{10b^3}{81a^4x}\right) X \sqrt[3]{X} - \frac{10b^4}{243a^4} \int \frac{\partial x \sqrt[3]{X}}{x}$$


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$$\int \frac{\partial x \sqrt[3]{X^2}}{x} = \frac{3}{2} \sqrt[3]{X^2} + a \int \frac{\partial x}{x \sqrt[3]{X}}$$

$$\int \frac{\partial x \sqrt[3]{X^2}}{x^2} = -\frac{X \sqrt[3]{X^2}}{ax} + \frac{2b}{3a} \int \frac{\partial x \sqrt[3]{X^2}}{x}$$

$$\int \frac{\partial x \sqrt[3]{X^2}}{x^3} = \left(-\frac{1}{2ax^2} + \frac{b}{6a^2x}\right) X \sqrt[3]{X^2} - \frac{b^2}{9a^2} \int \frac{\partial x \sqrt[3]{X^2}}{x}$$

$$\int \frac{\partial x \sqrt[3]{X^2}}{x^4} = \left(-\frac{1}{3ax^3} + \frac{2b}{9a^2x^2} - \frac{2b^2}{27a^3x}\right) X \sqrt[3]{X^2} + \frac{4b^3}{81a^3} \int \frac{\partial x \sqrt[3]{X^2}}{x}$$

$$\int \frac{\partial x \sqrt[3]{X^2}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{7b}{36a^2x^3} - \frac{7b^2}{54a^3x^2} + \frac{7b^3}{162a^4x}\right) X \sqrt[3]{X^2} - \frac{7b^4}{243a^4} \int \frac{\partial x \sqrt[3]{X^2}}{x}$$



Taf. XXIII.

$$\int \frac{dx}{(a+bx^2)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{x}{a\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left( \frac{1}{3aX} + \frac{2}{3a^2} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left( \frac{1}{5aX^2} + \frac{4}{15a^2X} + \frac{8}{15a^3} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left( \frac{1}{7aX^3} + \frac{6}{35a^2X^2} + \frac{8}{35a^3X} + \frac{16}{35a^4} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left( \frac{1}{9aX^4} + \frac{8}{63a^2X^3} + \frac{16}{105a^3X^2} + \frac{64}{315a^4X} + \frac{128}{315a^5} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left( \frac{1}{11aX^5} + \frac{10}{99a^2X^4} + \frac{80}{693a^3X^3} + \frac{32}{231a^4X^2} + \frac{128}{693a^5X} + \frac{256}{693a^6} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left( \frac{1}{13aX^6} + \frac{12}{143a^2X^5} + \frac{40}{429a^3X^4} + \frac{320}{3003a^4X^3} + \frac{128}{1001a^5X^2} + \frac{512}{3003a^6X} + \frac{1024}{3003a^7} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left( \frac{1}{15aX^7} + \frac{14}{195a^2X^6} + \frac{56}{715a^3X^5} + \frac{112}{1287a^4X^4} + \frac{128}{1287a^5X^3} + \frac{256}{2145a^6X^2} + \frac{1024}{6435a^7X} + \frac{2048}{6435a^8} \right) \frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = \left( \frac{1}{17aX^8} + \frac{16}{255a^2X^7} + \frac{224}{3315a^3X^6} + \frac{896}{12155a^4X^5} + \frac{1792}{21879a^5X^4} + \frac{2048}{21879a^6X^3} + \frac{4096}{56465a^7X^2} + \frac{16384}{109395a^8X} + \frac{52768}{109395a^9} \right) \frac{x}{\sqrt{X}}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{\partial x}{\sqrt{a+bx^2}} = \frac{1}{\sqrt{b}} \log [x\sqrt{b} + \sqrt{a+bx^2}] + \text{Const.}$$

$$\text{oder } \int \frac{\partial x}{\sqrt{a+bx^2}} = \frac{1}{\sqrt{-b}} \text{Arc Sin } x\sqrt{-\frac{b}{a}} + \text{Const.}$$

Der erste Ausdruck wird reell, wenn  $b$  positiv, der zweite wird es, wenn  $b$  negativ ist. Zugleich können  $a$  und  $b$  nicht negativ seyn. Hieraus ergibt sich:

$$\text{I. } \int \frac{\partial x}{\sqrt{(\pm a+bx^2)}} = \frac{1}{\sqrt{b}} \log [x\sqrt{b} + \sqrt{(\pm a+bx^2)}] + \text{Const.}$$

$$\begin{aligned} \text{II. } \int \frac{\partial x}{\sqrt{a-bx^2}} &= \frac{1}{\sqrt{b}} \text{Arc Sin } x\sqrt{\frac{b}{a}} = \frac{1}{\sqrt{b}} \text{Arc Cos } \sqrt{\frac{a-bx^2}{a}} \\ &= \frac{1}{2\sqrt{b}} \text{Arc Cos } \frac{a-2bx^2}{a} = \frac{1}{\sqrt{b}} \text{Arc Tang } \frac{x\sqrt{b}}{\sqrt{a-bx^2}} \\ &= \frac{1}{\sqrt{b}} \text{Arc Cot } \frac{\sqrt{a-bx^2}}{x\sqrt{b}} = \frac{1}{\sqrt{b}} \text{Arc Sec } \sqrt{\frac{a}{a-bx^2}} \\ &= \frac{1}{\sqrt{b}} \text{Arc Cosec } \sqrt{\frac{a}{bx^2}} = \frac{1}{2\sqrt{b}} \text{Arc Sin vers } \frac{2bx^2}{a}. \end{aligned}$$

(Die Kreisbogen sind hier sämtlich für  $x=0$  verschwindend genommen.)

Insbesondere ist

$$\int \frac{\partial x}{\sqrt{1+x^2}} = \log [x + \sqrt{1+x^2}] + \text{Const.}$$

$$\int \frac{\partial x}{\sqrt{x^2-1}} = \log [x + \sqrt{x^2-1}] + \text{Const.}$$

$$\begin{aligned} \int \frac{\partial x}{\sqrt{1-x^2}} &= \text{Arc Sin } x = \text{Arc Cos } \sqrt{1-x^2} = \frac{1}{2} \text{Arc Cos } (1-2x^2) \\ &= \text{Arc Tang } \frac{x}{\sqrt{1-x^2}} = \text{Arc Cot } \frac{\sqrt{1-x^2}}{x} = \text{Arc Sec } \frac{1}{\sqrt{1-x^2}} \\ &= \text{Arc Cosec } \frac{1}{x} = \frac{1}{2} \text{Arc Sin vers } 2x^2. \end{aligned}$$

Das Integral  $\int \frac{\partial x}{\sqrt{(\pm a+bx^2)}}$  kann nur alsdann, wenn das obere Zeichen gilt, für  $x=0$  verschwinden, und in diesem Falle ist

$$\int \frac{\partial x}{\sqrt{(\pm a+bx^2)}} = \frac{1}{\sqrt{b}} \log \left( x\sqrt{\frac{b}{a}} + \sqrt{\frac{a+bx^2}{a}} \right)$$

Taf. XXIV.

$$\int \frac{x^m dx}{V(a+bx^2)}$$

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$$VZ. \quad a + bx^2 = X$$


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$$\int \frac{dx}{VX} = \int \frac{dx}{VX} \quad (\text{Man s. die vorhergehende Seite.})$$

$$\int \frac{x dx}{VX} = \frac{VX}{b}$$

$$\int \frac{x^2 dx}{VX} = \frac{xVX}{2b} - \frac{a}{2b} \int \frac{dx}{VX}$$

$$\int \frac{x^3 dx}{VX} = \left( \frac{x^2}{3b} - \frac{2a}{3b^2} \right) VX$$

$$\int \frac{x^4 dx}{VX} = \left( \frac{x^3}{4b} - \frac{3ax}{8b^2} \right) VX + \frac{3a^2}{8b^2} \int \frac{dx}{VX}$$

$$\int \frac{x^5 dx}{VX} = \left( \frac{x^4}{5b} - \frac{4ax^2}{15b^2} + \frac{8a^2}{15b^3} \right) VX$$

$$\int \frac{x^6 dx}{VX} = \left( \frac{x^5}{6b} - \frac{5ax^3}{24b^2} + \frac{5a^2x}{16b^3} \right) VX - \frac{5a^3}{16b^3} \int \frac{dx}{VX}$$

$$\int \frac{x^7 dx}{VX} = \left( \frac{x^6}{7b} - \frac{6ax^4}{35b^2} + \frac{8a^2x^2}{35b^3} - \frac{16a^3}{35b^4} \right) VX$$

$$\int \frac{x^8 dx}{VX} = \left( \frac{x^7}{8b} - \frac{7ax^5}{48b^2} + \frac{35a^2x^3}{192b^3} - \frac{35a^3x}{128b^4} \right) VX + \frac{35a^4}{128b^4} \int \frac{dx}{VX}$$

$$\int \frac{x^9 dx}{VX} = \left( \frac{x^8}{9b} - \frac{8ax^6}{63b^2} + \frac{16a^2x^4}{105b^3} - \frac{64a^3x^2}{315b^4} + \frac{128a^4}{315b^5} \right) VX$$

$$\int \frac{x^{10} dx}{VX} = \left( \frac{x^9}{10b} - \frac{9ax^7}{80b^2} + \frac{21a^2x^5}{160b^3} - \frac{21a^3x}{128b^4} + \frac{63a^4x}{256b^5} \right) VX - \frac{63a^5}{256b^5} \int \frac{dx}{VX}$$

$$\int \frac{x^{11} dx}{VX} = \left( \frac{x^{10}}{11b} - \frac{10ax^8}{99b^2} + \frac{80a^2x^6}{693b^3} - \frac{32a^3x^4}{231b^4} + \frac{128a^4x^2}{693b^5} - \frac{256a^5}{693b^6} \right) VX$$

$$\int \frac{x^{12} dx}{VX} = \left( \frac{x^{11}}{12b} - \frac{11ax^9}{120b^2} + \frac{33a^2x^7}{320b^3} - \frac{77a^3x^5}{640b^4} + \frac{77a^4x^3}{512b^5} - \frac{231a^5x}{1024b^6} \right) VX + \frac{231a^6}{1024b^6} \int \frac{dx}{VX}$$

$$\int \frac{x^m dx}{\sqrt{1-x^2}}$$

Taf. XXV.

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$$\text{VZ. } 1 - x^2 = X$$


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$$\int \frac{\partial x}{\sqrt{X}} = \text{Arc Sin } x \quad (\text{Die andern Formen Seite 141})$$

$$\int \frac{x \partial x}{\sqrt{X}} = -\sqrt{X}$$

$$\int \frac{x^2 \partial x}{\sqrt{X}} = -\frac{1}{2}x\sqrt{X} + \frac{1}{2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^3 \partial x}{\sqrt{X}} = -\left(\frac{1}{3}x^2 + \frac{2}{3}\right)\sqrt{X}$$

$$\int \frac{x^4 \partial x}{\sqrt{X}} = -\left(\frac{1}{4}x^3 + \frac{3}{8}x\right)\sqrt{X} + \frac{5}{8}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^5 \partial x}{\sqrt{X}} = -\left(\frac{1}{5}x^4 + \frac{4}{15}x^2 + \frac{8}{15}\right)\sqrt{X}$$

$$\int \frac{x^6 \partial x}{\sqrt{X}} = -\left(\frac{1}{6}x^5 + \frac{5}{24}x^3 + \frac{5}{16}x\right)\sqrt{X} + \frac{5}{16}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^7 \partial x}{\sqrt{X}} = -\left(\frac{1}{7}x^6 + \frac{6}{55}x^4 + \frac{8}{55}x^2 + \frac{16}{35}\right)\sqrt{X}$$

$$\int \frac{x^8 \partial x}{\sqrt{X}} = -\left(\frac{1}{8}x^7 + \frac{7}{48}x^5 + \frac{55}{192}x^3 + \frac{55}{128}x\right)\sqrt{X} + \frac{55}{128}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^9 \partial x}{\sqrt{X}} = -\left(\frac{1}{9}x^8 + \frac{8}{63}x^6 + \frac{16}{105}x^4 + \frac{64}{315}x^2 + \frac{128}{315}\right)\sqrt{X}$$

$$\int \frac{x^{10} \partial x}{\sqrt{X}} = -\left(\frac{1}{10}x^9 + \frac{9}{80}x^7 + \frac{21}{160}x^5 + \frac{21}{128}x^3 + \frac{63}{256}x\right)\sqrt{X} + \frac{63}{256}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^{11} \partial x}{\sqrt{X}} = -\left(\frac{1}{11}x^{10} + \frac{10}{99}x^8 + \frac{80}{693}x^6 + \frac{52}{231}x^4 + \frac{128}{693}x^2 + \frac{256}{693}\right)\sqrt{X}$$

$$\int \frac{x^{12} \partial x}{\sqrt{X}} = -\left(\frac{1}{12}x^{11} + \frac{11}{120}x^9 + \frac{33}{520}x^7 + \frac{77}{640}x^5 + \frac{77}{512}x^3 + \frac{231}{1024}x\right)\sqrt{X} + \frac{231}{1024}\int \frac{\partial x}{\sqrt{X}}$$

Taf. XXVI.

$$\int \frac{\partial x}{x^2 \sqrt{a+bx^2}}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{\partial x}{x \sqrt{X}} = \int \frac{\partial x}{x \sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{\partial x}{x^2 \sqrt{X}} = -\frac{\sqrt{X}}{ax}$$

$$\int \frac{\partial x}{x^3 \sqrt{X}} = -\frac{\sqrt{X}}{2ax^2} - \frac{b}{2a} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^4 \sqrt{X}} = \left( -\frac{1}{3ax^3} + \frac{2b}{3a^2x} \right) \sqrt{X}$$

$$\int \frac{\partial x}{x^5 \sqrt{X}} = \left( -\frac{1}{4ax^4} + \frac{3b}{8a^2x^2} \right) \sqrt{X} + \frac{3b^2}{8a^2} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^6 \sqrt{X}} = \left( -\frac{1}{5ax^5} + \frac{4b}{15a^2x^3} - \frac{8b^2}{15a^3x} \right) \sqrt{X}$$

$$\int \frac{\partial x}{x^7 \sqrt{X}} = \left( -\frac{1}{6ax^6} + \frac{5b}{24a^2x^4} - \frac{5b^2}{16a^3x^2} \right) \sqrt{X} - \frac{5b^3}{16a^3} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^8 \sqrt{X}} = \left( -\frac{1}{7ax^7} + \frac{6b}{35a^2x^5} - \frac{8b^2}{55a^3x^3} + \frac{16b^3}{35a^4x} \right) \sqrt{X}$$

$$\int \frac{\partial x}{x^9 \sqrt{X}} = \left( -\frac{1}{8ax^8} + \frac{7b}{48a^2x^6} - \frac{35b^2}{192a^3x^4} + \frac{35b^3}{128a^4x^2} \right) \sqrt{X} + \frac{35b^3}{128a^4} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^{10} \sqrt{X}} = \left( -\frac{1}{9ax^9} + \frac{8b}{63a^2x^7} - \frac{16b^2}{105a^3x^5} + \frac{64b^3}{315a^4x^3} - \frac{128b^4}{315a^5x} \right) \sqrt{X}$$

$$\int \frac{\partial x}{x^{11} \sqrt{X}} = \left( -\frac{1}{10ax^{10}} + \frac{9b}{80a^2x^8} - \frac{21b^2}{160a^3x^6} + \frac{21b^3}{128a^4x^4} - \frac{65b^4}{256a^5x^2} \right) \sqrt{X} - \frac{63b^4}{256a^5} \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x}{x^{12} \sqrt{X}} = \left( -\frac{1}{11ax^{11}} + \frac{10b}{99a^2x^9} - \frac{80b^2}{693a^3x^7} + \frac{52b^3}{231a^4x^5} - \frac{128b^4}{693a^5x^3} + \frac{256b^5}{695a^6x} \right) \sqrt{X}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{\partial x}{x\sqrt{a+bx^2}} = \frac{1}{2\sqrt{a}} \log \frac{\sqrt{a+bx^2} - \sqrt{a}}{\sqrt{a+bx^2} + \sqrt{a}} + \text{Const.}$$

$$\text{oder } \int \frac{\partial x}{x\sqrt{a+bx^2}} = \frac{1}{\sqrt{-a}} \text{Arc Sec } x\sqrt{-\frac{b}{a}} + \text{Const.}$$

Der erste Ausdruck wird reell, wenn  $a$  positiv, der zweite, wenn  $a$  negativ ist. Zugleich können  $b$  und  $a$  nicht negativ seyn. Hieraus ergibt sich

$$\begin{aligned} \text{I. } \int \frac{\partial x}{x\sqrt{a+bx^2}} &= \frac{1}{2\sqrt{a}} \log \frac{\sqrt{a+bx^2} - \sqrt{a}}{\sqrt{a+bx^2} + \sqrt{a}} + \text{Const.} \\ &= \frac{1}{\sqrt{a}} \log \frac{\sqrt{a+bx^2} - \sqrt{a}}{x} + \text{Const.} \end{aligned}$$

wo  $\sqrt{a}$  sowohl positiv als negativ genommen werden kann. Dieses Integral kann nicht für  $x=0$  verschwinden.

$$\begin{aligned} \text{II. } \int \frac{\partial x}{x\sqrt{-a+bx^2}} &= \frac{1}{\sqrt{a}} \text{Arc Sec } x\sqrt{\frac{b}{a}} = \frac{1}{\sqrt{a}} \text{Arc Tang } \sqrt{\frac{bx^2-a}{a}} \\ &= \frac{1}{\sqrt{a}} \text{Arc Cot } \sqrt{\frac{a}{bx^2-a}} = \frac{1}{\sqrt{a}} \text{Arc Cosec } \frac{x\sqrt{b}}{\sqrt{bx^2-a}} \\ &= \frac{1}{\sqrt{a}} \text{Arc Cos } \frac{\sqrt{a}}{x\sqrt{b}} = \frac{1}{2\sqrt{a}} \text{Arc Cos } \frac{2a-bx^2}{bx^2} \\ &= \frac{1}{\sqrt{a}} \text{Arc Sin } \frac{\sqrt{bx^2-a}}{x\sqrt{b}} = \frac{1}{2\sqrt{a}} \text{Arc Sin vers } \frac{2(bx^2-a)}{bx^2} \end{aligned}$$

Diese Ausdrücke verschwinden sämmtlich für  $x = \sqrt{\frac{a}{b}}$ ; für  $x=0$  kann das Integral nicht verschwinden.

Insbesondere ist

$$\int \frac{\partial x}{x\sqrt{1+x^2}} = \log \frac{\sqrt{1+x^2}-1}{x} + \text{Const.}$$

$$\int \frac{\partial x}{x\sqrt{1-x^2}} = \log \frac{\sqrt{1-x^2}-1}{x} + \text{Const.} = \log \frac{1-\sqrt{1-x^2}}{x} + \text{Const.}$$

$$\begin{aligned} \int \frac{\partial x}{x\sqrt{x^2-1}} &= \text{Arc Sec } x = \text{Arc Tang } \sqrt{x^2-1} = \text{Arc Cot } \sqrt{\frac{1}{x^2-1}} \\ &= \text{Arc Cosec } \frac{x}{\sqrt{x^2-1}} = \text{Arc Cos } \frac{1}{x} = \frac{1}{2} \text{Arc Cos } \frac{2-x^2}{x^2} \\ &= \text{Arc Sin } \frac{\sqrt{x^2-1}}{x} = \frac{1}{2} \text{Arc Sin vers } \frac{2(x^2-1)}{x^2}. \end{aligned}$$

Taf. XXVII.

$$\int \frac{x^m dx}{(a+bx^2)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \frac{x}{a\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = -\frac{1}{b\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = -\frac{x}{b\sqrt{X}} + \frac{1}{b} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left(\frac{x^2}{b} + \frac{2a}{b^2}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left(\frac{x^3}{2b} + \frac{3ax}{2b^2}\right) \frac{1}{\sqrt{X}} - \frac{3a}{2b^2} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left(\frac{x^4}{3b} - \frac{4ax^2}{3b^2} - \frac{8a^2}{3b^3}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left(\frac{x^5}{4b} - \frac{5ax^3}{8b^2} - \frac{15a^2x}{8b^3}\right) \frac{1}{\sqrt{X}} + \frac{15a^2}{8b^3} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left(\frac{x^6}{5b} - \frac{2ax^4}{5b^2} + \frac{8a^2x^2}{5b^3} + \frac{16a^3}{5b^4}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left(\frac{x^7}{6b} - \frac{7ax^5}{24b^2} + \frac{35a^2x^3}{48b^3} + \frac{35a^3x}{16b^4}\right) \frac{1}{\sqrt{X}} - \frac{35a^3}{16b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left(\frac{x^8}{7b} - \frac{8ax^6}{35b^2} + \frac{16a^2x^4}{35b^3} - \frac{64a^3x^2}{55b^4} - \frac{128a^4}{35b^5}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{1}{2}}} = \left(\frac{x^9}{8b} - \frac{3ax^7}{16b^2} + \frac{21a^2x^5}{64b^3} - \frac{105a^3x^3}{128b^4} - \frac{315a^4x}{128b^5}\right) \frac{1}{\sqrt{X}} + \frac{315a^4}{128b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{11} dx}{X^{\frac{1}{2}}} = \left(\frac{x^{10}}{9b} - \frac{10ax^8}{63b^2} + \frac{16a^2x^6}{63b^3} - \frac{32a^3x^4}{63b^4} + \frac{128a^4x^2}{63b^5} + \frac{256a^5}{63b^6}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^{\frac{1}{2}}}$$

Taf. XXVIII.

$$\text{VL. } a + bx^2 = X$$

$$\int \frac{\partial x}{xX^{\frac{1}{2}}} = \frac{1}{a\sqrt{X}} + \frac{1}{a} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{ax} - \frac{2bx}{a^2}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{2ax^2} - \frac{3b}{2a^2}\right) \frac{1}{\sqrt{X}} - \frac{3b}{2a^2} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{3ax^3} + \frac{4b}{3a^2x} + \frac{8b^2x}{3a^3}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^2} + \frac{15b^2}{8a^3}\right) \frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^3} - \frac{8b^2}{5a^3x} - \frac{16b^3x}{5a^4}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{6ax^6} + \frac{7b}{24a^2x^4} - \frac{35b^2}{48a^3x^2} - \frac{35b^3}{16a^4}\right) \frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^8X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^7} + \frac{8b}{35a^2x^5} - \frac{16b^2}{35a^3x^3} + \frac{64b^3}{35a^4x} + \frac{128b^4x}{35a^5}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{8ax^8} + \frac{3b}{16a^2x^6} - \frac{21b^2}{64a^3x^4} + \frac{105b^3}{128a^4x^2} + \frac{315b^4}{128a^5}\right) \frac{1}{\sqrt{X}} + \frac{315b^4}{128a^5} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{1}{2}}} = \left(-\frac{1}{9ax^9} + \frac{10b}{63a^2x^7} - \frac{16b^2}{63a^3x^5} + \frac{32b^3}{63a^4x^3} - \frac{128b^4}{63a^5x} - \frac{256b^5x}{63a^6}\right) \frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^{11}X^{\frac{1}{2}}} = \left(\frac{1}{10ax^{10}} + \frac{11b}{80a^2x^8} - \frac{33b^2}{160a^3x^6} + \frac{231b^3}{640a^4x^4} - \frac{231b^4}{256a^5x^2} - \frac{693b^5}{256a^6}\right) \frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6} \int \frac{\partial x}{x\sqrt{X}}$$



Taf. XXIX.

$$\int \frac{x^m dx}{(a + bx^2)^{\frac{1}{2}}}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \left( \frac{2bx^3}{3a^2} + \frac{x}{a} \right) \frac{1}{X\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = - \frac{1}{3bX\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \frac{x^3}{3aX\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( -\frac{x^2}{b} - \frac{2a}{3b^2} \right) \frac{1}{X\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left( -\frac{4x^3}{3b} - \frac{ax}{b^2} \right) \frac{1}{X\sqrt{X}} + \frac{1}{b^2} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left( \frac{x^4}{b} + \frac{4ax^2}{b^2} + \frac{8a^2}{3b^3} \right) \frac{1}{X\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left( \frac{x^5}{2b} + \frac{10ax^3}{3b^2} + \frac{5a^2x}{2b^3} \right) \frac{1}{X\sqrt{X}} - \frac{5a}{2b^3} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left( \frac{x^6}{3b} - \frac{2ax^4}{b^2} - \frac{8a^2x^2}{b^3} - \frac{16a^3}{3b^4} \right) \frac{1}{X\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{x^7}{4b} - \frac{7ax^5}{8b^2} - \frac{35a^2x^3}{6b^3} - \frac{35a^3x}{8b^4} \right) \frac{1}{X\sqrt{X}} + \frac{35a^2}{8b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left( \frac{x^8}{5b} - \frac{8ax^6}{15b^2} + \frac{16a^2x^4}{5b^3} + \frac{64a^3x^2}{5b^4} + \frac{128a^4}{15b^5} \right) \frac{1}{X\sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{1}{2}}} = \left( \frac{x^9}{6b} - \frac{3ax^7}{8b^2} + \frac{21a^2x^5}{16b^3} + \frac{35a^3x^3}{4b^4} + \frac{105a^4x}{16b^5} \right) \frac{1}{X\sqrt{X}} - \frac{105a^3}{16b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{11} dx}{X^{\frac{1}{2}}} = \left( \frac{x^{10}}{7b} - \frac{2ax^8}{7b^2} + \frac{16a^2x^6}{21b^3} - \frac{32a^3x^4}{7b^4} - \frac{128a^4x^2}{7b^5} - \frac{256a^5}{21b^6} \right) \frac{1}{X\sqrt{X}}$$

$$\int \frac{dx}{x^n(a+bx^2)^{\frac{1}{2}}}$$

Taf. XXX.

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$$\text{VZ. } a+bx^2 = X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left(\frac{4}{3a} + \frac{bx^2}{a^2}\right) \frac{1}{X\sqrt{X}} + \frac{1}{a^2} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = -\frac{1}{axX\sqrt{X}} - \frac{4b}{a} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = -\frac{1}{2ax^2X\sqrt{X}} - \frac{5b}{2a} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{3ax^3} + \frac{2b}{a^2x}\right) \frac{1}{X\sqrt{X}} + \frac{8b^2}{a^2} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{4ax^4} + \frac{7b}{8a^2x^2}\right) \frac{1}{X\sqrt{X}} + \frac{35b^2}{8a^2} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^5} + \frac{8b}{15a^2x^3} - \frac{16b^2}{5a^3x}\right) \frac{1}{X\sqrt{X}} - \frac{64b^3}{5a^3} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{6ax^6} + \frac{3b}{8a^2x^4} - \frac{21b^2}{16a^3x^2}\right) \frac{1}{X\sqrt{X}} - \frac{105b^3}{16a^3} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^7} + \frac{2b}{7a^2x^5} - \frac{16b^2}{21a^3x^3} + \frac{32b^3}{7a^4x}\right) \frac{1}{X\sqrt{X}} + \frac{128b^4}{7a^4} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{8ax^8} + \frac{11b}{48a^2x^6} - \frac{33b^2}{64a^3x^4} + \frac{231b^3}{128a^4x^2}\right) \frac{1}{X\sqrt{X}} + \frac{1155b^4}{128a^4} \int \frac{dx}{xX^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{1}{2}}} = \left(-\frac{1}{9ax^9} + \frac{4b}{21a^2x^7} - \frac{8b^2}{21a^3x^5} + \frac{64b^3}{63a^4x^3} - \frac{128b^4}{21a^5x}\right) \frac{1}{X\sqrt{X}} - \frac{512b^5}{21a^5} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^{11}X^{\frac{1}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{13b}{80a^2x^8} - \frac{143b^2}{480a^3x^6} + \frac{429b^3}{640a^4x^4} - \frac{3003b^4}{1280a^5x^2}\right) \frac{1}{X\sqrt{X}} - \frac{3003b^5}{256a^5} \int \frac{dx}{xX^{\frac{1}{2}}}$$

Taf. XXXI.

$$\int \frac{x^m dx}{(a + bx^2)^{\frac{1}{2}}}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \left( \frac{8b^2x^5}{15a^3} + \frac{4bx^3}{3a^2} + \frac{x}{a} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = - \frac{1}{5bX^2 \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left( \frac{2bx^5}{15a^2} + \frac{x^3}{3a} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( -\frac{x^2}{3b} - \frac{2a}{15b^2} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \frac{x^5}{5aX^2 \sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left( -\frac{x^4}{b} - \frac{4ax^2}{3b^2} - \frac{8a^2}{15b^3} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left( -\frac{23x^5}{15b} - \frac{7ax^3}{3b^2} - \frac{a^2x}{b^3} \right) \frac{1}{X^2 \sqrt{X}} + \frac{1}{b^3} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left( \frac{x^6}{b} + \frac{6ax^4}{b^2} + \frac{8a^2x^2}{b^3} + \frac{16a^3}{5b^4} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{x^7}{2b} + \frac{161ax^5}{30b^2} + \frac{49a^2x^3}{6b^3} + \frac{7a^3x}{2b^4} \right) \frac{1}{X^2 \sqrt{X}} - \frac{7a}{2b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left( \frac{x^8}{3b} - \frac{8ax^6}{3b^2} - \frac{16a^2x^4}{b^3} - \frac{64a^3x^2}{3b^4} - \frac{128a^4}{15b^5} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{1}{2}}} = \left( \frac{x^9}{4b} - \frac{9ax^7}{8b^2} - \frac{483a^2x^5}{40b^3} - \frac{147a^3x^3}{8b^4} - \frac{63a^4x}{8b^5} \right) \frac{1}{X^2 \sqrt{X}} + \frac{65a^2}{8b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{11} dx}{X^{\frac{1}{2}}} = \left( \frac{x^{10}}{5b} - \frac{2ax^8}{5b^2} + \frac{16a^2x^6}{3b^3} + \frac{32a^3x^4}{b^4} + \frac{128a^4x}{3b^5} + \frac{256a^5}{15b^6} \right) \frac{1}{X^2 \sqrt{X}}$$

$$\int \frac{dx}{x^m(a+bx^2)^{\frac{1}{2}}}$$

Taf. XXXII.

$$\text{VL. } a + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left( \frac{23}{15a} + \frac{7bx^2}{3a^2} + \frac{b^2x^4}{a^3} \right) \frac{1}{X^2\sqrt{X}} + \frac{1}{a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{6b}{a^2} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = -\frac{1}{2ax^2X^2\sqrt{X}} - \frac{7b}{2a^2} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left( -\frac{1}{3ax^3} + \frac{8b}{3a^2x} \right) \frac{1}{X^2\sqrt{X}} + \frac{16b^2}{a^2} \int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left( -\frac{1}{4ax^4} + \frac{9b}{8a^2x^2} \right) \frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2} \int \frac{dx}{xX^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left( -\frac{1}{5ax^5} + \frac{2b}{3a^2x^3} - \frac{16b^2}{3a^3x} \right) \frac{1}{X^2\sqrt{X}} - \frac{32b^3}{a^3} \int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left( -\frac{1}{6ax^6} + \frac{11b}{24a^2x^4} - \frac{33b^2}{16a^3x^2} \right) \frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3} \int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = \left( -\frac{1}{7ax^7} + \frac{12b}{35a^2x^5} - \frac{8b^2}{7a^3x^3} + \frac{64b^3}{7a^4x} \right) \frac{1}{X^2\sqrt{X}} + \frac{384b^4}{7a^4} \int \frac{dx}{X^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left( -\frac{1}{8ax^8} + \frac{13b}{48a^2x^6} - \frac{143b^2}{192a^3x^4} + \frac{429b^3}{128a^4x^2} \right) \frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4} \int \frac{dx}{xX^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{1}{2}}} = \left( -\frac{1}{9ax^9} + \frac{2b}{9a^2x^7} - \frac{8b^2}{15a^3x^5} + \frac{16b^3}{9a^4x^3} - \frac{128b^4}{9a^5x} \right) \frac{1}{X^2\sqrt{X}} - \frac{256b^5}{3a^5} \int \frac{dx}{X^{\frac{11}{2}}}$$

$$\int \frac{dx}{x^{11}X^{\frac{1}{2}}} = \left( -\frac{1}{10ax^{10}} + \frac{3b}{16a^2x^8} - \frac{13b^2}{32a^3x^6} + \frac{143b^3}{128a^4x^4} - \frac{1287b^4}{256a^5x^2} \right) \frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5} \int \frac{dx}{xX^{\frac{11}{2}}}$$

Taf. XXXIII.  $\int \frac{x^m dx}{(a+bx^2)^{\frac{3}{2}}}, \int \frac{dx}{x^m(a+bx^2)^{\frac{3}{2}}}$

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VZ.  $a + bx^2 = X$

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$$\int \frac{dx}{X^{\frac{3}{2}}} = \left( \frac{16b^3 x^7}{35a^4} + \frac{8b^2 x^5}{5a^3} + \frac{2bx^3}{a^2} + \frac{x}{a} \right) \frac{1}{X^3 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = -\frac{1}{7bX^3 \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \left( \frac{8b^2 x^7}{105a^3} + \frac{4bx^5}{15a^2} + \frac{x^3}{3a} \right) \frac{1}{X^3 \sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left( -\frac{x^2}{5b} - \frac{2a}{35b^2} \right) \frac{1}{X^3 \sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left( \frac{2bx^7}{35a^2} + \frac{x^5}{5a} \right) \frac{1}{X^3 \sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left( -\frac{x^4}{3b} - \frac{4ax^2}{15b^2} - \frac{8a^2}{105b^3} \right) \frac{1}{X^3 \sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \frac{x^7}{7aX^3 \sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{3}{2}}} = \left( -\frac{x^6}{b} - \frac{2ax^4}{b^2} - \frac{8a^2 x^2}{5b^3} - \frac{16a^3}{35b^4} \right) \frac{1}{X^3 \sqrt{X}}$$

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$$\int \frac{dx}{xX^{\frac{3}{2}}} = \left( \frac{176}{105a} + \frac{58bx^2}{15a^2} + \frac{10b^2 x^4}{3a^3} + \frac{b^3 x^6}{a^4} \right) \frac{1}{X^3 \sqrt{X}} + \frac{1}{a^4} \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx}{x^2 X^{\frac{3}{2}}} = -\frac{1}{axX^3 \sqrt{X}} - \frac{8b}{a} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3 X^{\frac{3}{2}}} = -\frac{1}{2ax^2 X^3 \sqrt{X}} - \frac{9b}{2a} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4 X^{\frac{3}{2}}} = \left( -\frac{1}{3ax^3} + \frac{10b}{3a^2 x} \right) \frac{1}{X^3 \sqrt{X}} + \frac{80b^2}{3a^2} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^5 X^{\frac{3}{2}}} = \left( -\frac{1}{4ax^4} + \frac{11b}{8a^2 x^2} \right) \frac{1}{X^3 \sqrt{X}} + \frac{99b^2}{8a^2} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^6 X^{\frac{3}{2}}} = \left( -\frac{1}{5ax^5} + \frac{4b}{5a^2 x^3} - \frac{8b^2}{a^3 x} \right) \frac{1}{X^3 \sqrt{X}} - \frac{64b^3}{a^3} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^7 X^{\frac{3}{2}}} = \left( -\frac{1}{6ax^6} + \frac{13b}{24a^2 x^4} - \frac{145b^2}{48a^3 x^2} \right) \frac{1}{X^3 \sqrt{X}} - \frac{429b^3}{16a^3} \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$\int x^m dx \sqrt{a+bx^2}$$

Taf. XXXIV.

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$$\text{VL. } a + bx^2 = X$$


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$$\int dx \sqrt{X} = \frac{x\sqrt{X}}{2} + \frac{a}{2} \int \frac{dx}{\sqrt{X}} \quad \text{S. 147. 228}$$

$$\int x dx \sqrt{X} = \frac{X\sqrt{X}}{3b}$$

$$\int x^2 dx \sqrt{X} = \frac{xX\sqrt{X}}{4b} - \frac{a}{4b} \int dx \sqrt{X}$$

$$\int x^3 dx \sqrt{X} = \left( \frac{x^2}{5b} - \frac{2a}{15b^2} \right) X\sqrt{X}$$

$$\int x^4 dx \sqrt{X} = \left( \frac{x^3}{6b} - \frac{ax}{8b^2} \right) X\sqrt{X} + \frac{a^2}{8b^2} \int dx \sqrt{X}$$

$$\int x^5 dx \sqrt{X} = \left( \frac{x^4}{7b} - \frac{4ax^2}{35b^2} + \frac{8a^2}{105b^3} \right) X\sqrt{X}$$

$$\int x^6 dx \sqrt{X} = \left( \frac{x^5}{8b} - \frac{5ax^3}{48b^2} + \frac{5a^2x}{64b^3} \right) X\sqrt{X} - \frac{5a^3}{64b^3} \int dx \sqrt{X}$$

$$\int x^7 dx \sqrt{X} = \left( \frac{x^6}{9b} - \frac{2ax^4}{21b^2} + \frac{8a^2x^2}{105b^3} - \frac{16a^3}{315b^4} \right) X\sqrt{X}$$

$$\int x^8 dx \sqrt{X} = \left( \frac{x^7}{10b} - \frac{7ax^5}{80b^2} + \frac{7a^2x^3}{96b^3} - \frac{7a^3x}{128b^4} \right) X\sqrt{X} + \frac{7a^4}{128b^4} \int dx \sqrt{X}$$

$$\int x^9 dx \sqrt{X} = \left( \frac{x^8}{11b} - \frac{8ax^6}{99b^2} + \frac{16a^2x^4}{231b^3} - \frac{64a^3x^2}{1155b^4} + \frac{128a^4}{3465b^5} \right) X\sqrt{X}$$

$$\int x^{10} dx \sqrt{X} = \left( \frac{x^9}{12b} - \frac{3ax^7}{40b^2} + \frac{21a^2x^5}{320b^3} - \frac{7a^3x^3}{128b^4} + \frac{21a^4x}{512b^5} \right) X\sqrt{X} - \frac{21a^5}{512b^5} \int dx \sqrt{X}$$

$$\int x^{11} dx \sqrt{X} = \left( \frac{x^{10}}{13b} - \frac{10ax^8}{143b^2} + \frac{80a^2x^6}{1287b^3} - \frac{160a^3x^4}{3003b^4} + \frac{128a^4x^2}{3003b^5} - \frac{256a^5}{9009b^6} \right) X\sqrt{X}$$

Taf. XXXV.

$$\int \frac{\partial x V(a+bx^2)}{x^m}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x V X}{x} = V X + a \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^2} = -\frac{V X}{x} + b \int \frac{\partial x}{V X}$$

$$\int \frac{\partial x V X}{x^3} = -\frac{V X}{2x^2} + \frac{b}{2} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^4} = -\frac{X V X}{3ax^3}$$

$$\int \frac{\partial x V X}{x^5} = -\frac{X V X}{4ax^4} + \frac{b V X}{8ax^2} - \frac{b^2}{8a} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^6} = \left(-\frac{1}{5ax^5} + \frac{2b}{15a^2x^3}\right) X V X$$

$$\int \frac{\partial x V X}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{8a^2x^4}\right) X V X - \frac{b^2 V X}{16a^2x^2} + \frac{b^3}{16a^2} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^8} = \left(-\frac{1}{7ax^7} + \frac{4b}{35a^2x^5} - \frac{8b^2}{105a^3x^3}\right) X V X$$

$$\int \frac{\partial x V X}{x^9} = \left(-\frac{1}{8ax^8} + \frac{5b}{48a^2x^6} - \frac{5b^2}{64a^3x^4}\right) X V X + \frac{5b^3 V X}{128a^3x^2} - \frac{5b^4}{128a^3} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{2b}{21a^2x^7} - \frac{8b^2}{105a^3x^5} + \frac{16b^3}{315a^4x^3}\right) X V X$$

$$\int \frac{\partial x V X}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{7b}{80a^2x^8} - \frac{7b^2}{96a^3x^6} + \frac{7b^3}{128a^4x^4}\right) X V X - \frac{7b^4 V X}{256a^4x^2} + \frac{7b^5}{256a^4} \int \frac{\partial x}{x V X}$$

$$\int \frac{\partial x V X}{x^{12}} = \left(-\frac{1}{11ax^{11}} + \frac{8b}{99a^2x^9} - \frac{16b^2}{231a^3x^7} + \frac{64b^3}{1155a^4x^5} - \frac{128b^4}{3465a^5x^3}\right) X V X$$

$$\int x^n dx (a + bx^2)^{\frac{1}{2}}$$

Taf. XXXVI.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X}{4} + \frac{3a}{8} \right) x \sqrt{X} + \frac{3a^2}{8} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^2 \sqrt{X}}{5b}$$

$$\int x^2 dx X^{\frac{1}{2}} = \frac{x X^2 \sqrt{X}}{6b} - \frac{a}{6b} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{7b} - \frac{2a}{35b^2} \right) X^2 \sqrt{X}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{8b} - \frac{ax}{16b^2} \right) X^2 \sqrt{X} + \frac{a^2}{16b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{9b} - \frac{4ax^2}{63b^2} + \frac{8a^2}{315b^3} \right) X^2 \sqrt{X}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{10b} - \frac{ax^3}{16b^2} + \frac{a^2 x}{32b^3} \right) X^2 \sqrt{X} - \frac{a^3}{32b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{x^6}{11b} - \frac{2ax^4}{33b^2} + \frac{8a^2 x^2}{231b^3} - \frac{16a^3}{1155b^4} \right) X^2 \sqrt{X}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{x^7}{12b} - \frac{7ax^5}{120b^2} + \frac{7a^2 x^3}{192b^3} - \frac{7a^3 x}{384b^4} \right) X^2 \sqrt{X} + \frac{7a^4}{384b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{13b} - \frac{8ax^6}{143b^2} + \frac{16a^2 x^4}{429b^3} - \frac{64a^3 x^2}{3003b^4} + \frac{128a^4}{15015b^5} \right) X^2 \sqrt{X}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{14b} - \frac{3ax^7}{56b^2} + \frac{3a^2 x^5}{80b^3} - \frac{3a^3 x^3}{128b^4} + \frac{3a^4 x}{256b^5} \right) X^2 \sqrt{X} - \frac{3a^5}{256b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^{11} dx X^{\frac{1}{2}} = \left( \frac{x^{10}}{15b} - \frac{2ax^8}{39b^2} + \frac{16a^2 x^6}{429b^3} - \frac{32a^3 x^4}{1287b^4} + \frac{128a^4 x^2}{9009b^5} - \frac{256a^5}{45045b^6} \right) X^2 \sqrt{X}$$



Taf. XXXVII.

$$\int \frac{\partial x(a+bx^2)^{\frac{1}{2}}}{x^n}$$

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$$\text{VZ. } a+bx^2 = X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left(\frac{X}{3} + a\right) \sqrt{X} + a^2 \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X^2 \sqrt{X}}{ax} + \frac{4b}{a} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = -\frac{X^2 \sqrt{X}}{2ax^2} + \frac{3b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2x}\right) X^2 \sqrt{X} + \frac{8b^2}{3a^2} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{b}{8a^2x^2}\right) X^2 \sqrt{X} + \frac{3b^2}{8a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = -\frac{X^2 \sqrt{X}}{5ax^5}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{24a^2x^4} + \frac{b^2}{48a^3x^2}\right) X^2 \sqrt{X} - \frac{b^3}{16a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = \left(-\frac{1}{7ax^7} + \frac{2b}{35a^2x^5}\right) X^2 \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{b}{16a^2x^6} - \frac{b^2}{64a^3x^4} - \frac{b^3}{128a^4x^2}\right) X^2 \sqrt{X} + \frac{3b^4}{128a^4} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{4b}{63a^2x^7} - \frac{8b^2}{315a^3x^5}\right) X^2 \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{b}{16a^2x^8} - \frac{b^2}{32a^3x^6} + \frac{b^3}{128a^4x^4} + \frac{b^4}{256a^5x^2}\right) X^2 \sqrt{X} - \frac{3b^5}{256a^5} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int x^n dx (a + bx^2)^{\frac{1}{2}}$$

Taf. XXXVIII

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$$\text{VZ. } a + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^2}{6} + \frac{5aX}{24} + \frac{5a^2}{16} \right) x \sqrt{X} + \frac{5a^2}{16} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^2 \sqrt{X}}{7b}$$

$$\int x^2 dx X^{\frac{1}{2}} = \frac{x X^2 \sqrt{X}}{8b} - \frac{a}{8b} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{9b} - \frac{2a}{63b^2} \right) X^2 \sqrt{X}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{10b} - \frac{3ax}{80b^2} \right) X^2 \sqrt{X} + \frac{3a^2}{80b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{11b} - \frac{4ax^2}{99b^2} + \frac{8a^2}{693b^3} \right) X^2 \sqrt{X}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{12b} - \frac{ax^3}{24b^2} + \frac{a^2x}{64b^3} \right) X^2 \sqrt{X} - \frac{a^3}{64b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{x^6}{13b} - \frac{6ax^4}{143b^2} + \frac{8a^2x^2}{429b^3} - \frac{16a^3}{3003b^4} \right) X^2 \sqrt{X}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{x^7}{14b} - \frac{ax^5}{24b^2} + \frac{a^2x^3}{48b^3} - \frac{a^3x}{128b^4} \right) X^2 \sqrt{X} + \frac{a^4}{128b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{15b} - \frac{8ax^6}{195b^2} + \frac{16a^2x^4}{715b^3} - \frac{64a^3x^2}{6435b^4} + \frac{128a^4}{45045b^5} \right) X^2 \sqrt{X}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{16b} - \frac{9ax^7}{224b^2} + \frac{3a^2x^5}{128b^3} - \frac{3a^3x^3}{256b^4} + \frac{9a^4x}{2048b^5} \right) X^2 \sqrt{X} - \frac{9a^5}{2048b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^{11} dx X^{\frac{1}{2}} = \left( \frac{x^{10}}{17b} - \frac{2ax^8}{51b^2} + \frac{16a^2x^6}{663b^3} - \frac{32a^3x^4}{2431b^4} + \frac{128a^4x^2}{21879b^5} - \frac{256a^5}{153153b^6} \right) X^2 \sqrt{X}$$

Taf. XXXIX.

$$\int \frac{dx(a+bx^2)^{\frac{5}{2}}}{x^m}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx X^{\frac{5}{2}}}{x} = \left( \frac{X^2}{5} + \frac{aX}{3} + a^2 \right) \sqrt{X} + a^3 \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^2} = -\frac{X^3 \sqrt{X}}{ax} + \frac{6b}{a} \int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^3} = -\frac{X^3 \sqrt{X}}{2ax^2} + \frac{5b}{2a} \int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^4} = \left( -\frac{1}{5ax^3} - \frac{4b}{3a^2x} \right) X^3 \sqrt{X} + \frac{8b^2}{a^2} \int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{3b}{8a^2x^2} \right) X^3 \sqrt{X} + \frac{15b^2}{8a^2} \int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^6} = \left( -\frac{1}{5ax^5} - \frac{2b}{15a^2x^3} - \frac{8b^2}{15a^3x} \right) X^3 \sqrt{X} + \frac{16b^3}{5a^3} \int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^7} = \left( -\frac{1}{6ax^6} - \frac{b}{24a^2x^4} - \frac{b^2}{16a^3x^2} \right) X^3 \sqrt{X} + \frac{5b^3}{16a^3} \int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^8} = -\frac{X^3 \sqrt{X}}{7ax^7}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^9} = \left( -\frac{1}{8ax^8} + \frac{b}{48a^2x^6} + \frac{b^2}{192a^3x^4} + \frac{b^3}{128a^4x^2} \right) X^3 \sqrt{X} - \frac{5b^4}{128a^4} \int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{10}} = \left( -\frac{1}{9ax^9} + \frac{2b}{63a^2x^7} \right) X^3 \sqrt{X}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{11}} = \left( -\frac{1}{10ax^{10}} + \frac{3b}{80a^2x^8} - \frac{b^2}{160a^3x^6} - \frac{b^3}{640a^4x^4} - \frac{5b^4}{1280a^5x^2} \right) X^3 \sqrt{X}$$

$$+ \frac{5b^5}{256a^5} \int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{12}} = \left( -\frac{1}{11ax^{11}} + \frac{4b}{99a^2x^9} - \frac{8b^2}{693a^3x^7} \right) X^3 \sqrt{X}$$

$$\int x^n dx (a + bx^2)^{\frac{1}{2}}$$

Taf. XL.

$$\text{VZ. } a + bx^2 = X$$

$$\int dx X^{\frac{1}{2}} = \left( \frac{X^3}{8} + \frac{7aX^2}{48} + \frac{35a^2X}{192} + \frac{35a^3}{128} \right) x \sqrt{X} + \frac{35a^4}{128} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^4 \sqrt{X}}{96}$$

$$\int x^2 dx X^{\frac{1}{2}} = \frac{x X^4 \sqrt{X}}{10b} - \frac{a}{10b} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{11b} - \frac{2a}{99b^2} \right) X^4 \sqrt{X}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{12b} - \frac{ax}{40b^2} \right) X^4 \sqrt{X} + \frac{a^2}{40b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{13b} - \frac{4ax^2}{143b^2} + \frac{8a^2}{1287b^3} \right) X^4 \sqrt{X}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{14b} - \frac{5ax^3}{168b^2} + \frac{a^2x}{112b^3} \right) X^4 \sqrt{X} - \frac{a^3}{112b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{x^6}{15b} - \frac{2ax^4}{65b^2} + \frac{8a^2x^2}{715b^3} - \frac{16a^3}{6435b^4} \right) X^4 \sqrt{X}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{x^7}{16b} - \frac{ax^5}{32b^2} + \frac{5a^2x^3}{384b^3} - \frac{a^3x}{256b^4} \right) X^4 \sqrt{X} + \frac{7a^4}{256b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{17b} - \frac{8ax^6}{255b^2} + \frac{16a^2x^4}{1105b^3} - \frac{64a^3x^2}{12155b^4} + \frac{128b^5}{109395b^5} \right) X^4 \sqrt{X}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{18b} - \frac{ax^7}{32b^2} + \frac{a^2x^5}{64b^3} - \frac{5a^3x^3}{768b^4} + \frac{a^4x}{512b^5} \right) X^4 \sqrt{X}$$

$$- \frac{7a^5}{512b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^{11} dx X^{\frac{1}{2}} = \left( \frac{x^{10}}{19b} - \frac{10ax^8}{323b^2} + \frac{16a^2x^6}{969b^3} - \frac{32a^3x^4}{4199b^4} + \frac{128a^4x^2}{46189b^5} \right. \\ \left. - \frac{256a^5}{415701b^6} \right) X^4 \sqrt{X}$$

Taf. XLI.

$$\int \frac{\partial x (a + bx^2)^{\frac{7}{2}}}{x^m}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\begin{aligned} \int \frac{\partial x X^{\frac{7}{2}}}{x} &= \left( \frac{X^3}{7} + \frac{aX^2}{5} + \frac{a^2X}{3} + a^3 \right) \sqrt{X} + a^4 \int \frac{\partial x}{x \sqrt{X}} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^2} &= -\frac{X^{\frac{3}{2}} \sqrt{X}}{ax} + \frac{8b}{a} \int \partial x X^{\frac{7}{2}} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^3} &= -\frac{X^{\frac{3}{2}} \sqrt{X}}{2ax^2} + \frac{7b}{2a} \int \frac{\partial x X^{\frac{7}{2}}}{x} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^4} &= \left( -\frac{1}{3ax^3} - \frac{2b}{a^2x} \right) X^{\frac{3}{2}} \sqrt{X} + \frac{16b^2}{a^2} \int \partial x X^{\frac{7}{2}} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^5} &= \left( -\frac{1}{4ax^4} - \frac{5b}{8a^2x^2} \right) X^{\frac{3}{2}} \sqrt{X} + \frac{35b^2}{8a^2} \int \frac{\partial x X^{\frac{7}{2}}}{x} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^6} &= \left( -\frac{1}{5ax^5} - \frac{4b}{15a^2x^3} - \frac{8b^2}{5a^3x} \right) X^{\frac{3}{2}} \sqrt{X} + \frac{64b^3}{5a^3} \int \partial x X^{\frac{7}{2}} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^7} &= \left( -\frac{1}{6ax^6} - \frac{b}{8a^2x^4} - \frac{5b^2}{16a^3x^2} \right) X^{\frac{3}{2}} \sqrt{X} + \frac{35b^3}{16a^3} \int \frac{\partial x X^{\frac{7}{2}}}{x} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^8} &= \left( -\frac{1}{7ax^7} - \frac{2b}{35a^2x^5} - \frac{8b^2}{105a^3x^3} - \frac{16b^3}{35a^4x} \right) X^{\frac{3}{2}} \sqrt{X} \\ &\quad + \frac{128b^4}{35a^4} \int \partial x X^{\frac{7}{2}} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^9} &= \left( -\frac{1}{8ax^8} - \frac{b}{48a^2x^6} - \frac{b^2}{64a^3x^4} - \frac{5b^3}{128a^4x^2} \right) X^{\frac{3}{2}} \sqrt{X} \\ &\quad + \frac{35b^4}{128a^4} \int \frac{\partial x X^{\frac{7}{2}}}{x} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^{10}} &= -\frac{X^{\frac{3}{2}} \sqrt{X}}{9ax^9} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^{11}} &= \left( -\frac{1}{10ax^{10}} + \frac{b}{80a^2x^8} + \frac{b^2}{480a^3x^6} + \frac{b^3}{640a^4x^4} + \frac{b^4}{256a^5x^2} \right) X^{\frac{3}{2}} \sqrt{X} \\ &\quad - \frac{7b^5}{256a^5} \int \frac{\partial x X^{\frac{7}{2}}}{x} \\ \int \frac{\partial x X^{\frac{7}{2}}}{x^{12}} &= \left( -\frac{1}{11ax^{11}} + \frac{2b}{99a^2x^9} \right) X^{\frac{3}{2}} \sqrt{X} \end{aligned}$$

$$\int x^m dx (a + bx^2)^{\frac{1}{2}}, \int \frac{dx(a + bx^2)^{\frac{1}{2}}}{x^m} \quad \text{Taf. XLII.}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^4}{10} + \frac{9aX^3}{80} + \frac{21a^2X^2}{160} + \frac{21a^3X}{128} + \frac{63a^4}{256} \right) \sqrt{X} + \frac{63a^5}{256} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^{\frac{1}{2}} \sqrt{X}}{11b}$$

$$\int x^2 dx X^{\frac{1}{2}} = \frac{x X^{\frac{1}{2}} \sqrt{X}}{12b} - \frac{a}{12b} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{13b} - \frac{2a}{143b^2} \right) X^{\frac{1}{2}} \sqrt{X}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{14b} - \frac{ax}{56b^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{a^2}{56b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{15b} - \frac{4ax^2}{195b^2} + \frac{8a^2}{2145b^3} \right) X^{\frac{1}{2}} \sqrt{X}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{16b} - \frac{5ax^3}{224b^2} + \frac{5a^2x}{896b^3} \right) X^{\frac{1}{2}} \sqrt{X} - \frac{5a^3}{896b^3} \int dx X^{\frac{1}{2}}$$


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$$\int \frac{dx X^{\frac{1}{2}}}{x} = \left( \frac{X^4}{9} + \frac{aX^3}{7} + \frac{a^2X^2}{5} + \frac{a^3X}{3} + a^4 \right) \sqrt{X} + a^5 \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^2} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{ax} + \frac{10b}{a} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^3} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{2ax^2} + \frac{9b}{2a} \int \frac{dx X^{\frac{1}{2}}}{x}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^4} = \left( -\frac{1}{3ax^3} - \frac{8b}{3a^2x} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{80b^2}{3a^2} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{7b}{8a^2x^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{63b^2}{8a^2} \int \frac{dx X^{\frac{1}{2}}}{x}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^6} = \left( -\frac{1}{5ax^5} - \frac{2b}{5a^2x^3} - \frac{16b^2}{5a^3x} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{32b^3}{a^3} \int dx X^{\frac{1}{2}}$$

Taf. XLIII.

$$\int \frac{dx}{(ax+bx^2)^{\frac{1}{2}}}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(-\frac{1}{3X} + \frac{8b}{3a^2}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(-\frac{1}{5X^2} + \frac{4^2b}{15a^2X} - \frac{2 \cdot 4^3b^2}{15a^4}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left(-\frac{1}{7X^3} + \frac{6 \cdot 4b}{35a^2X^2} - \frac{2 \cdot 4^3b^2}{35a^4X} + \frac{4^5b^3}{35a^6}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left(-\frac{1}{9X^4} + \frac{2 \cdot 4^2b}{63a^2X^3} - \frac{4^4b^2}{105a^4X^2} + \frac{4^6b^3}{315a^6X} - \frac{2 \cdot 4^7b^4}{315a^8}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left(-\frac{1}{11X^5} + \frac{10 \cdot 4b}{99a^2X^4} - \frac{5 \cdot 4^4b^2}{693a^4X^3} + \frac{2 \cdot 4^5b^3}{231a^6X^2} - \frac{2 \cdot 4^7b^4}{693a^8X} + \frac{4^9b^5}{693a^{10}}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left(-\frac{1}{13X^6} + \frac{3 \cdot 4^2b}{143a^2X^5} - \frac{10 \cdot 4^3b^2}{429a^4X^4} + \frac{5 \cdot 4^6b^3}{3003a^6X^3} - \frac{2 \cdot 4^7b^4}{1001a^8X^2} + \frac{2 \cdot 4^9b^5}{3003a^{10}X} - \frac{4^{11}b^6}{3003a^{12}}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left(-\frac{1}{15X^7} + \frac{14b}{195a^2X^6} - \frac{14 \cdot 4^3b^2}{715a^4X^5} + \frac{7 \cdot 4^5b^3}{1287a^6X^4} - \frac{2 \cdot 4^7b^4}{1287a^8X^3} + \frac{4^9b^5}{2145a^{10}X^2} - \frac{4^{11}b^6}{6435a^{12}X} + \frac{2 \cdot 4^{12}b^7}{6435a^{14}}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = -\frac{2(2bx+a)}{17a^2X^{\frac{17}{2}}} - \frac{4^3b}{17a^2} \int \frac{dx}{X^{\frac{17}{2}}}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{dx}{\sqrt{(ax+bx^2)}} = \frac{1}{\sqrt{b}} \log \frac{\sqrt{(ax+bx^2)} + x\sqrt{b}}{\sqrt{(ax+bx^2)} - x\sqrt{b}}$$

oder  $\int \frac{dx}{\sqrt{(ax+bx^2)}} = \frac{2}{\sqrt{-b}} \text{Arc Tang} \frac{x\sqrt{-b}}{\sqrt{(ax+bx^2)}}$ ,

woraus sich ergibt, daß in jedem Falle der erste Ausdruck reell wird, wenn  $b$  positiv, der zweite, wenn  $b$  negativ ist. Hieraus erhält man

$$\begin{aligned} \text{I. } \int \frac{dx}{\sqrt{(ax+bx^2)}} &= \pm \frac{1}{\sqrt{b}} \log \frac{\sqrt{(ax+bx^2)} \pm x\sqrt{b}}{\sqrt{(ax+bx^2)} \mp x\sqrt{b}} \\ &= \pm \frac{1}{\sqrt{b}} \log \frac{\sqrt{(a+bx)} \pm \sqrt{bx}}{\sqrt{(a+bx)} \mp \sqrt{bx}} \\ &= \pm \frac{1}{\sqrt{b}} \log \frac{2bx+a \pm 2\sqrt{b} \cdot \sqrt{(ax+bx^2)}}{a} \\ &= \pm \frac{2}{\sqrt{b}} \log \frac{\sqrt{(a+bx)} \pm \sqrt{bx}}{\sqrt{a}}. \end{aligned}$$

Die oberen Zeichen gehören hier zusammen, und eben so die unteren.

$$\begin{aligned} \text{II. } \int \frac{dx}{\sqrt{(ax-bx^2)}} &= \frac{2}{\sqrt{b}} \text{Arc Tang} \frac{x\sqrt{b}}{\sqrt{(ax-bx^2)}} = \frac{2}{\sqrt{b}} \text{Arc Tang} \sqrt{\frac{bx}{a-bx}} \\ &= \frac{2}{\sqrt{b}} \text{Arc Cot} \sqrt{\frac{a-bx}{bx}} = \frac{2}{\sqrt{b}} \text{Arc Sec} \sqrt{\frac{a}{a-bx}} \\ &= \frac{2}{\sqrt{b}} \text{Arc Cosec} \sqrt{\frac{a}{bx}} = \frac{2}{\sqrt{b}} \text{Arc Sin} \sqrt{\frac{bx}{a}} \\ &= \frac{2}{\sqrt{b}} \text{Arc Cos} \sqrt{\frac{a-bx}{a}} = \frac{1}{\sqrt{b}} \text{Arc Cos} \frac{a-2bx}{a} \\ &= \frac{1}{\sqrt{b}} \text{Arc Sin vers} \frac{2bx}{a}. \end{aligned}$$

Sämmtliche Integrale auf dieser Seite verschwinden für  $x=0$ .

Inbesondere ist

$$\begin{aligned} \int \frac{dx}{\sqrt{(x^2+x)}} &= \pm \log [2x+1 \pm 2\sqrt{(x^2+x)}] \\ \int \frac{dx}{\sqrt{(x^2-x)}} &= \pm \log [1-2x \mp 2\sqrt{(x^2-x)}]. \end{aligned}$$



Taf. XLIV.

$$\int \frac{x^2 dx}{V(ax+bx^2)}$$

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$$VZ. \quad ax + bx^2 = X$$


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$$\int \frac{dx}{VX} = \int \frac{dx}{VX} \quad (\text{Man s. die vorhergehende Seite.})$$

$$\int \frac{x dx}{VX} = \frac{VX}{b} - \frac{a}{2b} \int \frac{dx}{VX}$$

$$\int \frac{x^2 dx}{VX} = \left( \frac{x}{2b} - \frac{3a}{4b^2} \right) VX + \frac{3a^2}{8b^2} \int \frac{dx}{VX}$$

$$\int \frac{x^3 dx}{VX} = \left( \frac{x^2}{3b} - \frac{5ax}{12b^2} + \frac{5a^2}{8b^3} \right) VX - \frac{5a^3}{16b^3} \int \frac{dx}{VX}$$

$$\int \frac{x^4 dx}{VX} = \left( \frac{x^3}{4b} - \frac{7ax^2}{24b^2} + \frac{35a^2x}{96b^3} - \frac{35a^3}{64b^4} \right) VX + \frac{35a^4}{128b^4} \int \frac{dx}{VX}$$

$$\int \frac{x^5 dx}{VX} = \left( \frac{x^4}{5b} - \frac{9ax^3}{40b^2} + \frac{21a^2x^2}{80b^3} - \frac{21a^3x}{64b^4} + \frac{63a^4}{128b^5} \right) VX - \frac{63a^5}{256b^5} \int \frac{dx}{VX}$$

$$\int \frac{x^6 dx}{VX} = \left( \frac{x^5}{6b} - \frac{11ax^4}{60b^2} + \frac{33a^2x^3}{160b^3} - \frac{77a^3x^2}{320b^4} + \frac{77a^4x}{256b^5} - \frac{231a^5}{512b^6} \right) VX + \frac{231a^6}{1024b^6} \int \frac{dx}{VX}$$

$$\int \frac{x^7 dx}{VX} = \left( \frac{x^6}{7b} - \frac{13ax^5}{84b^2} + \frac{143a^2x^4}{840b^3} - \frac{429a^3x^3}{2240b^4} + \frac{143a^4x^2}{640b^5} - \frac{143a^5x}{512b^6} + \frac{429a^6}{1024b^7} \right) VX - \frac{429a^7}{2048b^7} \int \frac{dx}{VX}$$

$$\int \frac{x^8 dx}{VX} = \left( \frac{x^7}{8b} - \frac{15ax^6}{112b^2} + \frac{65a^2x^5}{448b^3} - \frac{143a^3x^4}{896b^4} + \frac{1287a^4x^3}{7168b^5} - \frac{429a^5x^2}{2048b^6} + \frac{2145a^6x}{8192b^7} - \frac{6435a^7}{16384b^8} \right) VX + \frac{6435a^8}{32768b^8} \int \frac{dx}{VX}$$

$$\int \frac{x^9 dx}{VX} = \frac{x^8 VX}{9b} - \frac{17a}{18b} \int \frac{x^8 dx}{VX}$$

$$\int \frac{x^{10} dx}{VX} = \left( \frac{x^9}{10b} - \frac{19ax^8}{180b^2} \right) VX + \frac{323a^2}{360b^2} \int \frac{x^8 dx}{VX}$$

$$\int \frac{\partial x}{x^m \sqrt{ax + bx^2}}$$

Taf. XLV.

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{\partial x}{x \sqrt{X}} = -\frac{2\sqrt{X}}{ax}$$

$$\int \frac{\partial x}{x^2 \sqrt{X}} = \left(-\frac{1}{3ax^2} + \frac{2b}{3a^2x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^3 \sqrt{X}} = \left(-\frac{1}{5ax^3} + \frac{4b}{15a^2x^2} - \frac{8b^2}{15a^3x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^4 \sqrt{X}} = \left(-\frac{1}{7ax^4} + \frac{6b}{35a^2x^3} - \frac{8b^2}{35a^3x^2} + \frac{16b^3}{35a^4x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^5 \sqrt{X}} = \left(-\frac{1}{9ax^5} + \frac{8b}{63a^2x^4} - \frac{16b^2}{105a^3x^3} + \frac{64b^3}{315a^4x^2} - \frac{128b^4}{315a^5x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^6 \sqrt{X}} = \left(-\frac{1}{11ax^6} + \frac{10b}{99a^2x^5} - \frac{80b^2}{693a^3x^4} + \frac{320b^3}{231a^4x^3} - \frac{128b^4}{693a^5x^2} + \frac{256b^5}{693a^6x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^7 \sqrt{X}} = \left(-\frac{1}{13ax^7} + \frac{12b}{143a^2x^6} - \frac{40b^2}{429a^3x^5} + \frac{320b^3}{3003a^4x^4} - \frac{128b^4}{1001a^5x^3} + \frac{512b^5}{3003a^6x^2} - \frac{1024b^6}{3003a^7x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^8 \sqrt{X}} = \left(-\frac{1}{15ax^8} + \frac{14b}{195a^2x^7} - \frac{56b^2}{715a^3x^6} + \frac{112b^3}{1287a^4x^5} - \frac{128b^4}{1287a^5x^4} + \frac{256b^5}{2145a^6x^3} - \frac{1024b^6}{6435a^7x^2} + \frac{2048b^7}{6435a^8x}\right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^9 \sqrt{X}} = -\frac{2\sqrt{X}}{17ax^9} - \frac{16b}{17a} \int \frac{\partial x}{x^8 \sqrt{X}}$$

$$\int \frac{\partial x}{x^{10} \sqrt{X}} = \left(-\frac{1}{19ax^{10}} + \frac{18b}{323a^2x^9}\right) 2\sqrt{X} + \frac{288b^2}{323a^2} \int \frac{\partial x}{x^8 \sqrt{X}}$$

Taf. XLVI

$$\int \frac{x^m dx}{(ax + bx^2)^{\frac{1}{2}}}$$

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$$\sqrt{X} \quad ax + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = -\frac{2(2bx + a)}{a^2 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = \frac{2x}{a \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = -\frac{2x}{b \sqrt{X}} + \frac{1}{b} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( \frac{x^2}{b} + \frac{3ax}{b^2} \right) \frac{1}{\sqrt{X}} - \frac{3a}{2b^2} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left( \frac{x^3}{2b} - \frac{5ax^2}{4b^2} + \frac{15a^2 x}{4b^3} \right) \frac{1}{\sqrt{X}} + \frac{15a^2}{8b^3} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left( \frac{x^4}{3b} - \frac{7ax^3}{12b^2} + \frac{35a^2 x^2}{24b^3} + \frac{35a^3 x}{8b^4} \right) \frac{1}{\sqrt{X}} - \frac{55a^3}{16b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left( \frac{x^5}{4b} - \frac{3ax^4}{8b^2} + \frac{21a^2 x^3}{32b^3} - \frac{105a^3 x^2}{64b^4} + \frac{515a^4 x}{64b^5} \right) \frac{1}{\sqrt{X}} + \frac{515a^4}{128b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left( \frac{x^6}{5b} - \frac{11ax^5}{40b^2} + \frac{53a^2 x^4}{80b^3} - \frac{231a^3 x^3}{320b^4} + \frac{231a^4 x^2}{128b^5} + \frac{695a^5 x}{128b^6} \right) \frac{1}{\sqrt{X}} - \frac{695a^5}{256b^6} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{x^7}{6b} - \frac{13ax^6}{60b^2} + \frac{143a^2 x^5}{480b^3} - \frac{143a^3 x^4}{520b^4} + \frac{1001a^4 x^3}{1280b^5} - \frac{1001a^5 x^2}{512b^6} - \frac{3003a^6 x}{512b^7} \right) \frac{1}{\sqrt{X}} + \frac{3003a^6}{1024b^7} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \frac{x^8}{7b \sqrt{X}} - \frac{15a}{14b} \int \frac{x^8 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^{10} dx}{X^{\frac{1}{2}}} = \left( \frac{x^9}{8b} - \frac{17a}{112b^2} \right) \frac{1}{\sqrt{X}} - \frac{255a^2}{224b^3} \int \frac{x^9 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{\partial x}{x^m(ax+bx^2)^{\frac{1}{2}}}$$

Taf. XLVII.

$$\text{VZ. } ax+bx^2=X$$

$$\int \frac{\partial x}{xX^{\frac{1}{2}}} = -\frac{2}{3ax\sqrt{X}} - \frac{4b}{3a} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^2} + \frac{2b}{5a^2x}\right) \frac{2}{\sqrt{X}} + \frac{8b^2}{5a^2} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^3} + \frac{8b}{35a^2x^2} - \frac{16b^2}{35a^3x}\right) \frac{2}{\sqrt{X}} - \frac{64b^3}{35a^3} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{9ax^4} + \frac{10b}{63a^2x^3} - \frac{16b^2}{63a^3x^2} + \frac{32b^3}{63a^4x}\right) \frac{2}{\sqrt{X}} + \frac{128b^4}{63a^4} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{11ax^5} + \frac{4b}{33a^2x^4} - \frac{40b^2}{231a^3x^3} + \frac{64b^3}{231a^4x^2} - \frac{128b^4}{231a^5x}\right) \frac{2}{\sqrt{X}} - \frac{512b^5}{231a^5} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{13ax^6} + \frac{14b}{143a^2x^5} - \frac{56b^2}{429a^3x^4} + \frac{80b^3}{429a^4x^3} - \frac{128b^4}{429a^5x^2} + \frac{256b^5}{429a^6x}\right) \frac{2}{\sqrt{X}} + \frac{1024b^6}{429a^6} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{15ax^7} + \frac{16b}{195a^2x^6} - \frac{224b^2}{2145a^3x^5} + \frac{896b^3}{6435a^4x^4} - \frac{256b^4}{1287a^5x^3} + \frac{2048b^5}{6435a^6x^2} - \frac{4096b^6}{6435a^7x}\right) \frac{2}{\sqrt{X}} - \frac{16384b^7}{6435a^7} \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{1}{2}}} = -\frac{2}{17ax^8\sqrt{X}} - \frac{18b}{17a} \int \frac{\partial x}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{19ax^9} + \frac{20b}{323a^2x^8}\right) \frac{2}{\sqrt{X}} + \frac{360b^2}{323a^2} \int \frac{\partial x}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{1}{2}}} = \left(-\frac{1}{21a^2x^{10}} + \frac{22b}{399a^2x^9} - \frac{440b^2}{6783a^3x^8}\right) \frac{2}{\sqrt{X}} - \frac{2640b^3}{2261a^3} \int \frac{\partial x}{x^7X^{\frac{3}{2}}}$$

Taf. XXXIX.

$$\int \frac{\partial x(a+bx^2)^{\frac{1}{2}}}{x^m}$$

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left( \frac{X^2}{5} + \frac{aX}{3} + a^2 \right) \sqrt{X} + a^3 \int \frac{\partial x}{x \sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X^2 \sqrt{X}}{ax} + \frac{6b}{a} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = -\frac{X^3 \sqrt{X}}{2ax^2} + \frac{5b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left( -\frac{1}{3ax^3} - \frac{4b}{3a^2x} \right) X^3 \sqrt{X} + \frac{8b^2}{a^2} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{3b}{8a^2x^2} \right) X^3 \sqrt{X} + \frac{15b^2}{8a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = \left( -\frac{1}{5ax^5} - \frac{2b}{15a^2x^3} - \frac{8b^2}{15a^3x} \right) X^3 \sqrt{X} + \frac{16b^3}{5a^3} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left( -\frac{1}{6ax^6} - \frac{b}{24a^2x^4} - \frac{b^2}{16a^3x^2} \right) X^3 \sqrt{X} + \frac{5b^3}{16a^3} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = -\frac{X^3 \sqrt{X}}{7ax^7}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left( -\frac{1}{8ax^8} + \frac{b}{48a^2x^6} + \frac{b^2}{192a^3x^4} + \frac{b^3}{128a^4x^2} \right) X^3 \sqrt{X} - \frac{5b^4}{128a^4} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} = \left( -\frac{1}{9ax^9} + \frac{2b}{63a^2x^7} \right) X^3 \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{11}} = \left( \frac{1}{10ax^{10}} + \frac{3b}{80a^2x^8} - \frac{b^2}{160a^3x^6} - \frac{b^3}{640a^4x^4} - \frac{3b^4}{1280a^5x^2} \right) X^3 \sqrt{X} + \frac{3b^5}{256a^5} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{12}} = \left( -\frac{1}{11ax^{11}} + \frac{4b}{99a^2x^9} - \frac{8b^2}{693a^3x^7} \right) X^3 \sqrt{X}$$

$$\int x^n dx (a + bx^2)^{\frac{1}{2}}$$

Taf. XL.

$$\text{VZ. } a + bx^2 = X$$

$$\int dx X^{\frac{1}{2}} = \left( \frac{X^3}{8} + \frac{7aX^2}{48} + \frac{35a^2X}{192} + \frac{35a^3}{128} \right) x \sqrt{X} + \frac{35a^4}{128} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^4 \sqrt{X}}{96}$$

$$\int x^2 dx X^{\frac{1}{2}} = \frac{x X^4 \sqrt{X}}{10b} - \frac{a}{10b} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{11b} - \frac{2a}{99b^2} \right) X^4 \sqrt{X}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{12b} - \frac{ax}{40b^2} \right) X^4 \sqrt{X} + \frac{a^2}{40b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{13b} - \frac{4ax^2}{143b^2} + \frac{8a^2}{1287b^3} \right) X^4 \sqrt{X}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{14b} - \frac{5ax^3}{168b^2} + \frac{a^2x}{112b^3} \right) X^4 \sqrt{X} - \frac{a^3}{112b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{x^6}{15b} - \frac{2ax^4}{65b^2} + \frac{8a^2x^2}{715b^3} - \frac{16a^3}{6435b^4} \right) X^4 \sqrt{X}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{x^7}{16b} - \frac{ax^5}{32b^2} + \frac{5a^2x^3}{384b^3} - \frac{a^3x}{256b^4} \right) X^4 \sqrt{X} + \frac{7a^4}{256b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{17b} - \frac{8ax^6}{255b^2} + \frac{16a^2x^4}{1105b^3} - \frac{64a^3x^2}{12155b^4} + \frac{128b^5}{109395b^5} \right) X^4 \sqrt{X}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{18b} - \frac{ax^7}{32b^2} + \frac{a^2x^5}{64b^3} - \frac{5a^3x^3}{768b^4} + \frac{a^4x}{512b^5} \right) X^4 \sqrt{X} - \frac{7a^5}{512b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^{11} dx X^{\frac{1}{2}} = \left( \frac{x^{10}}{19b} - \frac{10ax^8}{323b^2} + \frac{16a^2x^6}{969b^3} - \frac{32a^3x^4}{4199b^4} + \frac{128a^4x^2}{46189b^5} - \frac{256a^5}{415701b^6} \right) X^4 \sqrt{X}$$

Taf. XLI.

$$\int \frac{\partial x(a+bx^2)^{\frac{7}{2}}}{x^m}$$

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$$\text{VZ. } a+bx^2 = X$$


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$$\int \frac{\partial x X^{\frac{7}{2}}}{x} = \left( \frac{X^3}{7} + \frac{aX^2}{5} + \frac{a^2X}{3} + a^3 \right) \sqrt{X} + a^4 \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^2} = -\frac{X^2 \sqrt{X}}{ax} + \frac{8b}{a} \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^3} = -\frac{X^2 \sqrt{X}}{2ax^2} + \frac{7b}{2a} \int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^4} = \left( -\frac{1}{3ax^3} - \frac{2b}{a^2x} \right) X^2 \sqrt{X} + \frac{16b^2}{a^2} \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{5b}{8a^2x^2} \right) X^2 \sqrt{X} + \frac{35b^2}{8a^2} \int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^6} = \left( -\frac{1}{5ax^5} - \frac{4b}{15a^2x^3} - \frac{8b^2}{5a^3x} \right) X^2 \sqrt{X} + \frac{64b^3}{5a^3} \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^7} = \left( -\frac{1}{6ax^6} - \frac{b}{8a^2x^4} - \frac{5b^2}{16a^3x^2} \right) X^2 \sqrt{X} + \frac{35b^3}{16a^3} \int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^8} = \left( -\frac{1}{7ax^7} - \frac{2b}{35a^2x^5} - \frac{8b^2}{105a^3x^3} - \frac{16b^3}{35a^4x} \right) X^2 \sqrt{X} + \frac{128b^4}{35a^4} \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^9} = \left( -\frac{1}{8ax^8} - \frac{b}{48a^2x^6} - \frac{b^2}{64a^3x^4} - \frac{5b^3}{128a^4x^2} \right) X^2 \sqrt{X} + \frac{35b^4}{128a^4} \int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{10}} = -\frac{X^2 \sqrt{X}}{9ax^9}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{11}} = \left( -\frac{1}{10ax^{10}} + \frac{b}{80a^2x^8} + \frac{b^2}{480a^3x^6} + \frac{b^3}{640a^4x^4} + \frac{b^4}{256a^5x^2} \right) X^2 \sqrt{X} - \frac{7b^5}{256a^5} \int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{12}} = \left( -\frac{1}{11ax^{11}} + \frac{2b}{99a^2x^9} \right) X^2 \sqrt{X}$$

$$\int x^m dx (a + bx^2)^{\frac{1}{2}}, \quad \int \frac{dx(a + bx^2)^{\frac{1}{2}}}{x^m} \quad \text{Taf. XLII.}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^4}{10} + \frac{9aX^3}{80} + \frac{21a^2X^2}{160} + \frac{21a^3X}{128} + \frac{63a^4}{256} \right) x \sqrt{X} + \frac{63a^5}{256} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^{\frac{1}{2}} \sqrt{X}}{11b}$$

$$\int x^2 dx X^{\frac{1}{2}} = \frac{x X^{\frac{1}{2}} \sqrt{X}}{12b} - \frac{a}{12b} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{13b} - \frac{2a}{143b^2} \right) X^{\frac{1}{2}} \sqrt{X}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{14b} - \frac{ax}{56b^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{a^2}{56b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{15b} - \frac{4ax^2}{195b^2} + \frac{8a^2}{2145b^3} \right) X^{\frac{1}{2}} \sqrt{X}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{16b} - \frac{5ax^3}{224b^2} + \frac{5a^2x}{896b^3} \right) X^{\frac{1}{2}} \sqrt{X} - \frac{5a^3}{896b^3} \int dx X^{\frac{1}{2}}$$


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$$\int \frac{dx X^{\frac{1}{2}}}{x} = \left( \frac{X^4}{9} + \frac{aX^3}{7} + \frac{a^2X^2}{5} + \frac{a^3X}{3} + a^4 \right) \sqrt{X} + a^5 \int \frac{dx}{x \sqrt{X}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^2} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{ax} + \frac{10b}{a} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^3} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{2ax^2} + \frac{9b}{2a} \int \frac{dx X^{\frac{1}{2}}}{x}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^4} = \left( -\frac{1}{3ax^3} - \frac{8b}{3a^2x} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{80b^2}{3a^2} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^5} = \left( -\frac{1}{4ax^4} - \frac{7b}{8a^2x^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{63b^2}{8a^2} \int \frac{dx X^{\frac{1}{2}}}{x}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^6} = \left( -\frac{1}{5ax^5} - \frac{2b}{5a^2x^3} - \frac{16b^2}{5a^3x} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{32b^3}{a^3} \int dx X^{\frac{1}{2}}$$



Taf. XLIII.

$$\int \frac{dx}{(ax+bx^2)^{\frac{1}{2}}}$$

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$$\text{VL. } ax + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(-\frac{1}{3X} + \frac{8b}{3a^2}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(-\frac{1}{5X^2} + \frac{4^2b}{15a^2X} - \frac{2 \cdot 4^3b^2}{15a^4}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left(-\frac{1}{7X^3} + \frac{6 \cdot 4b}{35a^2X^2} - \frac{2 \cdot 4^3b^2}{35a^4X} + \frac{4^5b^3}{35a^6}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left(-\frac{1}{9X^4} + \frac{2 \cdot 4^2b}{63a^2X^3} - \frac{4^4b^2}{105a^4X^2} + \frac{4^6b^3}{315a^6X} - \frac{2 \cdot 4^7b^4}{315a^8}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left(-\frac{1}{11X^5} + \frac{10 \cdot 4b}{99a^2X^4} - \frac{5 \cdot 4^4b^2}{693a^4X^3} + \frac{2 \cdot 4^5b^3}{231a^6X^2} - \frac{2 \cdot 4^7b^4}{693a^8X} + \frac{4^9b^5}{693a^{10}}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left(-\frac{1}{13X^6} + \frac{3 \cdot 4^2b}{143a^2X^5} - \frac{10 \cdot 4^3b^2}{429a^4X^4} + \frac{5 \cdot 4^6b^3}{3003a^6X^3} - \frac{2 \cdot 4^7b^4}{1001a^8X^2} + \frac{2 \cdot 4^9b^5}{3003a^{10}X} - \frac{4^{11}b^6}{3003a^{12}}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left(-\frac{1}{15X^7} + \frac{14b}{195a^2X^6} - \frac{14 \cdot 4^3b^2}{715a^4X^5} + \frac{7 \cdot 4^5b^3}{1287a^6X^4} - \frac{2 \cdot 4^7b^4}{1287a^8X^3} + \frac{4^9b^5}{2145a^{10}X^2} - \frac{4^{11}b^6}{6435a^{12}X} + \frac{2 \cdot 4^{12}b^7}{6435a^{14}}\right) \frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = -\frac{2(2bx+a)}{17a^2X^{\frac{17}{2}}} - \frac{4^3b}{17a^2} \int \frac{dx}{X^{\frac{17}{2}}}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{\partial x}{\sqrt{(ax+bx^2)}} = \frac{1}{\sqrt{b}} \log \frac{\sqrt{(ax+bx^2)} + x\sqrt{b}}{\sqrt{(ax+bx^2)} - x\sqrt{b}}$$

oder

$$\int \frac{\partial x}{\sqrt{(ax+bx^2)}} = \frac{2}{\sqrt{-b}} \text{Arc Tang} \frac{x\sqrt{-b}}{\sqrt{(ax+bx^2)}},$$

woraus sich ergibt, daß in jedem Falle der erste Ausdruck reell wird, wenn  $b$  positiv, der zweite, wenn  $b$  negativ ist. Hieraus erhält man

$$\begin{aligned} \text{I. } \int \frac{\partial x}{\sqrt{(ax+bx^2)}} &= \pm \frac{1}{\sqrt{b}} \log \frac{\sqrt{(ax+bx^2)} \pm x\sqrt{b}}{\sqrt{(ax+bx^2)} \mp x\sqrt{b}} \\ &= \pm \frac{1}{\sqrt{b}} \log \frac{\sqrt{(a+bx)} \pm \sqrt{bx}}{\sqrt{(a+bx)} \mp \sqrt{bx}} \\ &= \pm \frac{1}{\sqrt{b}} \log \frac{2bx+a \pm 2\sqrt{b} \cdot \sqrt{(ax+bx^2)}}{a} \\ &= \pm \frac{2}{\sqrt{b}} \log \frac{\sqrt{(a+bx)} \pm \sqrt{bx}}{\sqrt{a}}. \end{aligned}$$

Die oberen Zeichen gehören hier zusammen, und eben so die unteren.

$$\begin{aligned} \text{II. } \int \frac{\partial x}{\sqrt{(ax-bx^2)}} &= \frac{2}{\sqrt{b}} \text{Arc Tang} \frac{x\sqrt{b}}{\sqrt{(ax-bx^2)}} = \frac{2}{\sqrt{b}} \text{Arc Tang} \sqrt{\frac{bx}{a-bx}} \\ &= \frac{2}{\sqrt{b}} \text{Arc Cot} \sqrt{\frac{a-bx}{bx}} = \frac{2}{\sqrt{b}} \text{Arc Sec} \sqrt{\frac{a}{a-bx}} \\ &= \frac{2}{\sqrt{b}} \text{Arc Cosec} \sqrt{\frac{a}{bx}} = \frac{2}{\sqrt{b}} \text{Arc Sin} \sqrt{\frac{bx}{a}} \\ &= \frac{2}{\sqrt{b}} \text{Arc Cos} \sqrt{\frac{a-bx}{a}} = \frac{1}{\sqrt{b}} \text{Arc Cos} \frac{a-2bx}{a} \\ &= \frac{1}{\sqrt{b}} \text{Arc Sin vers} \frac{2bx}{a}. \end{aligned}$$

Sämmtliche Integrale auf dieser Seite verschwinden für  $x = a$ .

Inshesondere ist

$$\begin{aligned} \int \frac{\partial x}{\sqrt{(x^2+x)}} &= \pm \log [2x+1 \pm 2\sqrt{(x^2+x)}] \\ \int \frac{\partial x}{\sqrt{(x^2-x)}} &= \pm \log [1-2x \mp 2\sqrt{(x^2-x)}]. \end{aligned}$$

Taf. XLIV.

$$\int \frac{x^m dx}{V(ax+bx^2)}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{dx}{VX} = \int \frac{dx}{VX} \quad (\text{Man s. die vorhergehende Seite.})$$

$$\int \frac{x dx}{VX} = \frac{VX}{b} - \frac{a}{2b} \int \frac{dx}{VX}$$

$$\int \frac{x^2 dx}{VX} = \left( \frac{x}{2b} - \frac{3a}{4b^2} \right) VX + \frac{3a^2}{8b^2} \int \frac{dx}{VX}$$

$$\int \frac{x^3 dx}{VX} = \left( \frac{x^2}{3b} - \frac{5ax}{12b^2} + \frac{5a^2}{8b^3} \right) VX - \frac{5a^3}{16b^3} \int \frac{dx}{VX}$$

$$\int \frac{x^4 dx}{VX} = \left( \frac{x^3}{4b} - \frac{7ax^2}{24b^2} + \frac{35a^2x}{96b^3} - \frac{35a^3}{64b^4} \right) VX + \frac{35a^4}{128b^4} \int \frac{dx}{VX}$$

$$\int \frac{x^5 dx}{VX} = \left( \frac{x^4}{5b} - \frac{9ax^3}{40b^2} + \frac{21a^2x^2}{80b^3} - \frac{21a^3x}{64b^4} + \frac{63a^4}{128b^5} \right) VX - \frac{63a^5}{256b^5} \int \frac{dx}{VX}$$

$$\int \frac{x^6 dx}{VX} = \left( \frac{x^5}{6b} - \frac{11ax^4}{60b^2} + \frac{33a^2x^3}{160b^3} - \frac{77a^3x^2}{320b^4} + \frac{77a^4x}{256b^5} - \frac{231a^5}{512b^6} \right) VX + \frac{231a^6}{1024b^6} \int \frac{dx}{VX}$$

$$\int \frac{x^7 dx}{VX} = \left( \frac{x^6}{7b} - \frac{13ax^5}{84b^2} + \frac{143a^2x^4}{840b^3} - \frac{429a^3x^3}{2240b^4} + \frac{143a^4x^2}{640b^5} - \frac{143a^5x}{512b^6} + \frac{429a^6}{1024b^7} \right) VX - \frac{429a^7}{2048b^7} \int \frac{dx}{VX}$$

$$\int \frac{x^8 dx}{VX} = \left( \frac{x^7}{8b} - \frac{15ax^6}{112b^2} + \frac{65a^2x^5}{448b^3} - \frac{143a^3x^4}{896b^4} + \frac{1287a^4x^3}{7168b^5} - \frac{429a^5x^2}{2048b^6} + \frac{2145a^6x}{8192b^7} - \frac{6435a^7}{16384b^8} \right) VX + \frac{6435a^8}{32768b^8} \int \frac{dx}{VX}$$

$$\int \frac{x^9 dx}{VX} = \frac{x^8 VX}{9b} - \frac{17a}{18b} \int \frac{x^8 dx}{VX}$$

$$\int \frac{x^{10} dx}{VX} = \left( \frac{x^9}{10b} - \frac{19ax^8}{180b^2} \right) VX + \frac{323a^2}{360b^2} \int \frac{x^8 dx}{VX}$$

$$\int \frac{\partial x}{x^m \sqrt{ax + bx^2}}$$

Taf. XLV.

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{\partial x}{x \sqrt{X}} = -\frac{2\sqrt{X}}{ax}$$

$$\int \frac{\partial x}{x^2 \sqrt{X}} = \left( -\frac{1}{3ax^2} + \frac{2b}{3a^2x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^3 \sqrt{X}} = \left( -\frac{1}{5ax^3} + \frac{4b}{15a^2x^2} - \frac{8b^2}{15a^3x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^4 \sqrt{X}} = \left( -\frac{1}{7ax^4} + \frac{6b}{35a^2x^3} - \frac{8b^2}{35a^3x^2} + \frac{16b^3}{35a^4x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^5 \sqrt{X}} = \left( -\frac{1}{9ax^5} + \frac{8b}{63a^2x^4} - \frac{16b^2}{105a^3x^3} + \frac{64b^3}{315a^4x^2} - \frac{128b^4}{315a^5x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^6 \sqrt{X}} = \left( -\frac{1}{11ax^6} + \frac{10b}{99a^2x^5} - \frac{80b^2}{693a^3x^4} + \frac{32b^3}{231a^4x^3} - \frac{128b^4}{693a^5x^2} + \frac{256b^5}{693a^6x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^7 \sqrt{X}} = \left( -\frac{1}{13ax^7} + \frac{12b}{143a^2x^6} - \frac{40b^2}{429a^3x^5} + \frac{320b^3}{3003a^4x^4} - \frac{128b^4}{1001a^5x^3} + \frac{512b^5}{3003a^6x^2} - \frac{1024b^6}{3003a^7x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^8 \sqrt{X}} = \left( -\frac{1}{15ax^8} + \frac{14b}{195a^2x^7} - \frac{56b^2}{715a^3x^6} + \frac{112b^3}{1287a^4x^5} - \frac{128b^4}{1287a^5x^4} + \frac{256b^5}{2145a^6x^3} - \frac{1024b^6}{6435a^7x^2} + \frac{2048b^7}{6435a^8x} \right) 2\sqrt{X}$$

$$\int \frac{\partial x}{x^9 \sqrt{X}} = -\frac{2\sqrt{X}}{17ax^9} - \frac{16b}{17a} \int \frac{\partial x}{x^8 \sqrt{X}}$$

$$\int \frac{\partial x}{x^{10} \sqrt{X}} = \left( -\frac{1}{19ax^{10}} + \frac{18b}{323a^2x^9} \right) 2\sqrt{X} + \frac{288b^2}{323a^2} \int \frac{\partial x}{x^8 \sqrt{X}}$$

Taf. XLVI

$$\int \frac{x^n dx}{(ax + bx^2)^{\frac{1}{2}}}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = -\frac{2(2bx + a)}{a^2 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = \frac{2x}{a \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = -\frac{2x}{b \sqrt{X}} + \frac{1}{b} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( \frac{x^2}{b} + \frac{3ax}{b^2} \right) \frac{1}{\sqrt{X}} - \frac{3a}{2b^2} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left( \frac{x^3}{2b} - \frac{5ax^2}{4b^2} - \frac{15a^2x}{4b^3} \right) \frac{1}{\sqrt{X}} + \frac{15a^2}{8b^3} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left( \frac{x^4}{3b} - \frac{7ax^3}{12b^2} + \frac{35a^2x^2}{24b^3} + \frac{35a^3x}{8b^4} \right) \frac{1}{\sqrt{X}} - \frac{35a^3}{16b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left( \frac{x^5}{4b} - \frac{3ax^4}{8b^2} + \frac{21a^2x^3}{32b^3} - \frac{105a^3x^2}{64b^4} - \frac{515a^4x}{64b^5} \right) \frac{1}{\sqrt{X}} + \frac{515a^4}{128b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left( \frac{x^6}{5b} - \frac{11ax^5}{40b^2} + \frac{53a^2x^4}{80b^3} - \frac{251a^3x^3}{320b^4} + \frac{251a^4x^2}{128b^5} + \frac{695a^5x}{128b^6} \right) \frac{1}{\sqrt{X}} - \frac{693a^5}{256b^6} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{x^7}{6b} - \frac{13ax^6}{60b^2} + \frac{143a^2x^5}{480b^3} - \frac{143a^3x^4}{320b^4} + \frac{1001a^4x^3}{1280b^5} - \frac{1001a^5x^2}{512b^6} - \frac{3003a^6x}{512b^7} \right) \frac{1}{\sqrt{X}} + \frac{3003a^6}{1024b^7} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \frac{x^8}{7b \sqrt{X}} - \frac{15a}{14b} \int \frac{x^8 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^{10} dx}{X^{\frac{1}{2}}} = \left( \frac{x^9}{8b} - \frac{17a}{112b^2} \right) \frac{1}{\sqrt{X}} - \frac{255a^2}{224b^2} \int \frac{x^8 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^m(ax+bx^2)^{\frac{1}{2}}}$$

Taf. XLVII.

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$$\text{VZ. } ax+bx^2=X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = -\frac{2}{3ax\sqrt{X}} - \frac{4b}{3a} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^2} + \frac{2b}{5a^2x}\right) \frac{2}{\sqrt{X}} + \frac{8b^2}{5a^2} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^3} + \frac{8b}{35a^2x^2} - \frac{16b^2}{35a^3x}\right) \frac{2}{\sqrt{X}} - \frac{64b^3}{35a^3} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{9ax^4} + \frac{10b}{63a^2x^3} - \frac{16b^2}{63a^3x^2} + \frac{32b^3}{63a^4x}\right) \frac{2}{\sqrt{X}} + \frac{128b^4}{63a^4} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{11ax^5} + \frac{4b}{33a^2x^4} - \frac{40b^2}{231a^3x^3} + \frac{64b^3}{231a^4x^2} - \frac{128b^4}{231a^5x}\right) \frac{2}{\sqrt{X}} - \frac{512b^5}{231a^5} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{13ax^6} + \frac{14b}{143a^2x^5} - \frac{56b^2}{429a^3x^4} + \frac{80b^3}{429a^4x^3} - \frac{128b^4}{429a^5x^2} + \frac{256b^5}{429a^6x}\right) \frac{2}{\sqrt{X}} + \frac{1024b^6}{429a^6} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{15ax^7} + \frac{16b}{195a^2x^6} - \frac{224b^2}{2145a^3x^5} + \frac{896b^3}{6435a^4x^4} - \frac{256b^4}{1287a^5x^3} + \frac{2048b^5}{6435a^6x^2} - \frac{4096b^6}{6435a^7x}\right) \frac{2}{\sqrt{X}} - \frac{16384b^7}{6435a^7} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = -\frac{2}{17ax^8\sqrt{X}} - \frac{18b}{17a} \int \frac{dx}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{19ax^9} + \frac{20b}{323a^2x^8}\right) \frac{2}{\sqrt{X}} + \frac{360b^2}{323a^2} \int \frac{dx}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{1}{2}}} = \left(-\frac{1}{21a^2x^{10}} + \frac{22b}{399a^2x^9} - \frac{440b^2}{6783a^3x^8}\right) \frac{2}{\sqrt{X}} - \frac{2640b^3}{2261a^3} \int \frac{dx}{x^7X^{\frac{3}{2}}}$$

Taf. XLVIII.

$$\int \frac{x^m dx}{(ax + bx^2)^{\frac{5}{2}}}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{5}{2}}} = \left( -\frac{2}{3X} + \frac{16b}{3a^2} \right) \frac{2bx+a}{a^2 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{5}{2}}} = \frac{2x}{3aX\sqrt{X}} - \frac{8(2bx+a)}{3a^3\sqrt{X}} = \left( \frac{1}{a+bx} - \frac{4(2bx+a)}{a^2} \right) \frac{2}{3a\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{5}{2}}} = \left( \frac{2x^2}{3aX} + \frac{4x}{3a^2} \right) \frac{1}{\sqrt{X}} = \left( \frac{x}{a+bx} + \frac{2x}{a} \right) \frac{2}{3a\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{5}{2}}} = \frac{2x^3}{3aX\sqrt{X}} = \frac{2x^3}{3a(a+bx)\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{5}{2}}} = \left( -\frac{8x^3}{3b} - \frac{2ax^2}{b^2} \right) \frac{1}{X\sqrt{X}} + \frac{1}{b^2} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{5}{2}}} = \left( \frac{x^4}{b} + \frac{20ax^3}{3b^2} + \frac{5a^2x^2}{b^3} \right) \frac{1}{X\sqrt{X}} - \frac{5a}{2b^3} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{5}{2}}} = \left( \frac{x^5}{2b} - \frac{7ax^4}{4b^2} - \frac{35a^2x^3}{3b^3} - \frac{35a^3x^2}{4b^4} \right) \frac{1}{X\sqrt{X}} + \frac{35a^2}{8b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{5}{2}}} = \left( \frac{x^6}{3b} - \frac{3ax^5}{4b^2} + \frac{21a^2x^4}{8b^3} + \frac{35a^3x^3}{2b^4} + \frac{105a^4x^2}{8b^5} \right) \frac{1}{X\sqrt{X}} - \frac{105a^3}{16b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{5}{2}}} = \left( \frac{x^7}{4b} - \frac{11ax^6}{24b^2} + \frac{33a^2x^5}{32b^3} - \frac{231a^3x^4}{64b^4} - \frac{385a^4x^3}{16b^5} - \frac{1155a^5x^2}{64b^6} \right) \frac{1}{X\sqrt{X}} + \frac{1155a^4}{128b^6} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{5}{2}}} = \left( \frac{x^8}{5b} - \frac{13ax^7}{40b^2} + \frac{143a^2x^6}{240b^3} - \frac{429a^3x^5}{320b^4} + \frac{3003a^4x^4}{640b^5} + \frac{1001a^5x^3}{32b^6} + \frac{3003a^6x^2}{128b^7} \right) \frac{1}{X\sqrt{X}} - \frac{3003a^5}{256b^7} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{5}{2}}} = \frac{x^9}{6bX\sqrt{X}} - \frac{5a}{4b} \int \frac{x^9 dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^m(ax+bx^2)^{\frac{1}{2}}}$$

Taf. XLIX.

$$\text{VZ. } ax + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{1}{2}}} = -\frac{2}{5axX\sqrt{X}} - \frac{8b}{5a} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{7ax^2} + \frac{2b}{7a^2x}\right) \frac{2}{X\sqrt{X}} + \frac{16b^2}{7a^2} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{9ax^3} + \frac{4b}{21a^2x^2} - \frac{8b^2}{21a^3x}\right) \frac{2}{X\sqrt{X}} - \frac{64b^3}{21a^3} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{11ax^4} + \frac{14b}{99a^2x^3} - \frac{8b^2}{33a^3x^2} + \frac{16b^3}{33a^4x}\right) \frac{2}{X\sqrt{X}} - \frac{128b^4}{33a^4} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{13ax^5} + \frac{16b}{143a^2x^4} - \frac{224b^2}{1287a^3x^3} + \frac{128b^3}{429a^4x^2} - \frac{256b^4}{429a^5x}\right) \frac{2}{X\sqrt{X}} - \frac{2048b^5}{429a^5} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{15ax^6} + \frac{6b}{65a^2x^5} - \frac{96b^2}{715a^3x^4} + \frac{448b^3}{2145a^4x^3} - \frac{256b^4}{715a^5x^2} + \frac{512b^5}{715a^6x}\right) \frac{2}{X\sqrt{X}} + \frac{4096b^6}{715a^6} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{17ax^7} + \frac{4b}{51a^2x^6} - \frac{24b^2}{221a^3x^5} + \frac{384b^3}{2431a^4x^4} - \frac{1792b^4}{7293a^5x^3} + \frac{1024b^5}{2431a^6x^2} - \frac{2048b^6}{2431a^7x}\right) \frac{2}{X\sqrt{X}} - \frac{16384b^7}{2431a^7} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = -\frac{2}{19ax^8X\sqrt{X}} - \frac{22b}{19a} \int \frac{dx}{x^7X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{21ax^9} + \frac{8b}{133a^2x^8}\right) \frac{2}{X\sqrt{X}} + \frac{176b^2}{133a^3} \int \frac{dx}{x^7X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{1}{2}}} = \left(-\frac{1}{23ax^{10}} + \frac{26b}{483a^2x^9} - \frac{208b^2}{3059a^3x^8}\right) \frac{2}{X\sqrt{X}} - \frac{4576b^3}{3059a^3} \int \frac{dx}{x^7X^{\frac{1}{2}}}$$



Taf. L.

$$\int \frac{x^n dx}{(ax + bx^2)^{\frac{7}{2}}}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{dx}{X^{\frac{7}{2}}} = \left( -\frac{1}{X^2} + \frac{16b}{3a^2 X} - \frac{128b^2}{3a^4} \right) \frac{2(2bx + a)}{5a^2 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{7}{2}}} = \frac{2x}{5aX^2 \sqrt{X}} - \left( \frac{1}{X} - \frac{8b}{a^2} \right) \frac{16(2bx + a)}{15a^3 \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{7}{2}}} = \left( \frac{x^2}{X^2} + \frac{2x}{aX} \right) \frac{2}{5a \sqrt{X}} - \frac{16(2bx + a)}{5a^4 \sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{7}{2}}} = \left( \frac{x^3}{X^2} + \frac{4x^2}{3aX} + \frac{8x}{3a^2} \right) \frac{2}{5a \sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{7}{2}}} = \left( \frac{x^4}{X^2} + \frac{2x^3}{3aX} \right) \frac{2}{5a \sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{7}{2}}} = \frac{2x^5}{5aX^2 \sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{7}{2}}} = \frac{2x^6}{5aX^2 \sqrt{X}} - \frac{2}{5a} \int \frac{x^5 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{7}{2}}} = -\frac{2x^6}{5bX^2 \sqrt{X}} + \frac{7}{5b} \int \frac{x^5 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{7}{2}}} = \left( \frac{x^7}{2b} + \frac{9ax^6}{10b^2} \right) \frac{1}{X^2 \sqrt{X}} - \frac{63a}{20b^2} \int \frac{x^5 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{7}{2}}} = \left( \frac{x^8}{3b} - \frac{11ax^7}{12b^2} - \frac{33a^2 x^6}{20b^3} \right) \frac{1}{X^2 \sqrt{X}} + \frac{231a^2}{40b^3} \int \frac{x^5 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^{10} dx}{X^{\frac{7}{2}}} = \left( \frac{x^9}{4b} - \frac{13ax^8}{24b^2} + \frac{143a^2 x^7}{96b^3} + \frac{429a^3 x^6}{160b^4} \right) \frac{1}{X^2 \sqrt{X}} - \frac{3003a^3}{320b^4} \int \frac{x^5 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^m(ax+bx^2)^{\frac{1}{2}}}$$

Taf. LI.

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$$\text{VZ. } ax+bx^2=X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = -\frac{2}{7axX^2\sqrt{X}} - \frac{12b}{7a} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{9ax^2} + \frac{2b}{9a^2x}\right) \frac{2}{X^2\sqrt{X}} + \frac{8b^2}{3a^2} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{11ax^3} + \frac{16b}{99a^2x^2} - \frac{32b^2}{99a^3x}\right) \frac{2}{X^3\sqrt{X}} - \frac{128b^3}{33a^3} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left(-\frac{1}{13ax^4} + \frac{18b}{143a^2x^3} - \frac{32b^2}{143a^3x^2} + \frac{64b^3}{143a^4x}\right) \frac{2}{X^4\sqrt{X}} + \frac{768b^4}{143a^4} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = \left(-\frac{1}{15ax^5} + \frac{4b}{39a^2x^4} - \frac{24b^2}{143a^3x^3} + \frac{128b^3}{429a^4x^2} - \frac{256b^4}{429a^5x}\right) \frac{2}{X^5\sqrt{X}} - \frac{1024b^5}{143a^5} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{17ax^6} + \frac{22b}{255a^2x^5} - \frac{88b^2}{663a^3x^4} + \frac{48b^3}{221a^4x^3} - \frac{256b^4}{663a^5x^2} + \frac{512b^5}{663a^6x}\right) \frac{2}{X^6\sqrt{X}} + \frac{2048b^6}{221a^6} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left(-\frac{1}{19ax^7} + \frac{24b}{323a^2x^6} - \frac{176b^2}{1615a^3x^5} + \frac{704b^3}{4199a^4x^4} - \frac{1152b^4}{4199a^5x^3} + \frac{2048b^5}{4199a^6x^2} - \frac{4096b^6}{4199a^7x}\right) \frac{2}{X^7\sqrt{X}} - \frac{49152b^7}{4199a^7} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = -\frac{2}{21ax^8X^2\sqrt{X}} - \frac{26b}{21a} \int \frac{dx}{x^7X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{23ax^9} + \frac{4b}{69a^2x^8}\right) \frac{2}{X^2\sqrt{X}} + \frac{104b^2}{69a^2} \int \frac{dx}{x^7X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{1}{2}}} = \left(-\frac{1}{25ax^{10}} + \frac{6b}{115a^2x^9} - \frac{8b^2}{115a^3x^8}\right) \frac{2}{X^3\sqrt{X}} - \frac{208b^3}{115a^3} \int \frac{dx}{x^7X^{\frac{1}{2}}}$$

Taf. LII

$$\int \frac{x^n dx}{(ax+bx^2)^{\frac{3}{2}}}, \quad \int \frac{dx}{x^n(ax+bx^2)^{\frac{3}{2}}}$$

$$\text{VZ. } ax + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \left( -\frac{1}{X^{\frac{3}{2}}} + \frac{24b}{5a^2 X^2} - \frac{128b^2}{5a^4 X} + \frac{1024b^3}{5a^6} \right) \frac{2(2bx+a)}{7a^2 \sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = \frac{2x}{7aX^{\frac{3}{2}} \sqrt{X}} - \left( \frac{1}{X^2} - \frac{16b}{3a^2 X} + \frac{128b^2}{3a^4} \right) \frac{24(2bx+a)}{35a^3 \sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \left( \frac{x^2}{X^3} + \frac{2x}{aX^2} \right) \frac{2}{7a \sqrt{X}} - \left( \frac{1}{X} - \frac{8b}{a^2} \right) \frac{32(2bx+a)}{21a^4 \sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left( \frac{x^3}{X^3} + \frac{8x^2}{5aX^2} + \frac{16x}{5a^2 X} \right) \frac{2}{7a \sqrt{X}} - \frac{128(2bx+a)}{35a^5 \sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left( \frac{x^4}{X^3} + \frac{6x^3}{5aX^2} + \frac{8x^2}{5a^2 X} + \frac{16x}{5a^3} \right) \frac{2}{7a \sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left( \frac{x^5}{X^3} + \frac{4x^4}{5aX^2} + \frac{8x^3}{15a^2 X} \right) \frac{2}{7a \sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \left( \frac{x^6}{X^3} + \frac{2x^5}{5aX^2} \right) \frac{2}{7a \sqrt{X}} = \left( \frac{1}{(a+bx)^3} + \frac{2}{5a(a+bx)^2} \right) \frac{2x^3}{7a \sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{3}{2}}} = \frac{2x^7}{7aX^3 \sqrt{X}} = \frac{2x^4}{7a(a+bx)^3 \sqrt{X}}$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = -\frac{2}{9axX^{\frac{3}{2}} \sqrt{X}} - \frac{16b}{9a} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^2 X^{\frac{3}{2}}} = \left( -\frac{1}{11ax^2} + \frac{2b}{11a^2 x} \right) \frac{2}{X^{\frac{3}{2}} \sqrt{X}} + \frac{32b^2}{11a^2} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3 X^{\frac{3}{2}}} = \left( -\frac{1}{13ax^3} + \frac{20b}{143a^2 x^2} - \frac{40b^2}{143a^3 x} \right) \frac{2}{X^{\frac{3}{2}} \sqrt{X}} - \frac{640b^3}{143a^3} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4 X^{\frac{3}{2}}} = \left( -\frac{1}{15ax^4} + \frac{22b}{195a^2 x^3} - \frac{8b^2}{39a^3 x^2} + \frac{16b^3}{39a^4 x} \right) \frac{2}{X^{\frac{3}{2}} \sqrt{X}} + \frac{256b^4}{39a^4} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^5 X^{\frac{3}{2}}} = \left( -\frac{1}{17ax^5} + \frac{8b}{85a^2 x^4} - \frac{176b^2}{1105a^3 x^3} + \frac{64b^3}{221a^4 x^2} - \frac{128b^4}{221a^5 x} \right) \frac{2}{X^{\frac{3}{2}} \sqrt{X}} - \frac{2048b^5}{221a^5} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int x^n dx \sqrt{ax + bx^2}$$

Taf. LIII.

$$VZ. \quad ax + bx^2 = X$$

$$\int dx \sqrt{X} = \left( \frac{x}{2} + \frac{a}{4b} \right) \sqrt{X} - \frac{a^2}{8b} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx \sqrt{X} = \frac{X \sqrt{X}}{3b} - \frac{a}{2b} \int dx \sqrt{X}$$

$$\int x^2 dx \sqrt{X} = \left( \frac{x}{4b} - \frac{5a}{24b^2} \right) X \sqrt{X} + \frac{5a^2}{16b^2} \int dx \sqrt{X}$$

$$\int x^3 dx \sqrt{X} = \left( \frac{x^2}{5b} - \frac{7ax}{40b^2} + \frac{7a^2}{48b^3} \right) X \sqrt{X} - \frac{7a^3}{32b^3} \int dx \sqrt{X}$$

$$\int x^4 dx \sqrt{X} = \left( \frac{x^3}{6b} - \frac{3ax^2}{20b^2} + \frac{21a^2x}{160b^3} - \frac{7a^3}{64b^4} \right) X \sqrt{X} + \frac{21a^4}{128b^4} \int dx \sqrt{X}$$

$$\int x^5 dx \sqrt{X} = \left( \frac{x^4}{7b} - \frac{11ax^3}{84b^2} + \frac{33a^2x^2}{280b^3} - \frac{33a^3x}{320b^4} + \frac{11a^4}{128b^5} \right) X \sqrt{X} - \frac{33a^5}{256b^5} \int dx \sqrt{X}$$

$$\int x^6 dx \sqrt{X} = \left( \frac{x^5}{8b} - \frac{13ax^4}{112b^2} + \frac{143a^2x^3}{1344b^3} - \frac{429a^3x^2}{4480b^4} + \frac{429a^4x}{5120b^5} - \frac{143a^5}{2048b^6} \right) X \sqrt{X} + \frac{429a^6}{4096b^6} \int dx \sqrt{X}$$

$$\int x^7 dx \sqrt{X} = \left( \frac{x^6}{9b} - \frac{5ax^5}{48b^2} + \frac{65a^2x^4}{672b^3} - \frac{715a^3x^3}{8064b^4} + \frac{715a^4x^2}{8960b^5} - \frac{143a^5x}{2048b^6} + \frac{715a^6}{12288b^7} \right) X \sqrt{X} - \frac{715a^7}{8192b^7} \int dx \sqrt{X}$$

$$\int x^8 dx \sqrt{X} = \frac{x^7 X \sqrt{X}}{10b} - \frac{17a}{20b} \int x^7 dx \sqrt{X}$$

$$\int x^9 dx \sqrt{X} = \left( \frac{x^8}{11b} - \frac{19ax^7}{220b^2} \right) X \sqrt{X} + \frac{323a^2}{440b^2} \int x^7 dx \sqrt{X}$$

$$\int x^{10} dx \sqrt{X} = \left( \frac{x^9}{12b} - \frac{7ax^8}{88b^2} + \frac{133a^2x^7}{1760b^3} \right) X \sqrt{X} - \frac{2261a^3}{3520b^3} \int x^7 dx \sqrt{X}$$

Taf. LIV.

$$\int \frac{\partial x \sqrt{ax + bx^2}}{x^m}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{\partial x \sqrt{X}}{x} = \sqrt{X} + \frac{a}{2} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + b \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^3} = -\frac{\sqrt{X}}{2ax^2} = -\frac{2(a+bx)\sqrt{X}}{3ax^2}$$

$$\int \frac{\partial x \sqrt{X}}{x^4} = \left(-\frac{1}{5ax^4} + \frac{2b}{15a^2x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^5} = \left(-\frac{1}{7ax^5} + \frac{4b}{35a^2x^4} - \frac{8b^2}{105a^3x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^6} = \left(-\frac{1}{9ax^6} + \frac{2b}{21a^2x^5} - \frac{8b^2}{105a^3x^4} + \frac{16b^3}{315a^4x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^7} = \left(-\frac{1}{11ax^7} + \frac{8b}{99a^2x^6} - \frac{16b^2}{231a^3x^5} + \frac{64b^3}{1155a^4x^4} - \frac{128b^4}{3465a^5x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^8} = \left(-\frac{1}{13ax^8} + \frac{10b}{143a^2x^7} - \frac{80b^2}{1287a^3x^6} + \frac{160b^3}{3003a^4x^5} - \frac{128b^4}{3003a^5x^4} + \frac{256b^5}{9009a^6x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^9} = \left(-\frac{1}{15ax^9} + \frac{4b}{65a^2x^8} - \frac{8b^2}{143a^3x^7} + \frac{64b^3}{1287a^4x^6} - \frac{128b^4}{3003a^5x^5} + \frac{512b^5}{15015a^6x^4} - \frac{1024b^6}{45045a^7x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^{10}} = -\frac{2X\sqrt{X}}{17ax^{10}} - \frac{14b}{17a} \int \frac{\partial x \sqrt{X}}{x^9}$$

$$\int \frac{\partial x \sqrt{X}}{x^{11}} = \left(-\frac{1}{19ax^{11}} + \frac{16b}{323a^2x^{10}}\right) 2X\sqrt{X} + \frac{224b^2}{323a^2} \int \frac{\partial x \sqrt{X}}{x^9}$$

$$\int x^m dx (a + bx^2)^{\frac{1}{2}}$$

Taf. LV.

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X}{b} - \frac{3a^2}{8b^2} \right) \frac{2bx+a}{8} \sqrt{X} + \frac{3a^4}{128b^2} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^2 \sqrt{X}}{5b} - \frac{a}{2b} \int dx X^{\frac{1}{2}}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{x}{6b} - \frac{7a}{60b^2} \right) X^2 \sqrt{X} + \frac{7a^2}{240b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{7b} - \frac{3ax}{28b^2} + \frac{3a^2}{40b^3} \right) X^2 \sqrt{X} - \frac{3a^3}{160b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{8b} - \frac{11ax^2}{112b^2} + \frac{33a^2x}{448b^3} - \frac{33a^3}{640b^4} \right) X^2 \sqrt{X} + \frac{33a^4}{2560b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{9b} - \frac{13ax^3}{144b^2} + \frac{143a^2x^2}{2016b^3} - \frac{143a^3x}{2688b^4} + \frac{143a^4}{3840b^5} \right) X^2 \sqrt{X} - \frac{143a^5}{15360b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{10b} - \frac{ax^4}{12b^2} + \frac{13a^2x^3}{192b^3} - \frac{143a^3x^2}{2688b^4} + \frac{143a^4x}{3584b^5} - \frac{143a^5}{5120b^6} \right) X^2 \sqrt{X} + \frac{143a^6}{20480b^6} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left( \frac{x^6}{11b} - \frac{17ax^5}{220b^2} + \frac{17a^2x^4}{264b^3} - \frac{221a^3x^3}{4224b^4} + \frac{221a^4x^2}{5376b^5} - \frac{221a^5x}{7168b^6} + \frac{221a^6}{10240b^7} \right) X^2 \sqrt{X} - \frac{221a^7}{40960b^7} \int dx X^{\frac{1}{2}}$$

$$\int x^8 dx X^{\frac{1}{2}} = \frac{x^7 X^2 \sqrt{X}}{12b} - \frac{19a}{24b} \int x^7 dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{13b} - \frac{7ax^7}{104b^2} \right) X^2 \sqrt{X} + \frac{133a^2}{208b^2} \int x^7 dx X^{\frac{1}{2}}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{14b} - \frac{23ax^8}{364b^2} + \frac{23a^2x^7}{416b^3} \right) X^2 \sqrt{X} - \frac{437a^3}{832b^3} \int x^7 dx X^{\frac{1}{2}}$$

Taf. LVI.

$$\int \frac{\partial x(ax+bx^2)^{\frac{1}{2}}}{x^m}$$

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$$\text{VL. } ax + bx^2 = X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \frac{X\sqrt{X}}{3} + \frac{a}{2} \int \partial x \sqrt{X}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^2} &= \frac{X\sqrt{X}}{2x} + \frac{3a}{4}\sqrt{X} + \frac{3a^2}{8} \int \frac{\partial x}{\sqrt{X}} \\ &= \left(\frac{5a}{4} + \frac{bx}{2}\right)\sqrt{X} + \frac{3a^2}{8} \int \frac{\partial x}{\sqrt{X}} \end{aligned}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^3} &= \frac{X\sqrt{X}}{x^2} - \frac{3a\sqrt{X}}{x} + \frac{3ab}{2} \int \frac{\partial x}{\sqrt{X}} \\ &= \left(b - \frac{2a}{x}\right)\sqrt{X} + \frac{3ab}{2} \int \frac{\partial x}{\sqrt{X}} \end{aligned}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^4} &= -\frac{2X^2\sqrt{X}}{3ax^4} + \frac{2b}{3a} \int \frac{\partial x X^{\frac{1}{2}}}{x^3} \\ &= -\left(\frac{2a}{3x^2} + \frac{8b}{3x}\right)\sqrt{X} + b^2 \int \frac{\partial x}{\sqrt{X}} \end{aligned}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = -\frac{2X^2\sqrt{X}}{5ax^5} = -\frac{2(a+bx)^2\sqrt{X}}{5ax^5}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = \left(-\frac{1}{7ax^6} + \frac{2b}{35a^2x^5}\right)2X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left(-\frac{1}{9ax^7} + \frac{4b}{63a^2x^6} - \frac{8b^2}{315a^3x^5}\right)2X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = \left(-\frac{1}{11ax^8} + \frac{2b}{33a^2x^7} - \frac{8b^2}{231a^3x^6} + \frac{16b^3}{155a^4x^5}\right)2X^2\sqrt{X}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^9} &= \left(-\frac{1}{13ax^9} + \frac{8b}{143a^2x^8} - \frac{16b^2}{429a^3x^7} + \frac{64b^3}{3003a^4x^6} \right. \\ &\quad \left. - \frac{128b^4}{15015a^5x^5}\right)2X^2\sqrt{X} \end{aligned}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} &= \left(-\frac{1}{15ax^{10}} + \frac{2b}{39a^2x^9} - \frac{16b^2}{429a^3x^8} + \frac{32b^3}{1287a^4x^7} \right. \\ &\quad \left. - \frac{128b^4}{9009a^5x^6} + \frac{256b^5}{45045a^6x^5}\right)2X^2\sqrt{X} \end{aligned}$$

$$\int x^n dx (ax + bx^2)^{\frac{1}{2}}$$

Taf. LVII.

$$\text{VZ. } ax + bx^2 = X$$

$$\int dx X^{\frac{1}{2}} = \left( \frac{X^2}{b} - \frac{5a^2 X}{16b^2} + \frac{15a^4}{128b^3} \right) \frac{2bx+a}{12} \sqrt{X} - \frac{5a^6}{1024b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^3 \sqrt{X}}{7b} - \frac{a}{2b} \int dx X^{\frac{1}{2}}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{x}{8b} - \frac{9a}{112b^2} \right) X^3 \sqrt{X} + \frac{9a^2}{32b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{9b} - \frac{11ax}{144b^2} + \frac{11a^2}{224b^3} \right) X^3 \sqrt{X} - \frac{11a^3}{64b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{10b} - \frac{13ax^2}{180b^2} + \frac{143a^2 x}{2880b^3} - \frac{143a^3}{4480b^4} \right) X^3 \sqrt{X} + \frac{143a^4}{1280b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{11b} - \frac{3ax^3}{44b^2} + \frac{39a^2 x^2}{792b^3} - \frac{39a^3 x}{1152b^4} + \frac{39a^4}{1792b^5} \right) X^3 \sqrt{X} - \frac{39a^5}{512b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{12b} - \frac{17ax^4}{264b^2} + \frac{17a^2 x^3}{352b^3} - \frac{221a^3 x^2}{6336b^4} + \frac{221a^4 x}{9216b^5} - \frac{221a^5}{14336b^6} \right) X^3 \sqrt{X} + \frac{221a^6}{4096b^6} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \frac{x^6 X^3 \sqrt{X}}{13b} - \frac{19a}{26b} \int x^6 dx X^{\frac{1}{2}}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{x^7}{14b} - \frac{3ax^6}{52b^2} \right) X^3 \sqrt{X} + \frac{57a^2}{104b^2} \int x^6 dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{15b} - \frac{23ax^7}{420b^2} + \frac{23a^2 x^6}{520b^3} \right) X^3 \sqrt{X} - \frac{437a^3}{1040b^3} \int x^6 dx X^{\frac{1}{2}}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{16b} - \frac{5ax^8}{96b^2} + \frac{115a^2 x^7}{2688b^3} - \frac{115a^3 x^6}{3328b^4} \right) X^3 \sqrt{X} + \frac{2185a^4}{6656b^4} \int x^6 dx X^{\frac{1}{2}}$$



Taf. LVIII.

$$\int \frac{\partial x(ax+bx^2)^{\frac{1}{2}}}{x^m}$$

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$$\text{VL. } ax + bx^2 = X^2$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \frac{X^2 \sqrt{X}}{5} + \frac{a}{2} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = \left( \frac{X^2}{4x} + \frac{5aX}{24} \right) \sqrt{X} + \frac{5a^2}{16} \int \partial x \sqrt{X}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^3} &= \left( \frac{X^2}{3x^2} + \frac{5aX}{12x} + \frac{5a^2}{8} \right) \sqrt{X} + \frac{5a^3}{16} \int \frac{\partial x}{\sqrt{X}} \\ &= \left( \frac{11a^2}{8} + \frac{13abx}{12} + \frac{b^2x^2}{3} \right) \sqrt{X} + \frac{5a^3}{16} \int \frac{\partial x}{\sqrt{X}} \end{aligned}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^4} &= \left( \frac{X^2}{2x^3} + \frac{5aX}{4x^2} - \frac{15a^2}{4x} \right) \sqrt{X} + \frac{15a^2b}{8} \int \frac{\partial x}{\sqrt{X}} \\ &= \left( -\frac{2a^2}{x} + \frac{9ab}{4} + \frac{b^2x}{2} \right) \sqrt{X} + \frac{15a^2b}{8} \int \frac{\partial x}{\sqrt{X}} \end{aligned}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = -\frac{2X^3 \sqrt{X}}{3ax^5} + \frac{4b}{3a} \int \frac{\partial x X^{\frac{1}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = \left( -\frac{1}{5ax^6} - \frac{2b}{15a^2x^5} \right) 2X^3 \sqrt{X} + \frac{8b^2}{15a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = -\frac{2X^3 \sqrt{X}}{7ax^7} = -\frac{2(a+bx)^3 \sqrt{X}}{7ax^4}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = \left( -\frac{1}{9ax^8} + \frac{2b}{63a^2x^7} \right) 2X^3 \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left( -\frac{1}{11ax^9} + \frac{4b}{99a^2x^8} - \frac{8b^2}{693a^3x^7} \right) 2X^3 \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{10}} = \left( -\frac{1}{13ax^{10}} + \frac{6b}{143a^2x^9} - \frac{8b^2}{429a^3x^8} + \frac{16b^3}{3003a^4x^7} \right) 2X^3 \sqrt{X}$$

$$\begin{aligned} \int \frac{\partial x X^{\frac{1}{2}}}{x^{11}} &= \left( -\frac{1}{15ax^{11}} + \frac{8b}{195a^2x^{10}} - \frac{16b^2}{715a^3x^9} + \frac{64b^3}{6435a^4x^8} \right. \\ &\quad \left. - \frac{128b^4}{45045a^5x^7} \right) 2X^3 \sqrt{X} \end{aligned}$$

$$\int x^n dx (ax + bx^2)^{\frac{1}{2}}$$

Taf. LIX.

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^3}{b} - \frac{7a^2 X^2}{24b^2} + \frac{35a^4 X}{384b^3} - \frac{35a^6}{1024b^4} \right) \frac{2bx + a}{16} \sqrt{X} + \frac{35a^8}{52768b^4} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^4 \sqrt{X}}{9b} - \frac{a}{2b} \int dx X^{\frac{1}{2}}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{x}{10b} - \frac{11a}{180b^2} \right) X^4 \sqrt{X} + \frac{11a^2}{40b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{11b} - \frac{13ax}{220b^2} + \frac{13a^2}{360b^3} \right) X^4 \sqrt{X} - \frac{13a^2}{80b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left( \frac{x^3}{12b} - \frac{5ax^2}{88b^2} + \frac{13a^2 x}{352b^3} - \frac{13a^3}{576b^4} \right) X^4 \sqrt{X} + \frac{13a^4}{128b^4} \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{13b} - \frac{17ax^3}{312b^2} + \frac{85a^2 x^2}{2288b^3} - \frac{17a^3 x}{704b^4} + \frac{17a^4}{1152b^5} \right) X^4 \sqrt{X} - \frac{17a^5}{256b^5} \int dx X^{\frac{1}{2}}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{14b} - \frac{19ax^4}{364b^2} + \frac{323a^2 x^3}{8736b^3} - \frac{1615a^3 x^2}{64064b^4} + \frac{323a^4 x}{19712b^5} - \frac{323a^5}{32256b^6} \right) X^4 \sqrt{X} + \frac{323a^6}{7168b^6} \int dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \frac{x^6 X^4 \sqrt{X}}{15b} - \frac{7a}{10b} \int x^6 dx X^{\frac{1}{2}}$$

$$\int x^8 dx X^{\frac{1}{2}} = \left( \frac{x^7}{16b} - \frac{23ax^6}{480b^2} \right) X^4 \sqrt{X} + \frac{161a^2}{320b^2} \int x^6 dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{17b} - \frac{25ax^7}{544b^2} + \frac{115a^2 x^6}{3264b^3} \right) X^4 \sqrt{X} - \frac{805a^3}{2176b^3} \int x^6 dx X^{\frac{1}{2}}$$

$$\int x^{10} dx X^{\frac{1}{2}} = \left( \frac{x^9}{18b} - \frac{3ax^8}{68b^2} + \frac{75a^2 x^7}{2176b^3} - \frac{345a^3 x^6}{13056b^4} \right) X^4 \sqrt{X} + \frac{2415a^4}{8704b^4} \int x^6 dx X^{\frac{1}{2}}$$

Taf. LX

$$\int \frac{\partial x(ax + bx^2)^{\frac{7}{2}}}{x^n}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int \frac{\partial x X^{\frac{7}{2}}}{x} = \frac{X^3 \sqrt{X}}{7} + \frac{a}{2} \int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^2} = \left( \frac{X^3}{6x} + \frac{7aX^2}{60} \right) \sqrt{X} + \frac{7a^2}{24} \int \partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^3} = \left( \frac{X^3}{5x^2} + \frac{7aX^2}{40x} + \frac{7a^2X}{48} \right) \sqrt{X} + \frac{7a^3}{32} \int \partial x \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^4} = \left( \frac{X^3}{4x^3} + \frac{7aX^2}{24x^2} + \frac{35a^2X}{96x} + \frac{35a^3}{64} \right) \sqrt{X} + \frac{35a^4}{128} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^5} = \left( \frac{X^3}{3x^4} + \frac{7aX^2}{12x^3} + \frac{35a^2X}{24x^2} - \frac{35a^3}{8x} \right) \sqrt{X} + \frac{35a^4}{16} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^6} = -\frac{2X^3 \sqrt{X}}{3x^5} + \frac{7b}{3} \int \frac{\partial x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^7} = \left( -\frac{1}{5x^6} - \frac{7b}{15ax^5} \right) 2X^3 \sqrt{X} + \frac{28b^2}{15a} \int \frac{\partial x X^{\frac{3}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^8} = \left( -\frac{1}{7x^7} - \frac{b}{5ax^6} - \frac{2b^2}{15a^2x^5} \right) 2X^3 \sqrt{X} + \frac{8b^3}{15a^2} \int \frac{\partial x X^{\frac{1}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^9} = -\frac{2X^4 \sqrt{X}}{9ax^9} = -\frac{2(a+bx)^4 \sqrt{X}}{9ax^9}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{10}} = \left( -\frac{1}{11ax^{10}} + \frac{2b}{99a^2x^9} \right) 2X^4 \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{11}} = \left( -\frac{1}{13ax^{11}} + \frac{4b}{143a^2x^{10}} - \frac{8b^2}{1287a^3x^9} \right) 2X^4 \sqrt{X}$$

$$\int x^m dx (ax + bx^2)^{\frac{1}{2}}, \int \frac{dx (ax + bx^2)^{\frac{1}{2}}}{x^m} \quad \text{Taf. LXL.}$$

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$$\text{VZ. } ax + bx^2 = X$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^4}{b} - \frac{9a^2 X^3}{32b^2} + \frac{21a^4 X^2}{256b^3} - \frac{105a^6 X}{4096b^4} + \frac{315a^8}{32768b^5} \right) \frac{2bx+a}{20} \sqrt{X} - \frac{63a^{10}}{262144b^5} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^{\frac{1}{2}} \sqrt{X}}{11b} - \frac{a}{2b} \int dx X^{\frac{1}{2}}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{x}{12b} - \frac{13a}{264b^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{13a^2}{48b^2} \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{13b} - \frac{5ax}{104b^2} + \frac{5a^2}{176b^3} \right) X^{\frac{1}{2}} \sqrt{X} - \frac{5a^3}{32b^3} \int dx X^{\frac{1}{2}}$$

$$\int x^4 dx X^{\frac{1}{2}} = \frac{x^3 X^{\frac{1}{2}} \sqrt{X}}{14b} - \frac{17a}{28b} \int x^3 dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \left( \frac{x^4}{15b} - \frac{19ax^3}{420b^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{323a^2}{840b^2} \int x^3 dx X^{\frac{1}{2}}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{16b} - \frac{7ax^4}{160b^2} + \frac{19a^2 x^3}{640b^3} \right) X^{\frac{1}{2}} \sqrt{X} - \frac{323a^3}{1280b^3} \int x^3 dx X^{\frac{1}{2}}$$

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$$\int \frac{dx X^{\frac{1}{2}}}{x} = \frac{X^{\frac{1}{2}} \sqrt{X}}{9} + \frac{a}{2} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^2} = \left( \frac{X^4}{8x} + \frac{9aX^3}{112} \right) \sqrt{X} + \frac{9a^2}{32} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^3} = \left( \frac{X^4}{7x^2} + \frac{3aX^3}{28x} + \frac{3a^2 X^2}{40} \right) \sqrt{X} + \frac{3a^3}{16} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^4} = \left( \frac{X^4}{6x^3} + \frac{3aX^3}{20x^2} + \frac{21a^2 X^2}{160x} + \frac{7a^3 X}{64} \right) \sqrt{X} + \frac{21a^4}{128} \int dx \sqrt{X}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^5} = \left( \frac{X^4}{5x^4} + \frac{9aX^3}{40x^3} + \frac{21a^2 X^2}{80x^2} + \frac{21a^3 X}{64x} + \frac{63a^4}{128} \right) \sqrt{X} + \frac{63a^5}{256} \int \frac{dx}{\sqrt{X}}$$

Taf. LXII.

$$\int \frac{dx}{(a + bx + cx^2)^{\frac{n}{2}}}$$

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$$\text{VZ. } a + bx + cx^2 = X, 4ac - b^2 = k$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{2(2cx + b)}{k\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left( \frac{1}{3kX} + \frac{8c}{3k^2} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left( \frac{1}{5kX^2} + \frac{4^2c}{15k^2X} + \frac{2 \cdot 4^3c^2}{15k^3} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left( \frac{1}{7kX^3} + \frac{6 \cdot 4c}{35k^2X^2} + \frac{2 \cdot 4^2c^2}{35k^3X} + \frac{4^3c^3}{35k^4} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left( \frac{1}{9kX^4} + \frac{2 \cdot 4^2c}{63k^2X^3} + \frac{4^4c^2}{105k^3X^2} + \frac{4^5c^3}{315k^4X} + \frac{2 \cdot 4^7c^4}{315k^5} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left( \frac{1}{11kX^5} + \frac{10 \cdot 4c}{99k^2X^4} + \frac{5 \cdot 4^2c^2}{693k^3X^3} + \frac{2 \cdot 4^3c^3}{231k^4X^2} + \frac{2 \cdot 4^5c^4}{693k^5X} + \frac{4^9c^5}{693k^6} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left( \frac{1}{13kX^6} + \frac{3 \cdot 4^2c}{143k^2X^5} + \frac{10 \cdot 4^3c^2}{429k^3X^4} + \frac{5 \cdot 4^6c^3}{3003k^4X^3} + \frac{2 \cdot 4^7c^4}{1001k^5X^2} + \frac{2 \cdot 4^9c^5}{3003k^6X} + \frac{4^{11}c^6}{3003k^7} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left( \frac{1}{15kX^7} + \frac{14 \cdot 4c}{195k^2X^6} + \frac{14 \cdot 4^3c^2}{715k^3X^5} + \frac{7 \cdot 4^5c^3}{1287k^4X^4} + \frac{2 \cdot 4^7c^4}{1287k^5X^3} + \frac{4^9c^5}{2145k^6X^2} + \frac{4^{11}c^6}{6435k^7X} + \frac{2 \cdot 4^{12}c^7}{6435k^8} \right) \frac{2(2cx + b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = \frac{2(2cx + b)}{17kX^8\sqrt{X}} + \frac{64c}{17k} \int \frac{dx}{X^{\frac{17}{2}}}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{\partial x}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \log [2cx + b + 2\sqrt{c} \cdot \sqrt{a+bx+cx^2}] + \text{Const.}$$

oder auch

$$\int \frac{\partial x}{\sqrt{a+bx+cx^2}} = \frac{-1}{\sqrt{-c}} \text{Arc Sin} \frac{2cx+b}{\sqrt{b^2-4ac}} + \text{Const.}$$

Die erste Form wird reell, wenn  $c$  positiv, die zweite, wenn  $c$  negativ ist. Hieraus ergibt sich:

$$\text{I. } \int \frac{\partial x}{\sqrt{X}} = \int \frac{\partial x}{\sqrt{a+bx+cx^2}} = \pm \frac{1}{\sqrt{c}} \log (2cx + b \pm 2\sqrt{c} \cdot \sqrt{X})$$

und wenn das Integral für  $x=0$  verschwinden soll,

$$\int \frac{\partial x}{\sqrt{X}} = \pm \frac{1}{\sqrt{c}} \log \frac{2cx + b \pm 2\sqrt{c} \cdot \sqrt{X}}{b \pm 2\sqrt{ac}}.$$

Die oberen Zeichen müssen hier zugleich genommen werden, und eben so die unteren.

$$\begin{aligned} \text{II. } \int \frac{\partial x}{\sqrt{X}} &= \int \frac{\partial x}{\sqrt{a+bx-cx^2}} = \frac{1}{\sqrt{c}} \text{Arc Sin} \frac{2cx-b}{\sqrt{b^2+4ac}} \\ &= \frac{1}{\sqrt{c}} \text{Arc Cos} \frac{2\sqrt{cX}}{\sqrt{b^2+4ac}} = \frac{1}{\sqrt{c}} \text{Arc Tang} \frac{2cx-b}{2\sqrt{cX}} \\ &= \frac{1}{\sqrt{c}} \text{Arc Cot} \frac{2\sqrt{cX}}{2cx-b} = \frac{1}{\sqrt{c}} \text{Arc Sec} \frac{\sqrt{b^2+4ac}}{2\sqrt{cX}} \\ &= \frac{1}{\sqrt{c}} \text{Arc Cosec} \frac{\sqrt{b^2+4ac}}{2cx-b} = \frac{1}{2\sqrt{c}} \text{Arc Sin vers} \frac{2(2cx-b)^2}{b^2+4ac}, \end{aligned}$$

und diese Kreisbogen verschwinden sämmtlich für  $x = \frac{b}{2c}$ . Sollen sie für  $x=0$  verschwinden, so ist

$$\begin{aligned} \int \frac{\partial x}{\sqrt{X}} &= \int \frac{\partial x}{\sqrt{a+bx-cx^2}} = \frac{1}{\sqrt{c}} \text{Arc Sin} \frac{2(2cx-b)\sqrt{ac}+2b\sqrt{cX}}{b^2+4ac} \\ &= \frac{1}{\sqrt{c}} \text{Arc Cos} \frac{4\sqrt{aX}-b(2cx-b)}{b^2+4ac} = \text{etc.} \end{aligned}$$

Im der Ausübung dürfte es besser seyn den Bogen mit der Constante nicht zusammen zu ziehen.

Taf. LXIII.

$$\int \partial x (a + bx + cx^2)^{\frac{1}{2}}$$

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$$VZ. \quad a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \partial x X^{\frac{1}{2}} = \frac{(2cx + b) \sqrt{X}}{4c} + \frac{k}{8c} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{3}{2}} = \left( \frac{X}{8c} + \frac{3k}{64c^2} \right) (2cx + b) \sqrt{X} + \frac{3k^2}{128c^2} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{5}{2}} = \left( \frac{X^2}{12c} + \frac{5kX}{192c^2} + \frac{5k^2}{512c^3} \right) (2cx + b) \sqrt{X} + \frac{5k^3}{1024c^3} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{7}{2}} = \left( \frac{X^3}{16c} + \frac{7kX^2}{6 \cdot 4^3 c^2} + \frac{35k^2 X}{6 \cdot 4^5 c^3} + \frac{35k^3}{4^7 c^4} \right) (2cx + b) \sqrt{X} + \frac{35k^4}{2 \cdot 4^7 c^4} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{9}{2}} = \left( \frac{X^4}{20c} + \frac{9kX^3}{10 \cdot 4^3 c^2} + \frac{21k^2 X^2}{5 \cdot 4^5 c^3} + \frac{21k^3 X}{4^7 c^4} + \frac{63k^4}{2 \cdot 4^9 c^5} \right) \times (2cx + b) \sqrt{X} + \frac{63k^5}{4^9 c^5} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{11}{2}} = \left( \frac{X^5}{24c} + \frac{11kX^4}{15 \cdot 4^3 c^2} + \frac{33k^2 X^3}{10 \cdot 4^5 c^3} + \frac{77k^3 X^2}{5 \cdot 4^7 c^4} + \frac{77k^4 X}{4^9 c^5} + \frac{231k^5}{2 \cdot 4^{11} c^6} \right) (2cx + b) \sqrt{X} + \frac{231k^6}{4^{11} c^6} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{13}{2}} = \left( \frac{X^6}{28c} + \frac{13kX^5}{21 \cdot 4^3 c^2} + \frac{143k^2 X^4}{210 \cdot 4^4 c^3} + \frac{429k^3 X^3}{35 \cdot 4^7 c^4} + \frac{143k^4 X^2}{10 \cdot 4^9 c^5} + \frac{143k^5 X}{2 \cdot 4^{11} c^6} + \frac{429k^6}{4^{12} c^7} \right) (2cx + b) \sqrt{X} + \frac{429k^7}{2 \cdot 4^{12} c^7} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{15}{2}} = \left( \frac{X^7}{32c} + \frac{15kX^6}{7 \cdot 4^4 c^2} + \frac{65k^2 X^5}{7 \cdot 4^6 c^3} + \frac{143k^3 X^4}{14 \cdot 4^7 c^4} + \frac{1287k^4 X^3}{7 \cdot 4^{10} c^5} + \frac{429k^5 X^2}{2 \cdot 4^{12} c^6} + \frac{2145k^6 X}{2 \cdot 4^{13} c^7} + \frac{6435k^7}{4^{15} c^8} \right) (2cx + b) \sqrt{X} + \frac{6435k^8}{2 \cdot 4^{15} c^8} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^n dx}{V(a+bx+cx^2)}$$

Taf. LXIV.

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$$VZ. \quad a + bx + cx^2 = X$$


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$$\int \frac{\partial x}{VX} = \int \frac{\partial x}{VX} \quad (\text{Seite 183.})$$

$$\int \frac{x \partial x}{VX} = \frac{VX}{c} - \frac{b}{2c} \int \frac{\partial x}{VX}$$

$$\int \frac{x^2 \partial x}{VX} = \left( \frac{x}{2c} - \frac{3b}{4c^2} \right) VX + \left( \frac{3b^2}{8c^2} - \frac{a}{2c} \right) \int \frac{\partial x}{VX}$$

$$\int \frac{x^3 \partial x}{VX} = \left( \frac{x^2}{3c} - \frac{5bx}{12c^2} + \frac{5b^2}{8c^3} - \frac{2a}{3c^2} \right) VX - \left( \frac{5b^3}{16c^3} - \frac{3ab}{4c^2} \right) \int \frac{\partial x}{VX}$$

$$\int \frac{x^4 \partial x}{VX} = \left[ \frac{x^3}{4c} - \frac{7bx^2}{24c^2} + \left( \frac{35b^2}{96c^3} - \frac{3a}{8c^2} \right) x - \frac{35b^3}{64c^4} + \frac{55ab}{48c^3} \right] VX$$

$$+ \left( \frac{35b^4}{128c^4} - \frac{15ab^3}{16c^3} + \frac{3a^2}{8c^2} \right) \int \frac{\partial x}{VX}$$

$$\int \frac{x^5 \partial x}{VX} = \frac{x^4 VX}{5c} - \frac{4a}{5c} \int \frac{x^3 \partial x}{VX} - \frac{9b}{10c} \int \frac{x^4 \partial x}{VX}$$

$$\int \frac{x^6 \partial x}{VX} = \left( \frac{x^5}{6c} - \frac{11bx^4}{60c^2} \right) VX + \frac{11ab}{15c^2} \int \frac{x^3 \partial x}{VX} + \left( \frac{33b^2}{40c^2} - \frac{5a}{6c} \right) \int \frac{x^4 \partial x}{VX}$$

$$\int \frac{x^7 \partial x}{VX} = \left[ \frac{x^6}{7c} - \frac{13bx^5}{84c^2} + \left( \frac{143b^2}{840c^3} - \frac{6a}{35c^2} \right) x^4 \right] VX$$

$$- \left( \frac{143ab^2}{210c^3} - \frac{24a^2}{35c^2} \right) \int \frac{x^3 \partial x}{VX} - \left( \frac{429b^3}{56c^3} - \frac{649ab}{420c^2} \right) \int \frac{x^4 \partial x}{VX}$$

$$\int \frac{x^8 \partial x}{VX} = \left[ \frac{x^7}{8c} - \frac{15bx^6}{112c^2} + \left( \frac{65b^2}{448c^3} - \frac{7a}{48c^2} \right) x^5 - \right.$$

$$\left. \left( \frac{143b^3}{896c^4} - \frac{1079ab}{3360c^3} \right) x^4 \right] VX + \left( \frac{143ab^2}{224c^4} - \frac{1079a^2b}{840c^3} \right) \int \frac{x^3 \partial x}{VX}$$

$$+ \left( \frac{6435b^4}{896c^4} - \frac{2431ab^2}{1120c^3} + \frac{35a^2}{48c^2} \right) \int \frac{x^4 \partial x}{VX}$$

$$\int \frac{x^9 \partial x}{VX} = \frac{x^8 VX}{9c} - \frac{8a}{9c} \int \frac{x^7 \partial x}{VX} - \frac{17b}{18c} \int \frac{x^8 \partial x}{VX}$$



Taf. LXV.

$$\int \frac{dx}{x^m \sqrt{a+bx+cx^2}}$$

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$$\text{VZ. } a + bx + cx^2 = X$$


---

$$\int \frac{dx}{x\sqrt{X}} = \int \frac{dx}{x\sqrt{X}} \quad (\text{Man s. die folgende Seite.})$$

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3\sqrt{X}} = \left(-\frac{1}{2ax^2} + \frac{3b}{4a^2x}\right)\sqrt{X} + \left(\frac{3b^2}{8a^2} - \frac{c}{2a}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4\sqrt{X}} = \left[-\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \left(\frac{5b^2}{8a^3} - \frac{2c}{3a^2}\right)\frac{1}{x}\right]\sqrt{X} - \left(\frac{5b^3}{16a^3} - \frac{3bc}{4a^2}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5\sqrt{X}} = \left[-\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \left(\frac{35b^2}{96a^3} - \frac{3c}{8a^2}\right)\frac{1}{x^2} + \left(\frac{35b^3}{64a^4} - \frac{55bc}{48a^3}\right)\frac{1}{x}\right]\sqrt{X} + \left(\frac{35b^4}{128a^4} - \frac{15b^2c}{16a^3} + \frac{3c^2}{8a^2}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6\sqrt{X}} = -\frac{\sqrt{X}}{5ax^5} - \frac{9b}{10a} \int \frac{dx}{x^5\sqrt{X}} - \frac{4c}{5a} \int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{11b}{60a^2x^5}\right)\sqrt{X} + \left(\frac{33b^2}{40a^2} - \frac{5c}{6a}\right) \int \frac{dx}{x^5\sqrt{X}} + \frac{11bc}{15a^2} \int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^8\sqrt{X}} = \left[-\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \left(\frac{143b^2}{840a^3} - \frac{6c}{35a^2}\right)\frac{1}{x^5}\right]\sqrt{X} - \left(\frac{429b^3}{360a^3} - \frac{649bc}{420a^2}\right) \int \frac{dx}{x^5\sqrt{X}} - \left(\frac{143b^2c}{210a^3} - \frac{24c^2}{35a^2}\right) \int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^9\sqrt{X}} = \left[-\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \left(\frac{65b^2}{448a^3} - \frac{7c}{48a^2}\right)\frac{1}{x^6} + \left(\frac{143b^3}{896a^4} - \frac{1079bc}{3360a^3}\right)\frac{1}{x^5}\right]\sqrt{X} + \left(\frac{1287b^4}{1792a^4} - \frac{2431b^2c}{1120a^3} + \frac{35c^2}{48a^2}\right) \int \frac{dx}{x^5\sqrt{X}} + \left(\frac{143b^3c}{224a^4} - \frac{1079bc^2}{840a^3}\right) \int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^{10}\sqrt{X}} = -\frac{\sqrt{X}}{9ax^9} - \frac{17b}{18a} \int \frac{dx}{x^9\sqrt{X}} - \frac{8c}{9a} \int \frac{dx}{x^8\sqrt{X}}$$

*Anmerkung zur vorhergehenden Tafel.*

Es ist im Allgemeinen

$$\int \frac{\partial x}{x\sqrt{X}} = \frac{1}{\sqrt{a}} \log \frac{2a + bx - 2\sqrt{a} \cdot \sqrt{X}}{x} + \text{Const.}$$

$$\text{oder } \int \frac{\partial x}{x\sqrt{X}} = \frac{1}{\sqrt{-a}} \text{ArcTang} \frac{2a + bx}{2\sqrt{-a} \cdot \sqrt{X}} + \text{Const.}$$

Die erste Form wird reell, wenn  $a$  positiv, die zweite, wenn  $a$  negativ ist. Hieraus ergibt sich:

$$\text{I. } \int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{(a+bx+cx^2)}} \\ = \pm \frac{1}{\sqrt{a}} \log \frac{2a + bx \mp 2\sqrt{aX}}{x} + \text{Const.} \\ = \pm \frac{1}{\sqrt{a}} \log \frac{2a + bx \mp 2\sqrt{aX}}{kx}.$$

Das  $k$  in dem letzteren Ausdrucke bezeichnet eine willkürliche Constante. Die oberen Zeichen gehören hier zusammen, und eben so die unteren. Für  $x=0$  kann das Integral nicht verschwinden.

$$\text{II. } \int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{(-a+bx+cx^2)}} = \frac{1}{\sqrt{a}} \text{ArcTang} \frac{bx-2a}{2\sqrt{aX}} \\ = \frac{1}{\sqrt{a}} \text{ArcCot} \frac{2\sqrt{aX}}{bx-2a} = \frac{1}{\sqrt{a}} \text{ArcSec} \frac{x\sqrt{(b^2+4ac)}}{2\sqrt{aX}} \\ = \frac{1}{\sqrt{a}} \text{ArcCosec} \frac{x\sqrt{(b^2+4ac)}}{bx-2a} = \frac{1}{\sqrt{a}} \text{ArcSin} \frac{bx-2a}{x\sqrt{(b^2+4ac)}} \\ = \frac{1}{\sqrt{a}} \text{ArcCos} \frac{2\sqrt{aX}}{x\sqrt{(b^2+4ac)}} = \frac{1}{2\sqrt{a}} \text{ArcSin vers} \frac{2(bx-2a)^2}{(b^2+4ac)x^2}.$$

Diese Kreisbogen verschwinden sämmtlich für  $x = \frac{2a}{b}$ . Für  $x=0$  können sie nicht verschwinden.

Taf. LXVI.

$$\int \frac{x^m dx}{(a+bx+cx^2)^{\frac{1}{2}}}$$

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$$\text{VZ. } a+bx+cx^2 = X, \quad 4ac - b^2 = k$$


---

$$\int \frac{dx}{X^{\frac{1}{2}}} = \frac{2(2cx+b)}{k\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = -\frac{2(a+bx)}{k\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = -\frac{(4ac-2b^2)x-2ab}{ck\sqrt{X}} + \frac{1}{c} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \frac{x^2}{c\sqrt{X}} - \frac{2a}{c} \int \frac{x dx}{X^{\frac{1}{2}}} - \frac{3b}{2c} \int \frac{x^2 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left( \frac{x^3}{2c} - \frac{5bx^2}{4c^2} \right) \frac{1}{\sqrt{X}} + \frac{5ab}{2c^2} \int \frac{x dx}{X^{\frac{1}{2}}} + \left( \frac{15b^2}{8c^2} - \frac{3a}{2c} \right) \int \frac{x^2 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left[ \frac{x^4}{3c} - \frac{7bx^3}{12c^2} + \left( \frac{35b^2}{24c^3} - \frac{4a}{3c^2} \right) x^2 \right] \frac{1}{\sqrt{X}} - \left( \frac{35ab^2}{12c^3} - \frac{8a^2}{3c^2} \right) \int \frac{x dx}{X^{\frac{1}{2}}} - \left( \frac{35b^3}{16c^3} - \frac{15ab}{4c^2} \right) \int \frac{x^2 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left[ \frac{x^5}{4c} - \frac{3bx^4}{8c^2} + \left( \frac{21b^2}{32c^3} - \frac{5a}{8c^2} \right) x^3 - \left( \frac{105b^3}{64c^4} - \frac{49ab}{16c^3} \right) x^2 \right] \frac{1}{\sqrt{X}} + \left( \frac{105ab^3}{32c^4} - \frac{49a^2b}{8c^3} \right) \int \frac{x dx}{X^{\frac{1}{2}}} + \left( \frac{315b^4}{128c^4} - \frac{105ab^2}{16c^3} + \frac{15a^2}{8c^2} \right) \int \frac{x^2 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \frac{x^6}{5c\sqrt{X}} - \frac{6a}{5c} \int \frac{x^5 dx}{X^{\frac{1}{2}}} - \frac{11b}{10c} \int \frac{x^6 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{x^7}{6c} - \frac{13bx^6}{60c^2} \right) \frac{1}{\sqrt{X}} + \frac{13ab}{10c^2} \int \frac{x^5 dx}{X^{\frac{1}{2}}} + \left( \frac{143b^2}{120c^2} - \frac{7a}{6c} \right) \int \frac{x^6 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left[ \frac{x^8}{7c} - \frac{5bx^7}{28c^2} + \left( \frac{13b^2}{56c^3} - \frac{8a}{35c^2} \right) x^6 \right] \frac{1}{\sqrt{X}} - \left( \frac{39ab^2}{28c^3} - \frac{48a^2}{35c^2} \right) \int \frac{x^5 dx}{X^{\frac{1}{2}}} - \left( \frac{143b^3}{112c^3} - \frac{351ab}{140c^2} \right) \int \frac{x^6 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^m(a+bx+cx^2)^{\frac{1}{2}}}$$

Taf. LXVII.

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$$\text{VL. } a + bx + cx^2 = X$$


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$$\int \frac{dx}{xX^{\frac{1}{2}}} = \frac{1}{a\sqrt{X}} - \frac{b}{2a} \int \frac{dx}{X^{\frac{1}{2}}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{2a^2}\right) \frac{1}{\sqrt{X}} + \left(\frac{3b^2}{4a^2} - \frac{2c}{a}\right) \int \frac{dx}{X^{\frac{1}{2}}} - \frac{3b}{2a^2} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{8a^3} - \frac{3c}{2a^2}\right) \frac{1}{\sqrt{X}} - \left(\frac{15b^3}{16a^4} - \frac{13bc}{4a^3}\right) \int \frac{dx}{X^{\frac{1}{2}}} + \left(\frac{15b^2}{8a^3} - \frac{3c}{2a^2}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left[-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \left(\frac{35b^2}{24a^3} - \frac{4c}{3a^2}\right) \frac{1}{x} - \left(\frac{35b^3}{16a^4} - \frac{15bc}{4a^3}\right)\right] \frac{1}{\sqrt{X}} + \left(\frac{35b^4}{32a^4} - \frac{115b^2c}{24a^3} + \frac{8c^2}{3a^2}\right) \int \frac{dx}{X^{\frac{1}{2}}} - \left(\frac{35b^3}{16a^4} - \frac{15bc}{4a^3}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = -\frac{1}{4ax^4\sqrt{X}} - \frac{9b}{8a} \int \frac{dx}{x^4X^{\frac{1}{2}}} - \frac{5c}{4a} \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4}\right) \frac{1}{\sqrt{X}} + \left(\frac{99b^2}{80a^2} - \frac{6c}{5a}\right) \int \frac{dx}{x^4X^{\frac{1}{2}}} + \frac{11bc}{8a^2} \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left[-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \left(\frac{143b^2}{480a^3} - \frac{7c}{24a^2}\right) \frac{1}{x^4}\right] \frac{1}{\sqrt{X}} - \left(\frac{429b^3}{320a^3} - \frac{269bc}{80a^2}\right) \int \frac{dx}{x^4X^{\frac{1}{2}}} - \left(\frac{143b^2c}{96a^3} - \frac{35c^2}{24a^2}\right) \int \frac{dx}{x^4X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = -\frac{1}{7ax^7\sqrt{X}} - \frac{15b}{14a} \int \frac{dx}{x^7X^{\frac{1}{2}}} - \frac{8c}{7a} \int \frac{dx}{x^6X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7}\right) \frac{1}{\sqrt{X}} + \left(\frac{255b^2}{224a^2} - \frac{9c}{8a}\right) \int \frac{dx}{x^7X^{\frac{1}{2}}} + \frac{17bc}{14a^2} \int \frac{dx}{x^6X^{\frac{1}{2}}}$$

Taf. LXVIII.

$$\int \frac{x^n dx}{(a + bx + cx^2)^{\frac{1}{2}}}$$

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$$\text{VZ. } a + bx + cx^2 = X, 4ac - b^2 = k$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \left( \frac{1}{3kX} + \frac{8c}{3k^2} \right) \frac{2(2cx + b)}{VX}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = -\frac{1}{3cXVX} - \frac{b}{2c} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left( -\frac{x}{2c} + \frac{b}{12c^2} \right) \frac{1}{XVX} + \left( \frac{b^2}{8c^2} + \frac{a}{2c} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( -\frac{x^2}{c} - \frac{bx}{4c^2} + \frac{b^2}{24c^3} - \frac{2a}{3c^2} \right) \frac{1}{XVX} + \left( \frac{b^3}{16c^3} - \frac{3ab}{4c^2} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \frac{1}{c} \int \frac{x^2 dx}{X^{\frac{1}{2}}} - \frac{a}{c} \int \frac{x^2 dx}{X^{\frac{1}{2}}} - \frac{b}{c} \int \frac{x^3 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \frac{x^4}{cXVX} - \frac{4a}{c} \int \frac{x^3 dx}{X^{\frac{1}{2}}} - \frac{5b}{2c} \int \frac{x^4 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \left( \frac{x^5}{2c} - \frac{7bx^4}{4c^2} \right) \frac{1}{XVX} + \frac{7ab}{c^2} \int \frac{x^3 dx}{X^{\frac{1}{2}}} + \left( \frac{35b^2}{8c^2} - \frac{5a}{2c} \right) \int \frac{x^4 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \left[ \frac{x^6}{3c} - \frac{3bx^5}{4c^2} + \left( \frac{21b^2}{8c^3} - \frac{2a}{c^2} \right) x^4 \right] \frac{1}{XVX} - \left( \frac{21ab^2}{2c^3} - \frac{8a^2}{c^2} \right) \int \frac{x^3 dx}{X^{\frac{1}{2}}} \\ - \left( \frac{105b^3}{16c^3} - \frac{35ab}{4c^2} \right) \int \frac{x^4 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \frac{x^7}{4cXVX} - \frac{7a}{4c} \int \frac{x^6 dx}{X^{\frac{1}{2}}} - \frac{11b}{8c} \int \frac{x^7 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left( \frac{x^8}{5c} - \frac{13bx^7}{40c^2} \right) \frac{1}{XVX} + \frac{91ab}{40c^2} \int \frac{x^6 dx}{X^{\frac{1}{2}}} + \left( \frac{143b^2}{80c^3} - \frac{8a}{5c} \right) \int \frac{x^7 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^2(a+bx+cx^2)^{\frac{1}{2}}}$$

Taf. LXIX.

$$\text{VZ. } a+bx+cx^2=X$$

$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left(\frac{1}{3aX} + \frac{1}{a^2}\right) \frac{1}{VX} - \frac{b}{2a} \int \frac{dx}{X^{\frac{1}{2}}} - \frac{b}{2a^2} \int \frac{dx}{X^{\frac{3}{2}}} + \frac{1}{a^2} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = -\frac{1}{axXVX} - \frac{5b}{2a} \int \frac{dx}{xX^{\frac{1}{2}}} - \frac{4c}{a} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left(-\frac{1}{2ax^2} + \frac{7b}{4a^2x}\right) \frac{1}{XVX} + \left(\frac{35b^2}{8a^2} - \frac{5c}{2a}\right) \int \frac{dx}{xX^{\frac{1}{2}}} + \frac{7bc}{a^2} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left[-\frac{1}{3ax^3} + \frac{3b}{4a^2x^2} - \left(\frac{21b^2}{8a^2} - \frac{2c}{a^2}\right) \frac{1}{x}\right] \frac{1}{XVX} - \left(\frac{105b^3}{16a^3} - \frac{35bc}{4a^2}\right) \int \frac{dx}{xX^{\frac{1}{2}}} - \left(\frac{21b^2c}{2a^3} - \frac{8c^2}{a^2}\right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = -\frac{1}{4ax^4XVX} - \frac{11b}{8a} \int \frac{dx}{x^4X^{\frac{1}{2}}} - \frac{7c}{4a} \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left(-\frac{1}{5ax^5} + \frac{13b}{40a^2x^4}\right) \frac{1}{XVX} + \left(\frac{143b^2}{80a^2} - \frac{8c}{5a}\right) \int \frac{dx}{x^4X^{\frac{1}{2}}} + \frac{91bc}{40a^2} \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left[-\frac{1}{6ax^6} + \frac{b}{4a^2x^5} - \left(\frac{13b^2}{32a^2} - \frac{3c}{8a^2}\right) \frac{1}{x^4}\right] \frac{1}{XVX} - \left(\frac{143b^3}{64a^3} - \frac{65bc}{16a^2}\right) \int \frac{dx}{x^4X^{\frac{1}{2}}} - \left(\frac{91b^2c}{64a^3} - \frac{21c^2}{8a^2}\right) \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = -\frac{1}{7ax^7XVX} - \frac{17b}{14a} \int \frac{dx}{x^7X^{\frac{1}{2}}} - \frac{10c}{7a} \int \frac{dx}{x^6X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left(-\frac{1}{8ax^8} + \frac{19b}{112a^2x^7}\right) \frac{1}{XVX} + \left(\frac{323b^2}{224a^2} - \frac{11c}{8a}\right) \int \frac{dx}{x^7X^{\frac{1}{2}}} + \frac{95bc}{56a^2} \int \frac{dx}{x^6X^{\frac{1}{2}}}$$

Taf. LXX.

$$\int \frac{x^n dx}{(a + bx + cx^2)^{\frac{1}{2}}}$$

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$$\text{VL. } a + bx + cx^2 = X, 4ac - b^2 = k$$


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$$\int \frac{dx}{X^{\frac{1}{2}}} = \left( \frac{1}{5kX^2} + \frac{16c}{15k^2X} + \frac{128c^2}{15k^3} \right) \frac{2(2cx+b)}{VX}$$

$$\int \frac{x dx}{X^{\frac{1}{2}}} = -\frac{1}{5cX^2VX} - \frac{b}{2c} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^2 dx}{X^{\frac{1}{2}}} = \left( -\frac{x}{4c} + \frac{3b}{40c^2} \right) \frac{1}{X^2VX} + \left( \frac{3b^2}{16c^2} + \frac{a}{4c} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^3 dx}{X^{\frac{1}{2}}} = \left( -\frac{x^2}{3c} + \frac{bx}{24c^2} - \frac{b^2}{80c^3} - \frac{2a}{15c^2} \right) \frac{1}{X^2VX} - \left( \frac{b^3}{32c^3} + \frac{3ab}{8c^2} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{1}{2}}} = \left[ -\frac{x^3}{2c} - \frac{bx^2}{12c^2} + \left( \frac{b^2}{96c^3} - \frac{3a}{8c^2} \right) x - \frac{b^3}{320c^4} + \frac{19ab}{240c^3} \right] \frac{1}{X^2VX} - \left( \frac{b^4}{128c^4} - \frac{3ab^2}{16c^3} - \frac{3a^2}{8c^2} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{1}{2}}} = \left[ -\frac{x^4}{c} - \frac{3bx^3}{4c^2} - \left( \frac{b^2}{8c^3} + \frac{4a}{3c^2} \right) x^2 + \left( \frac{b^3}{64c^4} - \frac{19ab}{48c^3} \right) x - \frac{3b^4}{640c^5} + \frac{11ab^2}{160c^4} - \frac{8a^2}{15c^3} \right] \frac{1}{X^2VX} - \left( \frac{3b^5}{256c^5} - \frac{5ab^3}{32c^4} + \frac{15a^2b}{16c^3} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^6 dx}{X^{\frac{1}{2}}} = \frac{1}{c} \int \frac{x^4 dx}{X^{\frac{1}{2}}} - \frac{a}{c} \int \frac{x^4 dx}{X^{\frac{1}{2}}} - \frac{b}{c} \int \frac{x^5 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{1}{2}}} = \frac{x^6}{cX^2VX} - \frac{6a}{c} \int \frac{x^5 dx}{X^{\frac{1}{2}}} - \frac{7b}{2c} \int \frac{x^6 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{1}{2}}} = \left( \frac{x^7}{2c} - \frac{9bx^6}{4c^2} \right) \frac{1}{X^2VX} + \frac{27ab}{2c^2} \int \frac{x^5 dx}{X^{\frac{1}{2}}} + \left( \frac{63b^2}{8c^2} - \frac{7a}{2c} \right) \int \frac{x^6 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{1}{2}}} = \left[ \frac{x^8}{3c} - \frac{11bx^7}{12c^2} + \left( \frac{33b^2}{8c^3} - \frac{8a}{3c^2} \right) x^6 \right] \frac{1}{X^2VX} - \left( \frac{231b^3}{16c^5} - \frac{65ab}{4c^4} \right) \int \frac{x^6 dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^m(a+bx+cx^2)^{\frac{1}{2}}}$$

Taf. LXXI.

$$\text{VZ. } a+bx+cx^2=X$$

$$\int \frac{dx}{xX^{\frac{1}{2}}} = \left( \frac{1}{5aX^2} + \frac{1}{3a^2X} + \frac{1}{a^3} \right) \frac{1}{\sqrt{X}} - \frac{b}{2a} \int \frac{dx}{X^{\frac{1}{2}}} - \frac{b}{2a^2} \int \frac{dx}{X^{\frac{3}{2}}} - \frac{b}{2a^3} \int \frac{dx}{X^{\frac{5}{2}}} + \frac{1}{a^3} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{1}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a} \int \frac{dx}{xX^{\frac{1}{2}}} - \frac{6c}{a} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{1}{2}}} = \left( -\frac{1}{2ax^2} + \frac{9b}{4a^2x} \right) \frac{1}{X^2\sqrt{X}} + \left( \frac{63b^2}{8a^2} - \frac{7c}{2a} \right) \int \frac{dx}{xX^{\frac{1}{2}}} + \frac{27bc}{2a^2} \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{1}{2}}} = \left[ -\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \left( \frac{33b^2}{8a^3} - \frac{8c}{3a^2} \right) \frac{1}{x} \right] \frac{1}{X^2\sqrt{X}} - \left( \frac{231b^3}{16a^3} - \frac{63bc}{4a^2} \right) \int \frac{dx}{xX^{\frac{1}{2}}} - \left( \frac{99b^2c}{4a^3} - \frac{16c^2}{a^2} \right) \int \frac{dx}{X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{1}{2}}} = -\frac{1}{4ax^4X^2\sqrt{X}} - \frac{13b}{8a} \int \frac{dx}{x^4X^{\frac{1}{2}}} - \frac{9c}{4a} \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{1}{2}}} = \left( -\frac{1}{5ax^5} + \frac{3b}{8a^2x^4} \right) \frac{1}{X^2\sqrt{X}} + \left( \frac{39b^2}{16a^2} - \frac{2c}{a} \right) \int \frac{dx}{x^4X^{\frac{1}{2}}} + \frac{27bc}{8a^2} \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{1}{2}}} = \left[ -\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \left( \frac{17b^2}{32a^3} - \frac{11c}{24a^2} \right) \frac{1}{x^4} \right] \frac{1}{X^2\sqrt{X}} - \left( \frac{221b^3}{64a^3} - \frac{93bc}{16a^2} \right) \int \frac{dx}{x^4X^{\frac{1}{2}}} - \left( \frac{153b^2c}{32a^3} - \frac{33c^2}{8a^2} \right) \int \frac{dx}{x^3X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{1}{2}}} = -\frac{1}{7ax^7X^2\sqrt{X}} - \frac{19b}{14a} \int \frac{dx}{x^7X^{\frac{1}{2}}} - \frac{12c}{7a} \int \frac{dx}{x^6X^{\frac{1}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{1}{2}}} = \left( -\frac{1}{8ax^8} + \frac{3b}{16a^2x^7} \right) \frac{1}{X^2\sqrt{X}} + \left( \frac{57b^2}{32a^2} - \frac{13c}{8a} \right) \int \frac{dx}{x^7X^{\frac{1}{2}}} + \frac{9bc}{4a^2} \int \frac{dx}{x^6X^{\frac{1}{2}}}$$



Taf. LXXII.  $\int \frac{x^m dx}{(a+bx+cx^2)^{\frac{3}{2}}}, \int \frac{dx}{x^m(a+bx+cx^2)^{\frac{3}{2}}}$

VZ.  $a+bx+cx^2 = X, 4ac - b^2 = k$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \left( \frac{1}{7kX^3} + \frac{24c}{35k^2X^2} + \frac{128c^2}{35k^3X} + \frac{1024c^3}{35k^4} \right) \frac{2(2cx+b)}{VX}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = -\frac{1}{7cX^3VX} - \frac{b}{2c} \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \left( -\frac{x}{6c} + \frac{5b}{84c^2} \right) \frac{1}{X^3VX} + \left( \frac{5b^2}{24c^2} + \frac{a}{6c} \right) \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left( -\frac{x^2}{5c} + \frac{bx}{20c^2} - \frac{b^2}{56c^3} - \frac{2a}{35c^2} \right) \frac{1}{X^3VX} - \left( \frac{b^3}{16c^3} + \frac{ab}{4c^2} \right) \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left[ -\frac{x^3}{4c} + \frac{bx^2}{40c^2} - \left( \frac{b^2}{160c^3} + \frac{a}{8c^2} \right) x + \frac{b^3}{448c^4} + \frac{29ab}{560c^3} \right] \frac{1}{X^3VX} \\ + \left( \frac{b^4}{128c^4} + \frac{3ab^2}{16c^3} + \frac{a^2}{8c^2} \right) \int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = -\frac{x^4}{3cX^3VX} + \frac{4a}{3c} \int \frac{x^3 dx}{X^{\frac{3}{2}}} + \frac{b}{6c} \int \frac{x^4 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \left( \frac{1}{7aX^3} + \frac{1}{5a^2X^2} + \frac{1}{3a^3X} + \frac{1}{a^4} \right) \frac{1}{VX} - \frac{b}{2a} \int \frac{dx}{X^{\frac{3}{2}}} \\ - \frac{b}{2a^2} \int \frac{dx}{X^{\frac{5}{2}}} - \frac{b}{2a^3} \int \frac{dx}{X^{\frac{7}{2}}} - \frac{b}{2a^4} \int \frac{dx}{X^{\frac{9}{2}}} + \frac{1}{a^4} \int \frac{dx}{xVX}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = -\frac{1}{axX^3VX} - \frac{9b}{2a} \int \frac{dx}{xX^{\frac{5}{2}}} - \frac{8c}{a} \int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left( -\frac{1}{2ax^2} + \frac{11b}{4a^2x} \right) \frac{1}{X^3VX} + \left( \frac{99b^2}{8a^2} - \frac{9c}{2a} \right) \int \frac{dx}{xX^{\frac{5}{2}}} \\ + \frac{29bc}{a^2} \int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left[ -\frac{1}{3ax^3} + \frac{13b}{12a^2x^2} - \left( \frac{143b^2}{24a^3} - \frac{10c}{3a^2} \right) \frac{1}{x} \right] \frac{1}{X^3VX} \\ - \left( \frac{429b^3}{16a^3} - \frac{99bc}{4a^2} \right) \int \frac{dx}{xX^{\frac{5}{2}}} - \left( \frac{143b^2c}{3a^3} - \frac{80c^2}{3a^2} \right) \int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int x^n dx \sqrt{a+bx+cx^2} \quad \text{Taf. LXXIII.}$$

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$$\text{VZ. } a + bx + cx^2 = X$$


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$$\int dx \sqrt{X} = \frac{(2cx+b)\sqrt{X}}{4c} + \frac{4ac-b^2}{8c} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx \sqrt{X} = \frac{X\sqrt{X}}{3c} - \frac{b}{2c} \int dx \sqrt{X}$$

$$\int x^2 dx \sqrt{X} = \left( \frac{x}{4c} - \frac{5b}{24c^2} \right) X\sqrt{X} + \left( \frac{5b^2}{6c^2} - \frac{a}{4c} \right) \int dx \sqrt{X}$$

$$\int x^3 dx \sqrt{X} = \left( \frac{x^2}{5c} - \frac{7bx}{40c^2} + \frac{7b^2}{48c^3} - \frac{2a}{15c^2} \right) X\sqrt{X} - \left( \frac{7b^3}{32c^3} - \frac{3ab}{8c^2} \right) \int dx \sqrt{X}$$

$$\int x^4 dx \sqrt{X} = \left[ \frac{x^3}{6c} - \frac{3bx^2}{20c^2} + \left( \frac{21b^2}{160c^3} - \frac{a}{8c^2} \right) x - \frac{7b^3}{64c^4} + \frac{49ab}{240c^3} \right] X\sqrt{X} \\ + \left( \frac{21b^4}{128c^4} - \frac{7ab^2}{16c^3} + \frac{a^2}{8c^2} \right) \int dx \sqrt{X}$$

$$\int x^5 dx \sqrt{X} = \frac{x^4 X \sqrt{X}}{7c} - \frac{4a}{7c} \int x^3 dx \sqrt{X} - \frac{11b}{14c} \int x^4 dx \sqrt{X}$$

$$\int x^6 dx \sqrt{X} = \left( \frac{x^5}{8c} - \frac{13bx^4}{112c^2} \right) X\sqrt{X} + \frac{13ab}{28c^2} \int x^3 dx \sqrt{X} \\ + \left( \frac{143b^2}{224c^2} - \frac{5a}{8c} \right) \int x^4 dx \sqrt{X}$$

$$\int x^7 dx \sqrt{X} = \left[ \frac{x^6}{9c} - \frac{5bx^5}{48c^2} + \left( \frac{65b^2}{672c^3} - \frac{2a}{21c^2} \right) x^4 \right] X\sqrt{X} \\ - \left( \frac{65ab^2}{168c^3} - \frac{8a^2}{21c^2} \right) \int x^3 dx \sqrt{X} - \left( \frac{715b^3}{1344c^3} - \frac{117ab}{112c^2} \right) \int x^4 dx \sqrt{X}$$

$$\int x^8 dx \sqrt{X} = \frac{x^7 X \sqrt{X}}{10c} - \frac{7a}{10c} \int x^6 dx \sqrt{X} - \frac{17b}{20c} \int x^7 dx \sqrt{X}$$

$$\int x^9 dx \sqrt{X} = \left( \frac{x^8}{11c} - \frac{19bx^7}{220c^2} \right) X\sqrt{X} + \frac{133ab}{220c^2} \int x^6 dx \sqrt{X} \\ + \left( \frac{323b^2}{440c^2} - \frac{8a}{11c} \right) \int x^7 dx \sqrt{X}$$

Taf. LXXIV.

$$\int \frac{dx \sqrt{a+bx+cx^2}}{x^m}$$

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$$\text{VZ. } a+bx+cx^2 = X$$


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$$\int \frac{dx \sqrt{X}}{x} = \sqrt{X} + a \int \frac{dx}{x\sqrt{X}} + \frac{b}{2} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{dx \sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2} \int \frac{dx}{x\sqrt{X}} + c \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{dx \sqrt{X}}{x^3} = -\left(\frac{1}{2x^2} + \frac{b}{4ax}\right)\sqrt{X} - \left(\frac{b^2}{8a} - \frac{c}{2}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx \sqrt{X}}{x^4} = -\frac{X\sqrt{X}}{3ax^3} + \left(\frac{b}{4ax^2} + \frac{b^2}{8a^2x}\right)\sqrt{X} + \left(\frac{b^3}{16a^2} - \frac{bc}{4a}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx \sqrt{X}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{5b}{24a^2x^3}\right)X\sqrt{X} - \left[\left(\frac{5b^2}{32a^2} - \frac{c}{8a}\right)\frac{1}{x^2} + \left(\frac{5b^3}{64a^3} - \frac{bc}{16a^2}\right)\frac{1}{x}\right]\sqrt{X} - \left(\frac{5b^4}{128a^3} - \frac{3b^2c}{16a^2} + \frac{c^2}{8a}\right) \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx \sqrt{X}}{x^6} = -\frac{X\sqrt{X}}{5ax^5} - \frac{7b}{10a} \int \frac{dx \sqrt{X}}{x^5} - \frac{2c}{5a} \int \frac{dx \sqrt{X}}{x^4}$$

$$\int \frac{dx \sqrt{X}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{3b}{20a^2x^5}\right)X\sqrt{X} + \left(\frac{21b^2}{40a^2} - \frac{c}{2a}\right) \int \frac{dx \sqrt{X}}{x^5} + \frac{3bc}{10a^2} \int \frac{dx \sqrt{X}}{x^4}$$

$$\int \frac{dx \sqrt{X}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{11b}{84a^2x^6} - \left(\frac{33b^2}{280a^3} - \frac{4c}{35a^2}\right)\frac{1}{x^5}\right]X\sqrt{X} - \left(\frac{33b^3}{80a^3} - \frac{111bc}{140a^2}\right) \int \frac{dx \sqrt{X}}{x^5} - \left(\frac{33b^2c}{140a^3} - \frac{8c^2}{35a^2}\right) \int \frac{dx \sqrt{X}}{x^4}$$

$$\int \frac{dx \sqrt{X}}{x^9} = -\frac{X\sqrt{X}}{8ax^8} - \frac{13b}{16a} \int \frac{dx \sqrt{X}}{x^8} - \frac{5c}{8a} \int \frac{dx}{x^7 \sqrt{X}}$$

$$\int \frac{dx \sqrt{X}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{5b}{48a^2x^8}\right)X\sqrt{X} + \left(\frac{65b^2}{96a^2} - \frac{2c}{3a}\right) \int \frac{dx \sqrt{X}}{x^8} + \frac{25bc}{48a^2} \int \frac{dx}{x^7 \sqrt{X}}$$

$$\int x^m \partial x (a + bx + cx^2)^{\frac{1}{2}} \quad \text{Taf. LXXV.}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \partial x X^{\frac{1}{2}} = \left( \frac{X}{8c} + \frac{3b}{64c^2} \right) (2cx + b) \sqrt{X} + \frac{3k^2}{128c^2} \int \frac{\partial x}{\sqrt{X}}$$

$$\int x \partial x X^{\frac{1}{2}} = \frac{X^2 \sqrt{X}}{5c} - \frac{b}{2c} \int \partial x X^{\frac{1}{2}}$$

$$\int x^2 \partial x X^{\frac{1}{2}} = \left( \frac{x}{6c} - \frac{7b}{60c^2} \right) X^2 \sqrt{X} + \left( \frac{7b^2}{24c^2} - \frac{a}{6c} \right) \int \partial x X^{\frac{1}{2}}$$

$$\begin{aligned} \int x^3 \partial x X^{\frac{1}{2}} = & \left( \frac{x^2}{7c} - \frac{3bx}{28c^2} + \frac{3b^2}{40c^3} - \frac{2a}{35c^2} \right) X^2 \sqrt{X} \\ & - \left( \frac{3b^3}{16c^3} - \frac{ab}{4c^2} \right) \int \partial x X^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \int x^4 \partial x X^{\frac{1}{2}} = & \left[ \frac{x^3}{8c} - \frac{11bx^2}{112c^2} + \left( \frac{33b^2}{448c^3} - \frac{a}{16c^2} \right) x - \frac{33b^2}{640c^4} \right. \\ & \left. + \frac{93ab}{1120c^3} \right] X^2 \sqrt{X} + \left( \frac{33b^4}{256c^4} - \frac{9ab^2}{32c^3} + \frac{a^2}{16c^2} \right) \int \partial x X^{\frac{1}{2}} \end{aligned}$$

$$\int x^5 \partial x X^{\frac{1}{2}} = \frac{x^4 X^2 \sqrt{X}}{9c} - \frac{4a}{9c} \int x^3 \partial x X^{\frac{1}{2}} - \frac{13b}{18c} \int x^4 \partial x X^{\frac{1}{2}}$$

$$\begin{aligned} \int x^6 \partial x X^{\frac{1}{2}} = & \left( \frac{x^5}{10c} - \frac{bx^4}{12c^2} \right) X^2 \sqrt{X} + \frac{ab}{3c^2} \int x^3 \partial x X^{\frac{1}{2}} \\ & + \left( \frac{13b^2}{24c^2} - \frac{a}{2c} \right) \int x^4 \partial x X^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \int x^7 \partial x X^{\frac{1}{2}} = & \left[ \frac{x^6}{11c} - \frac{17bx^5}{220c^2} + \left( \frac{17b^2}{264c^3} - \frac{2a}{33c^2} \right) x^4 \right] X^2 \sqrt{X} \\ & - \left( \frac{17ab^2}{66c^3} - \frac{8a^2}{33c^2} \right) \int x^3 \partial x X^{\frac{1}{2}} - \left( \frac{221b^3}{264c^3} - \frac{103ab}{132c^2} \right) \int x^4 \partial x X^{\frac{1}{2}} \end{aligned}$$

$$\int x^8 \partial x X^{\frac{1}{2}} = \frac{x^7 X^2 \sqrt{X}}{12c} - \frac{7a}{12c} \int x^6 \partial x X^{\frac{1}{2}} - \frac{19b}{24c} \int x^7 \partial x X^{\frac{1}{2}}$$

$$\begin{aligned} \int x^9 \partial x X^{\frac{1}{2}} = & \left( \frac{x^8}{13c} - \frac{7bx^7}{104c^2} \right) X^2 \sqrt{X} + \frac{49ab}{104c^2} \int x^6 \partial x X^{\frac{1}{2}} \\ & + \left( \frac{133b^2}{208c^2} - \frac{8a}{13c} \right) \int x^7 \partial x X^{\frac{1}{2}} \end{aligned}$$

Taf. LXXVI.

$$\int \frac{\partial x(a+bx+cx^2)^{\frac{1}{2}}}{x^m}$$

---


$$\text{VZ. } a+bx+cx^2 = X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left(\frac{X}{3} + a\right) \sqrt{X} + a^2 \int \frac{\partial x}{x \sqrt{X}} + \frac{ab}{2} \int \frac{\partial x}{\sqrt{X}} + \frac{b}{2} \int \partial x \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X \sqrt{X}}{ax} + \frac{3b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x} + \frac{4c}{a} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{b}{4a^2x}\right) X^2 \sqrt{X} + \left(\frac{3b^2}{8a^2} + \frac{3c}{2a}\right) \int \frac{\partial x X^{\frac{1}{2}}}{x} + \frac{bc}{a^2} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left[-\frac{1}{3ax^3} + \frac{b}{12a^2x^2} + \left(\frac{b^2}{24a^3} - \frac{2c}{3a^2}\right) \frac{1}{x}\right] X^2 \sqrt{X} - \left(\frac{b^3}{16a^3} - \frac{3bc}{4a^2}\right) \int \frac{\partial x X^{\frac{1}{2}}}{x} - \left(\frac{b^2c}{6a^3} - \frac{8c^2}{3a^2}\right) \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^5} = \left[-\frac{1}{4ax^4} + \frac{b}{8a^2x^3} - \left(\frac{b^2}{32a^3} + \frac{c}{8a^2}\right) \frac{1}{x^2} - \left(\frac{b^3}{64a^4} - \frac{3bc}{16a^3}\right) \frac{1}{x}\right] X^2 \sqrt{X} + \left(\frac{3b^4}{128a^4} - \frac{3b^2c}{16a^3} + \frac{3c^2}{8a^2}\right) \int \frac{\partial x X^{\frac{1}{2}}}{x} + \left(\frac{b^3c}{16a^4} - \frac{3bc^2}{4a^3}\right) \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^6} = -\frac{X^2 \sqrt{X}}{5ax^5} - \frac{b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x^5}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{7b}{60a^2x^5}\right) X^2 \sqrt{X} + \left(\frac{7b^2}{24a^2} - \frac{c}{6a}\right) \int \frac{\partial x X^{\frac{1}{2}}}{x^5}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{3b}{28a^2x^6} - \left(\frac{3b^2}{40a^3} - \frac{2c}{35a^2}\right) \frac{1}{x^5}\right] X^2 \sqrt{X} - \left(\frac{3b^3}{16a^3} - \frac{bc}{4a^2}\right) \int \frac{\partial x X^{\frac{1}{2}}}{x^5}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^9} = \left[-\frac{1}{8ax^8} + \frac{11b}{112a^2x^7} - \left(\frac{33b^2}{448a^3} - \frac{c}{16a^2}\right) \frac{1}{x^6} + \left(\frac{33b^3}{640a^4} - \frac{93bc}{1120a^3}\right) \frac{1}{x^5}\right] X^2 \sqrt{X} + \left(\frac{33b^4}{256a^4} - \frac{9b^2c}{32a^3} + \frac{c^2}{16a^2}\right) \int \frac{\partial x X^{\frac{1}{2}}}{x^5}$$

$$\int x^n dx (a + bx + cx^2)^{\frac{1}{2}} \quad \text{Taf. LXXXVII.}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^2}{12c} + \frac{5kX}{192c^2} + \frac{5k^2}{512c^3} \right) (2cx + b) \sqrt{X} + \frac{5k^3}{1024c^3} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^3 \sqrt{X}}{7c} - \frac{b}{2c} \int dx X^{\frac{1}{2}}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{x}{8c} - \frac{9b}{112c^2} \right) X^3 \sqrt{X} + \left( \frac{9b^2}{32c^2} - \frac{a}{8c} \right) \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^2}{9c} - \frac{11bx}{144c^2} + \frac{11b^2}{224c^3} - \frac{2a}{63c^2} \right) X^3 \sqrt{X} - \left( \frac{11b^3}{64c^3} - \frac{3ab}{16c^2} \right) \int dx X^{\frac{1}{2}}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left[ \frac{x^3}{10c} - \frac{13bx^2}{180c^2} + \left( \frac{143b^2}{2880c^3} - \frac{3a}{80c^2} \right) x - \frac{143b^3}{4480c^4} + \frac{451ab}{10080c^3} \right] X^3 \sqrt{X} + \left( \frac{143b^4}{1280c^4} - \frac{33ab^2}{160c^3} + \frac{3a^2}{80c^2} \right) \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \frac{x^4 X^3 \sqrt{X}}{11c} - \frac{4a}{11c} \int x^3 dx X^{\frac{1}{2}} - \frac{15b}{22c} \int x^4 dx X^{\frac{1}{2}}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^5}{12c} - \frac{17bx^4}{264c^2} \right) X^3 \sqrt{X} + \frac{17ab}{66c^2} \int x^3 dx X^{\frac{1}{2}} + \left( \frac{85b^2}{176c^2} - \frac{5a}{12c} \right) \int x^4 dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left[ \frac{x^6}{13c} - \frac{19bx^5}{312c^2} + \left( \frac{323b^2}{6864c^3} - \frac{6a}{143c^2} \right) x^4 \right] X^3 \sqrt{X} - \left( \frac{323ab^2}{1716c^3} - \frac{24a^2}{143c^2} \right) \int x^3 dx X^{\frac{1}{2}} - \left( \frac{1615b^3}{4576c^3} - \frac{2125ab}{3432c^2} \right) \int x^4 dx X^{\frac{1}{2}}$$

$$\int x^8 dx X^{\frac{1}{2}} = \frac{x^7 X^3 \sqrt{X}}{14c} - \frac{a}{2c} \int x^6 dx X^{\frac{1}{2}} - \frac{3b}{4c} \int x^7 dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^8}{15c} - \frac{23bx^7}{420c^2} \right) X^3 \sqrt{X} + \frac{23ab}{60c^2} \int x^6 dx X^{\frac{1}{2}} + \left( \frac{23b^2}{40c^2} - \frac{8a}{15c} \right) \int x^7 dx X^{\frac{1}{2}}$$

Taf. LXXXVIII.

$$\int \frac{\partial x(a + bx + cx^2)^{\frac{5}{2}}}{x^m}$$

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$$\text{VZ. } a + bx + cx^2 = X$$


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$$\int \frac{\partial x X^{\frac{5}{2}}}{x} = \left( \frac{X^2}{5} + \frac{aX}{3} + a^2 \right) \sqrt{X} + a^3 \int \frac{\partial x}{x \sqrt{X}} + \frac{a^2 b}{2} \int \frac{\partial x}{\sqrt{X}} \\ + \frac{ab}{2} \int \partial x \sqrt{X} + \frac{b}{2} \int \partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^2} = -\frac{X^3 \sqrt{X}}{ax} + \frac{5b}{2a} \int \frac{\partial x X^{\frac{5}{2}}}{x} + \frac{6c}{a} \int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^3} = \left( -\frac{1}{2ax^2} - \frac{3b}{4a^2 x} \right) X^3 \sqrt{X} + \left( \frac{15b^2}{8a^2} + \frac{5c}{2a} \right) \int \frac{\partial x X^{\frac{5}{2}}}{x} \\ + \frac{9bc}{2a^2} \int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^4} = \left[ -\frac{1}{3ax^3} - \frac{b}{12a^2 x^2} - \left( \frac{b^2}{8a^3} + \frac{4c}{3a^2} \right) \frac{1}{x} \right] X^3 \sqrt{X} \\ + \left( \frac{5b^3}{16a^3} + \frac{15bc}{4a^2} \right) \int \frac{\partial x X^{\frac{5}{2}}}{x} + \left( \frac{3b^2 c}{4a^3} + \frac{8c^2}{a^2} \right) \int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^5} = -\frac{X^3 \sqrt{X}}{4ax^4} - \frac{b}{8a} \int \frac{\partial x X^{\frac{5}{2}}}{x^4} + \frac{3c}{4a} \int \frac{\partial x X^{\frac{5}{2}}}{x^3}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^6} = \left( -\frac{1}{5ax^5} + \frac{3b}{40a^2 x^4} \right) X^3 \sqrt{X} + \left( \frac{3b^2}{80a^2} + \frac{2c}{5a} \right) \int \frac{\partial x X^{\frac{5}{2}}}{x^4} \\ - \frac{9bc}{40a^2} \int \frac{\partial x X^{\frac{5}{2}}}{x^3}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^7} = \left[ -\frac{1}{6ax^6} + \frac{b}{12a^2 x^5} - \left( \frac{b^2}{32a^3} + \frac{c}{24a^2} \right) \frac{1}{x^4} \right] X^3 \sqrt{X} \\ - \left( \frac{b^3}{64a^3} + \frac{3bc}{16a^2} \right) \int \frac{\partial x X^{\frac{5}{2}}}{x^4} + \left( \frac{3b^2 c}{32a^3} + \frac{c^2}{8a^2} \right) \int \frac{\partial x X^{\frac{5}{2}}}{x^3}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^8} = -\frac{X^3 \sqrt{X}}{7ax^7} - \frac{b}{2a} \int \frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^9} = \left( -\frac{1}{8ax^8} + \frac{9b}{112a^2 x^7} \right) X^3 \sqrt{X} + \left( \frac{9b^2}{32a^2} - \frac{c}{8a} \right) \int \frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$$\int x^n dx (a + bx + cx^2)^{\frac{1}{2}} \quad \text{Taf. LXXIX.}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int dx X^{\frac{1}{2}} = \left( \frac{X^3}{16c} + \frac{7kX^2}{384c^2} + \frac{35k^2X}{6144c^3} + \frac{35k^3}{16384c^4} \right) (2cx + b) \sqrt{X} + \frac{35k^4}{32768c^4} \int \frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{1}{2}} = \frac{X^4 \sqrt{X}}{9c} - \frac{b}{2c} \int dx X^{\frac{1}{2}}$$

$$\int x^2 dx X^{\frac{1}{2}} = \left( \frac{x}{10c} - \frac{11b}{180c^2} \right) X^4 \sqrt{X} + \left( \frac{11b^2}{40c^2} - \frac{a}{10c} \right) \int dx X^{\frac{1}{2}}$$

$$\int x^3 dx X^{\frac{1}{2}} = \left( \frac{x^3}{11c} - \frac{13bx}{220c^2} + \frac{13b^2}{360c^3} - \frac{2a}{99c^2} \right) X^4 \sqrt{X} - \left( \frac{13b^3}{80c^3} - \frac{3ab}{20c^2} \right) \int dx X^{\frac{1}{2}}$$

$$\int x^4 dx X^{\frac{1}{2}} = \left[ \frac{x^5}{12c} - \frac{5bx^3}{88c^2} + \left( \frac{13b^2}{352c^3} - \frac{a}{40c^2} \right) x - \frac{15b^3}{576c^4} + \frac{221ab}{7920c^3} \right] X^4 \sqrt{X} + \left( \frac{13b^4}{128c^4} - \frac{13ab^2}{80c^3} + \frac{a^2}{40c^2} \right) \int dx X^{\frac{1}{2}}$$

$$\int x^5 dx X^{\frac{1}{2}} = \frac{x^6 X^4 \sqrt{X}}{13c} - \frac{4a}{13c} \int x^3 dx X^{\frac{1}{2}} - \frac{17b}{26c} \int x^4 dx X^{\frac{1}{2}}$$

$$\int x^6 dx X^{\frac{1}{2}} = \left( \frac{x^7}{14c} - \frac{19bx^5}{364c^2} \right) X^4 \sqrt{X} + \frac{19ab}{91c^2} \int x^3 dx X^{\frac{1}{2}} + \left( \frac{323b^2}{728c^2} - \frac{5a}{14c} \right) \int x^4 dx X^{\frac{1}{2}}$$

$$\int x^7 dx X^{\frac{1}{2}} = \left[ \frac{x^8}{15c} - \frac{bx^6}{20c^2} + \left( \frac{19b^2}{520c^3} - \frac{2a}{65c^2} \right) x^4 \right] X^4 \sqrt{X} - \left( \frac{19ab^2}{130c^3} - \frac{8a^2}{65c^2} \right) \int x^3 dx X^{\frac{1}{2}} - \left( \frac{323b^3}{1040c^3} - \frac{133ab}{260c^2} \right) \int x^4 dx X^{\frac{1}{2}}$$

$$\int x^8 dx X^{\frac{1}{2}} = \frac{x^9 X^4 \sqrt{X}}{16c} - \frac{7a}{16c} \int x^6 dx X^{\frac{1}{2}} - \frac{23b}{32c} \int x^7 dx X^{\frac{1}{2}}$$

$$\int x^9 dx X^{\frac{1}{2}} = \left( \frac{x^{10}}{17c} - \frac{25bx^8}{544c^2} \right) X^4 \sqrt{X} + \frac{175ab}{544c^2} \int x^6 dx X^{\frac{1}{2}} + \left( \frac{575b^2}{1088c^2} - \frac{8a}{17c} \right) \int x^7 dx X^{\frac{1}{2}}$$



Taf. LXXX.

$$\int \frac{dx(a+bx+cx^2)^{\frac{1}{2}}}{x^n}$$

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$$\text{VL. } a+bx+cx^2 = X$$


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$$\int \frac{dx X^{\frac{1}{2}}}{x} = \left( \frac{X^2}{7} + \frac{aX^2}{5} + \frac{a^2 X}{3} + a^3 \right) \sqrt{X} + a^4 \int \frac{dx}{x\sqrt{X}} + \frac{a^3 b}{2} \int \frac{dx}{\sqrt{X}} \\ + \frac{a^2 b}{2} \int dx \sqrt{X} + \frac{ab}{2} \int dx X^{\frac{1}{2}} + \frac{b}{2} \int dx X^{\frac{3}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^2} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{ax} + \frac{7b}{2a} \int \frac{dx X^{\frac{1}{2}}}{x} + \frac{8c}{a} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^3} = \left( -\frac{1}{2ax^2} - \frac{5b}{4a^2 x} \right) X^{\frac{1}{2}} \sqrt{X} + \left( \frac{35b^2}{8a^2} + \frac{7c}{2a} \right) \int \frac{dx X^{\frac{1}{2}}}{x} \\ + \frac{10bc}{a^2} \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^4} = \left[ -\frac{1}{3ax^3} - \frac{b}{4a^2 x^2} - \left( \frac{5b^2}{8a^2} + \frac{2c}{a^2} \right) \frac{1}{x} \right] X^{\frac{1}{2}} \sqrt{X} \\ + \left( \frac{35b^3}{16a^3} + \frac{35bc}{4a^2} \right) \int \frac{dx X^{\frac{1}{2}}}{x} + \left( \frac{5b^2 c}{a^3} + \frac{16c^2}{a^2} \right) \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^5} = \left[ -\frac{1}{4ax^4} - \frac{b}{24a^2 x^3} - \left( \frac{b^2}{32a^3} + \frac{5c}{8a^2} \right) \frac{1}{x^2} - \left( \frac{5b^3}{64a^4} + \frac{29bc}{16a^3} \right) \frac{1}{x} \right] X^{\frac{1}{2}} \sqrt{X} \\ + \left( \frac{35b^4}{128a^4} + \frac{105b^2 c}{16a^3} + \frac{35c^2}{8a^2} \right) \int \frac{dx X^{\frac{1}{2}}}{x} + \left( \frac{5b^3 c}{8a^4} + \frac{29bc^2}{2a^3} \right) \int dx X^{\frac{1}{2}}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^6} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{5ax^5} - \frac{b}{10a^2} \int \frac{dx X^{\frac{1}{2}}}{x^5} + \frac{4c}{5a} \int \frac{dx X^{\frac{1}{2}}}{x^4}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^7} = \left( -\frac{1}{6ax^6} + \frac{b}{20a^2 x^5} \right) X^{\frac{1}{2}} \sqrt{X} + \left( \frac{b^2}{40a^2} + \frac{c}{2a} \right) \int \frac{dx X^{\frac{1}{2}}}{x^5} \\ - \frac{bc}{5a^2} \int \frac{dx X^{\frac{1}{2}}}{x^4}$$

$$\int \frac{dx X^{\frac{1}{2}}}{x^8} = \left[ -\frac{1}{7ax^7} + \frac{5b}{84a^2 x^6} - \left( \frac{b^2}{56a^3} + \frac{2c}{35a^2} \right) \frac{1}{x^5} \right] X^{\frac{1}{2}} \sqrt{X} \\ - \left( \frac{b^3}{112a^3} + \frac{29bc}{140a^2} \right) \int \frac{dx X^{\frac{1}{2}}}{x^5} + \left( \frac{b^2 c}{14a^3} + \frac{8c^2}{35a^2} \right) \int \frac{dx X^{\frac{1}{2}}}{x^4}$$

$$\int x^m \partial x (a + bx + cx^2)^{\frac{1}{2}}, \int \frac{\partial x (a + bx + cx^2)^{\frac{1}{2}}}{x^m} \quad \text{Taf. LXXXI.}$$

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$$\text{VZ. } a + bx + cx^2 = X, \quad 4ac - b^2 = k$$


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$$\int \partial x X^{\frac{1}{2}} = \int \partial x X^{\frac{1}{2}} \quad (\text{Seite 184.})$$

$$\int x \partial x X^{\frac{1}{2}} = \frac{X^{\frac{1}{2}} \sqrt{X}}{11c} - \frac{b}{2c} \int \partial x X^{\frac{1}{2}}$$

$$\int x^2 \partial x X^{\frac{1}{2}} = \left( \frac{x}{12c} - \frac{13b}{264c^2} \right) X^{\frac{1}{2}} \sqrt{X} + \left( \frac{13b^2}{48c^2} - \frac{a}{12c} \right) \int \partial x X^{\frac{1}{2}}$$

$$\int x^3 \partial x X^{\frac{1}{2}} = \left( \frac{x^2}{13c} - \frac{5bx}{104c^2} + \frac{5b^2}{176c^2} - \frac{2a}{143c^2} \right) X^{\frac{1}{2}} \sqrt{X} - \left( \frac{5b^3}{32c^3} - \frac{ab}{8c^2} \right) \int \partial x X^{\frac{1}{2}}$$

$$\int x^4 \partial x X^{\frac{1}{2}} = \frac{x^3 X^{\frac{1}{2}} \sqrt{X}}{14c} - \frac{3a}{14c} \int x^2 \partial x X^{\frac{1}{2}} - \frac{17b}{28c} \int x^3 \partial x X^{\frac{1}{2}}$$

$$\int x^5 \partial x X^{\frac{1}{2}} = \left( \frac{x^4}{15c} - \frac{19bx^3}{420c^2} \right) X^{\frac{1}{2}} \sqrt{X} + \frac{19ab}{140c^2} \int x^2 \partial x X^{\frac{1}{2}} + \left( \frac{323b^2}{840c^2} - \frac{4a}{15c} \right) \int x^3 \partial x X^{\frac{1}{2}}$$


---

$$\int \frac{\partial x X^{\frac{1}{2}}}{x} = \left( \frac{X^4}{9} + \frac{aX^3}{7} + \frac{a^2 X^2}{5} + \frac{a^3 X}{3} + a^4 \right) \sqrt{X} + a^5 \int \frac{\partial x}{x \sqrt{X}} + \frac{a^4 b}{2} \int \frac{\partial x}{\sqrt{X}} + \frac{a^3 b}{2} \int \partial x \sqrt{X} + \frac{a^2 b}{2} \int \partial x X^{\frac{1}{2}} + \frac{ab}{2} \int \partial x X^{\frac{3}{2}} + \frac{b}{2} \int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^2} = -\frac{X^{\frac{1}{2}} \sqrt{X}}{ax} + \frac{9b}{2a} \int \frac{\partial x X^{\frac{1}{2}}}{x} + \frac{10c}{a} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^3} = \left( -\frac{1}{2ax^2} - \frac{7b}{4a^2 x} \right) X^{\frac{1}{2}} \sqrt{X} + \left( \frac{63b^2}{8a^2} + \frac{9c}{2a} \right) \int \frac{\partial x X^{\frac{1}{2}}}{x} + \frac{7b^2 c}{4a^2} \int \partial x X^{\frac{1}{2}}$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^4} = \left[ -\frac{1}{3ax^3} - \frac{5b}{12a^2 x^2} - \left( \frac{35b^2}{24a^3} + \frac{8c}{3a^2} \right) \frac{1}{x} \right] X^{\frac{1}{2}} \sqrt{X} + \left( \frac{105b^3}{16a^3} + \frac{63bc}{4a^2} \right) \int \frac{\partial x X^{\frac{1}{2}}}{x} + \left( \frac{175b^2 c}{12a^3} + \frac{80c^2}{3a^2} \right) \int \partial x X^{\frac{1}{2}}$$

Taf. LXXXII.

$$\int \frac{\partial x}{(a+bx)^n \sqrt{x}}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x}{X \sqrt{x}} = \left\{ \begin{array}{l} \pm \frac{2}{\sqrt{ab}} \text{Arc Tang } \sqrt{\frac{bx}{a}} \\ \text{oder} \\ \frac{1}{\sqrt{-ab}} \log \frac{a - bx + 2\sqrt{x} \cdot \sqrt{-ab}}{X} \end{array} \right\} + \text{Const.} \quad *)$$

$$\int \frac{\partial x}{X^2 \sqrt{x}} = \frac{\sqrt{x}}{aX} + \frac{1}{2a} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 \sqrt{x}} = \left( \frac{1}{2aX^2} + \frac{3}{4a^2X} \right) \sqrt{x} + \frac{3}{8a^2} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^4 \sqrt{x}} = \left( \frac{1}{3aX^3} + \frac{5}{12a^2X^2} + \frac{5}{8a^3X} \right) \sqrt{x} + \frac{5}{16a^3} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^5 \sqrt{x}} = \left( \frac{1}{4aX^4} + \frac{7}{24a^2X^3} + \frac{35}{96a^3X^2} + \frac{35}{64a^4X} \right) \sqrt{x} + \frac{35}{128a^4} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^6 \sqrt{x}} = \left( \frac{1}{5aX^5} + \frac{9}{40a^2X^4} + \frac{21}{80a^3X^3} + \frac{21}{64a^4X^2} + \frac{63}{128a^5X} \right) \sqrt{x} + \frac{63}{256a^5} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^7 \sqrt{x}} = \left( \frac{1}{6aX^6} + \frac{11}{60a^2X^5} + \frac{33}{160a^3X^4} + \frac{77}{320a^4X^3} + \frac{77}{256a^5X^2} + \frac{231}{512a^6X} \right) \sqrt{x} + \frac{231}{1024a^6} \int \frac{\partial x}{X \sqrt{x}}$$

\*) Der erste Ausdruck wird genommen, wenn  $a$  und  $b$  dieselben Vorzeichen haben, und alsdann gilt das obere Zeichen für ein positives, das untere für ein negatives  $a$ ; der zweite Ausdruck hingegen wird genommen, wenn  $a$  und  $b$  verschiedene Vorzeichen haben. Beide Ausdrücke verschwinden für  $x = 0$ . Uebrigens ist  $\text{Arc Tang } \sqrt{\frac{bx}{a}} = \text{Arc Cot } \sqrt{\frac{a}{bx}} = \text{Arc Sec } \sqrt{\frac{a+bx}{a}}$   
 $= \text{Arc Cosec } \sqrt{\frac{a+bx}{bx}} = \text{Arc Cos } \sqrt{\frac{a}{a+bx}} = \frac{1}{2} \text{Arc Cos } \frac{a-bx}{a+bx} = \text{Arc Sin } \sqrt{\frac{bx}{a+bx}}$   
 $= \frac{1}{2} \text{Arc Sin } \frac{2\sqrt{abx}}{a+bx} = \frac{1}{2} \text{Arc Sin vers } \frac{2bx}{a+bx}.$

$$\int \frac{x^m \partial x \sqrt{x}}{a + bx}, \quad \int \frac{x^m \partial x \sqrt{x}}{(a + bx)^2} \quad \text{Taf. LXXXIII.}$$

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x \sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x \partial x \sqrt{x}}{X} = \left( \frac{x}{3b} - \frac{a}{b^2} \right) 2\sqrt{x} + \frac{a^2}{b^2} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X} = \left( \frac{x^2}{5b} - \frac{ax}{3b^2} + \frac{a^2}{b^3} \right) 2\sqrt{x} - \frac{a^3}{b^3} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X} = \left( \frac{x^3}{7b} - \frac{ax^2}{5b^2} + \frac{a^2x}{3b^3} - \frac{a^3}{b^4} \right) 2\sqrt{x} + \frac{a^4}{b^4} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X} = \left( \frac{x^4}{9b} - \frac{ax^3}{7b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3x}{3b^4} + \frac{a^4}{b^5} \right) 2\sqrt{x} - \frac{a^5}{b^5} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X} = \left( \frac{x^5}{11b} - \frac{ax^4}{9b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4x}{3b^5} - \frac{a^5}{b^6} \right) 2\sqrt{x} + \frac{a^6}{b^6} \int \frac{\partial x}{X\sqrt{x}}$$


---

$$\int \frac{\partial x \sqrt{x}}{X^2} = -\frac{\sqrt{x}}{bX} + \frac{1}{2b} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x \partial x \sqrt{x}}{X^2} = \frac{2x\sqrt{x}}{bX} - \frac{3a}{b} \int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X^2} = \left( \frac{x^2}{3b} - \frac{5ax}{3b^2} \right) \frac{2\sqrt{x}}{X} + \frac{5a^2}{b^2} \int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X^2} = \left( \frac{x^3}{5b} - \frac{7ax^2}{15b^2} + \frac{7a^2x}{3b^3} \right) \frac{2\sqrt{x}}{X} - \frac{7a^3}{b^3} \int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X^2} = \left( \frac{x^4}{7b} - \frac{9ax^3}{35b^2} + \frac{3a^2x^2}{5b^3} - \frac{3a^3x}{b^4} \right) \frac{2\sqrt{x}}{X} + \frac{9a^4}{b^4} \int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X^2} = \left( \frac{x^5}{9b} - \frac{11ax^4}{63b^2} + \frac{11a^2x^3}{35b^3} - \frac{11a^3x^2}{15b^4} + \frac{11a^4x}{3b^5} \right) \frac{2\sqrt{x}}{X} - \frac{11a^5}{b^5} \int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^6 \partial x \sqrt{x}}{X^2} = \left( \frac{x^6}{11b} - \frac{13ax^5}{99b^2} + \frac{13a^2x^4}{63b^3} - \frac{13a^3x^3}{35b^4} + \frac{13a^4x^2}{15b^5} - \frac{13a^5x}{3b^6} \right) \frac{2\sqrt{x}}{X} + \frac{13a^6}{b^6} \int \frac{\partial x \sqrt{x}}{X^2}$$

Taf. LXXXIV.  $\int \frac{x^n dx \sqrt{x}}{(a+bx)^3}, \int \frac{x^n dx \sqrt{x}}{(a+bx)^4}$

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VZ.  $a + bx = X$

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$$\int \frac{dx \sqrt{x}}{X^3} = \left( -\frac{1}{2bX^2} + \frac{1}{4abX} \right) \sqrt{x} + \frac{1}{8ab} \int \frac{dx}{X \sqrt{x}}$$

$$\int \frac{x dx \sqrt{x}}{X^3} = -\frac{2x \sqrt{x}}{bX^2} + \frac{3a}{b} \int \frac{dx \sqrt{x}}{X^3}$$

$$\int \frac{x^2 dx \sqrt{x}}{X^3} = \left( \frac{x^2}{b} + \frac{5ax}{b^2} \right) \frac{2\sqrt{x}}{X^2} - \frac{15a^2}{b^2} \int \frac{dx \sqrt{x}}{X^3}$$

$$\int \frac{x^3 dx \sqrt{x}}{X^3} = \left( \frac{x^3}{3b} - \frac{7ax^2}{3b^2} - \frac{35a^2x}{3b^3} \right) \frac{2\sqrt{x}}{X^2} + \frac{35a^3}{b^3} \int \frac{dx \sqrt{x}}{X^3}$$

$$\int \frac{x^4 dx \sqrt{x}}{X^3} = \left( \frac{x^4}{5b} - \frac{3ax^3}{5b^2} + \frac{21a^2x^2}{5b^3} + \frac{21a^3x}{b^4} \right) \frac{2\sqrt{x}}{X^2} - \frac{63a^4}{b^4} \int \frac{dx \sqrt{x}}{X^3}$$

$$\int \frac{x^5 dx \sqrt{x}}{X^3} = \left( \frac{x^5}{7b} - \frac{11ax^4}{35b^2} + \frac{33a^2x^3}{35b^3} - \frac{33a^3x^2}{5b^4} - \frac{33a^4x}{b^5} \right) \frac{2\sqrt{x}}{X^2} + \frac{99a^5}{b^5} \int \frac{dx \sqrt{x}}{X^3}$$

---


$$\int \frac{dx \sqrt{x}}{X^4} = \left( -\frac{1}{3bX^3} + \frac{1}{12abX^2} + \frac{1}{8a^2bX} \right) \sqrt{x} + \frac{1}{16a^2b} \int \frac{dx}{X \sqrt{x}}$$

$$\int \frac{x dx \sqrt{x}}{X^4} = -\frac{2x \sqrt{x}}{3bX^3} + \frac{a}{b} \int \frac{dx \sqrt{x}}{X^4}$$

$$\int \frac{x^2 dx \sqrt{x}}{X^4} = \left( -\frac{x^2}{b} - \frac{5ax}{3b^2} \right) \frac{2\sqrt{x}}{X^3} + \frac{5a^2}{b^2} \int \frac{dx \sqrt{x}}{X^4}$$

$$\int \frac{x^3 dx \sqrt{x}}{X^4} = \left( \frac{x^3}{b} + \frac{7ax^2}{b^2} + \frac{35a^2x}{3b^3} \right) \frac{2\sqrt{x}}{X^3} - \frac{35a^3}{b^3} \int \frac{dx \sqrt{x}}{X^4}$$

$$\int \frac{x^4 dx \sqrt{x}}{X^4} = \left( \frac{x^4}{3b} - \frac{3ax^3}{b^2} - \frac{21a^2x^2}{b^3} - \frac{35a^3x}{b^4} \right) \frac{2\sqrt{x}}{X^3} + \frac{105a^4}{b^4} \int \frac{dx \sqrt{x}}{X^4}$$

$$\int \frac{x^5 dx \sqrt{x}}{X^4} = \left( \frac{x^5}{5b} - \frac{11ax^4}{15b^2} + \frac{33a^2x^3}{5b^3} + \frac{231a^3x^2}{5b^4} + \frac{77a^4x}{b^5} \right) \frac{2\sqrt{x}}{X^3} - \frac{251a^5}{b^5} \int \frac{dx \sqrt{x}}{X^4}$$

$$\int \frac{\partial x}{(a + bx^2)^2 \sqrt{x}}$$

Taf. LXXXV.

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{\partial x}{X \sqrt{x}} = \int \frac{\partial x}{X \sqrt{x}} \quad *)$$

$$\int \frac{\partial x}{X^2 \sqrt{x}} = \frac{\sqrt{x}}{2aX} + \frac{3}{4a} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 \sqrt{x}} = \left( \frac{1}{4aX^2} + \frac{7}{16a^2X} \right) \sqrt{x} + \frac{21}{32a^2} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^4 \sqrt{x}} = \left( \frac{1}{6aX^3} + \frac{11}{48a^2X^2} + \frac{77}{192a^3X} \right) \sqrt{x} + \frac{77}{128a^3} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^5 \sqrt{x}} = \left( \frac{1}{8aX^4} + \frac{5}{32a^2X^3} + \frac{55}{256a^3X^2} + \frac{385}{1024a^4X} \right) \sqrt{x} + \frac{1155}{2048a^4} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{\partial x}{X^6 \sqrt{x}} = \left( \frac{1}{10aX^5} + \frac{19}{160a^2X^4} + \frac{19}{128a^3X^3} + \frac{209}{1024a^4X^2} + \frac{1463}{4096a^5X} \right) \sqrt{x} + \frac{4389}{8192a^5} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{bx}{X^7 \sqrt{x}} = \left( \frac{1}{12aX^6} + \frac{23}{240a^2X^5} + \frac{437}{3840a^3X^4} + \frac{437}{4352a^4X^3} + \frac{4807}{24576a^5X^2} + \frac{33649}{98304a^6X} \right) \sqrt{x} + \frac{100947}{196608a^6} \int \frac{\partial x}{X \sqrt{x}}$$

\*) Haben  $a$  und  $b$  dieselben Vorzeichen, so ist

$$\int \frac{\partial x}{X \sqrt{x}} = \frac{1}{bk^2 \sqrt{2}} \left[ \log \frac{x + k\sqrt{2x} + k^2}{\sqrt{x}} + \text{ArcTang} \frac{k\sqrt{2x}}{k^2 - x} \right]$$

wo alsdann  $k = \sqrt[4]{\frac{a}{b}}$ .

Haben  $a$  und  $b$  verschiedene Vorzeichen, so ist

$$\int \frac{\partial x}{X \sqrt{x}} = \frac{1}{2bk^2} \left[ \log \frac{k - \sqrt{x}}{k + \sqrt{x}} - 2 \text{ArcTang} \frac{\sqrt{x}}{k} \right]$$

und es ist  $k = \sqrt[4]{-\frac{a}{b}}$ .

Taf. LXXXVI.

$$\int \frac{x^m dx \sqrt{x}}{a + bx^2}$$

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$$\text{VZ. } a + bx^2 = X$$


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$$\int \frac{dx \sqrt{x}}{X} = \int \frac{dx \sqrt{x}}{X}^*)$$

$$\int \frac{x dx \sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^2 dx \sqrt{x}}{X} = \frac{2x\sqrt{x}}{3b} - \frac{a}{b} \int \frac{dx \sqrt{x}}{X}$$

$$\int \frac{x^3 dx \sqrt{x}}{X} = \left(\frac{x^2}{5b} - \frac{a}{b^2}\right) 2\sqrt{x} + \frac{a^2}{b^2} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^4 dx \sqrt{x}}{X} = \left(\frac{x^3}{7b} - \frac{ax}{3b^2}\right) 2\sqrt{x} + \frac{a^2}{b^2} \int \frac{dx \sqrt{x}}{X}$$

$$\int \frac{x^5 dx \sqrt{x}}{X} = \left(\frac{x^4}{9b} - \frac{ax^2}{5b^2} + \frac{a^2}{b^3}\right) 2\sqrt{x} - \frac{a^3}{b^3} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^6 dx \sqrt{x}}{X} = \left(\frac{x^5}{11b} - \frac{ax^3}{7b^2} + \frac{a^2x}{3b^3}\right) 2\sqrt{x} - \frac{a^3}{b^3} \int \frac{dx \sqrt{x}}{X}$$

$$\int \frac{x^7 dx \sqrt{x}}{X} = \left(\frac{x^6}{13b} - \frac{ax^4}{9b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3}{b^4}\right) 2\sqrt{x} + \frac{a^4}{b^4} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^8 dx \sqrt{x}}{X} = \left(\frac{x^7}{15b} - \frac{ax^5}{11b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x}{3b^4}\right) 2\sqrt{x} + \frac{a^4}{b^4} \int \frac{dx \sqrt{x}}{X}$$

$$\int \frac{x^9 dx \sqrt{x}}{X} = \left(\frac{x^8}{17b} - \frac{ax^6}{13b^2} + \frac{a^2x^4}{9b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4}{b^5}\right) 2\sqrt{x} - \frac{a^5}{b^5} \int \frac{dx}{X\sqrt{x}}$$

\*) Haben  $a$  und  $b$  dieselben Vorzeichen, so ist

$$\int \frac{dx \sqrt{x}}{X} = \frac{1}{b\sqrt{2}} \left[ -\log \frac{x + k^2 + k\sqrt{2x}}{\sqrt{x}} + \text{Arc Tang} \frac{k\sqrt{2x}}{k^2 - x} \right]$$

wo alsdann  $k = \sqrt{\frac{a}{b}}$ .

Haben  $a$  und  $b$  verschiedene Vorzeichen, so ist

$$\int \frac{dx \sqrt{x}}{X} = \frac{1}{2bk} \left[ \log \frac{k - \sqrt{x}}{k + \sqrt{x}} + 2 \text{Arc Tang} \frac{\sqrt{x}}{k} \right]$$

und es ist  $k = \sqrt{-\frac{a}{b}}$ .

$$\int \frac{x^n dx \sqrt{x}}{(a+bx^2)^2}, \int \frac{x^n dx \sqrt{x}}{(a+bx^2)^3} \text{ Taf. LXXXVII.}$$

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$$\text{VZ. } a+bx^2 = X$$


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$$\int \frac{\partial x \sqrt{x}}{X^2} = \frac{x \sqrt{x}}{2aX} + \frac{1}{4a} \int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x \partial x \sqrt{x}}{X^2} = -\frac{\sqrt{x}}{2bX} + \frac{1}{4b} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X^2} = -\frac{x \sqrt{x}}{2bX} + \frac{3}{4b} \int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X^2} = \left( \frac{2x^2}{b} + \frac{5a}{2b^2} \right) \frac{\sqrt{x}}{X} - \frac{5a}{4b^2} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X^2} = \left( \frac{2x^3}{3b} + \frac{7ax}{6b^2} \right) \frac{\sqrt{x}}{X} - \frac{7a}{4b^2} \int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X^2} = \left( \frac{2x^4}{5b} - \frac{18ax^2}{5b^2} - \frac{9a^2}{2b^3} \right) \frac{\sqrt{x}}{X} + \frac{9a^2}{4b^3} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{x^6 \partial x \sqrt{x}}{X^2} = \left( \frac{2x^5}{7b} - \frac{22ax^3}{21b^2} - \frac{11a^2x}{6b^3} \right) \frac{\sqrt{x}}{X} + \frac{11a^2}{4b^3} \int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^7 \partial x \sqrt{x}}{X^2} = \left( \frac{2x^6}{9b} - \frac{26ax^4}{45b^2} + \frac{26a^2x^2}{5b^3} + \frac{13a^3}{2b^4} \right) \frac{\sqrt{x}}{X} - \frac{13a^3}{4b^4} \int \frac{\partial x}{X \sqrt{x}}$$

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$$\int \frac{\partial x \sqrt{x}}{X^3} = \left( \frac{1}{4aX^2} + \frac{5}{16a^2X} \right) x \sqrt{x} + \frac{5}{32a^2} \int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x \partial x \sqrt{x}}{X^3} = \frac{(bx^2 - 3a) \sqrt{x}}{16abX^2} + \frac{3}{32ab} \int \frac{\partial x}{X \sqrt{x}}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X^3} = -\frac{2x \sqrt{x}}{5bX^2} + \frac{3a}{5b} \int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X^3} = -\frac{2x^2 \sqrt{x}}{3bX^2} + \frac{5a}{3b} \int \frac{x \partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X^3} = \left( -\frac{x^3}{b} - \frac{7ax}{5b^2} \right) \frac{2 \sqrt{x}}{X^2} + \frac{21a^2}{5b^2} \int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X^3} = \left( \frac{x^4}{b} + \frac{3ax^2}{b^2} \right) \frac{2 \sqrt{x}}{X^2} - \frac{15a^2}{b^2} \int \frac{x \partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^6 \partial x \sqrt{x}}{X^3} = \left( \frac{x^5}{3b} + \frac{11ax^3}{3b^2} + \frac{77a^2x}{15b^3} \right) \frac{2 \sqrt{x}}{X^2} - \frac{77a^3}{5b^3} \int \frac{\partial x \sqrt{x}}{X^3}$$



Taf. LXXXV III.  $\int \frac{dx}{(a+bx)x^m \sqrt{x}}, \int \frac{dx}{(a+bx)^2 x^m \sqrt{x}}$

VZ.  $a + bx = X$

$$\int \frac{dx}{Xx\sqrt{x}} = -\frac{2}{a\sqrt{x}} - \frac{b}{a} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{Xx^2\sqrt{x}} = \left(-\frac{1}{3ax} + \frac{b}{a^2}\right) \frac{2}{\sqrt{x}} + \frac{b^2}{a^2} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{Xx^3\sqrt{x}} = \left(-\frac{1}{5ax^2} + \frac{b}{3a^2x} - \frac{b^2}{a^3}\right) \frac{2}{\sqrt{x}} - \frac{b^3}{a^3} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{Xx^4\sqrt{x}} = \left(-\frac{1}{7ax^3} + \frac{b}{5a^2x^2} - \frac{b^2}{3a^3x} + \frac{b^3}{a^4}\right) \frac{2}{\sqrt{x}} + \frac{b^4}{a^4} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{Xx^5\sqrt{x}} = \left(-\frac{1}{9ax^4} + \frac{b}{7a^2x^3} - \frac{b^2}{5a^3x^2} + \frac{b^3}{3a^4x} - \frac{b^4}{a^5}\right) \frac{2}{\sqrt{x}} - \frac{b^5}{a^5} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{Xx^6\sqrt{x}} = \left(-\frac{1}{11ax^5} + \frac{b}{9a^2x^4} - \frac{b^2}{7a^3x^3} + \frac{b^3}{5a^4x^2} - \frac{b^4}{3a^5x} + \frac{b^5}{a^6}\right) \frac{2}{\sqrt{x}} + \frac{b^6}{a^6} \int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^2x\sqrt{x}} = -\frac{2}{aX\sqrt{x}} - \frac{3b}{a} \int \frac{dx}{X^2\sqrt{x}}$$

$$\int \frac{dx}{X^2x^2\sqrt{x}} = \left(-\frac{1}{3ax} + \frac{5b}{3a^2}\right) \frac{2}{X\sqrt{x}} + \frac{5b^2}{a^2} \int \frac{dx}{X^2\sqrt{x}}$$

$$\int \frac{dx}{X^2x^3\sqrt{x}} = \left(-\frac{1}{5ax^2} + \frac{7b}{15a^2x} - \frac{7b^2}{5a^3}\right) \frac{2}{X\sqrt{x}} - \frac{7b^3}{a^3} \int \frac{dx}{X^2\sqrt{x}}$$

$$\int \frac{dx}{X^2x^4\sqrt{x}} = \left(-\frac{1}{7ax^3} + \frac{9b}{35a^2x^2} - \frac{3b^2}{5a^3x} + \frac{3b^3}{a^4}\right) \frac{2}{X\sqrt{x}} + \frac{9b^4}{a^4} \int \frac{dx}{X^2\sqrt{x}}$$

$$\int \frac{dx}{X^2x^5\sqrt{x}} = \left(-\frac{1}{9ax^4} + \frac{11b}{63a^2x^3} - \frac{11b^2}{35a^3x^2} + \frac{11b^3}{15a^4x} - \frac{11b^4}{3a^5}\right) \frac{2}{X\sqrt{x}} - \frac{11b^5}{a^5} \int \frac{dx}{X^2\sqrt{x}}$$

$$\int \frac{dx}{X^2x^6\sqrt{x}} = \left(-\frac{1}{11ax^5} + \frac{13b}{99a^2x^4} - \frac{13b^2}{63a^3x^3} + \frac{13b^3}{35a^4x^2} - \frac{13b^4}{15a^5x} + \frac{13b^5}{3a^6}\right) \frac{2}{X\sqrt{x}} + \frac{13b^6}{a^6} \int \frac{dx}{X^2\sqrt{x}}$$

$$\int \frac{\partial x}{(a+bx)^2 x^m \sqrt{x}}, \int \frac{\partial x}{(a+bx)^4 x^m \sqrt{x}} \quad \text{Taf. LXXXIX.}$$

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{\partial x}{X^2 x \sqrt{x}} = -\frac{2}{aX^2 \sqrt{x}} - \frac{5b}{a} \int \frac{\partial x}{X^3 \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 x^2 \sqrt{x}} = \left(-\frac{1}{3ax} + \frac{7b}{3a^2}\right) \frac{2}{X^2 \sqrt{x}} + \frac{35b^2}{3a^2} \int \frac{\partial x}{X^3 \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 x^3 \sqrt{x}} = \left(-\frac{1}{5ax^2} + \frac{3b}{5a^2x} - \frac{21b^2}{5a^3}\right) \frac{2}{X^2 \sqrt{x}} + \frac{21b^3}{a^3} \int \frac{\partial x}{X^3 \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 x^4 \sqrt{x}} = \left(-\frac{1}{7ax^3} + \frac{11b}{35a^2x^2} - \frac{83b^2}{35a^3x} + \frac{33b^3}{5a^4}\right) \frac{2}{X^2 \sqrt{x}} + \frac{33b^4}{a^4} \int \frac{\partial x}{X^3 \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 x^5 \sqrt{x}} = \left(-\frac{1}{9ax^4} + \frac{13b}{63a^2x^3} - \frac{143b^2}{315a^3x^2} + \frac{143b^3}{105a^4x} - \frac{143b^4}{15a^5}\right) \frac{2}{X^2 \sqrt{x}} - \frac{143b^5}{3a^5} \int \frac{\partial x}{X^3 \sqrt{x}}$$

$$\int \frac{\partial x}{X^3 x^6 \sqrt{x}} = \left(-\frac{1}{11ax^5} + \frac{5b}{33a^2x^4} - \frac{65b^2}{231a^3x^3} + \frac{13b^3}{21a^4x^2} - \frac{13b^4}{7a^5x} + \frac{13b^5}{a^6}\right) \frac{2}{X^2 \sqrt{x}} + \frac{65b^6}{a^6} \int \frac{\partial x}{X^3 \sqrt{x}}$$


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$$\int \frac{\partial x}{X^4 x \sqrt{x}} = -\frac{2}{aX^4 \sqrt{x}} - \frac{7b}{a} \int \frac{\partial x}{X^4 \sqrt{x}}$$

$$\int \frac{\partial x}{X^4 x^2 \sqrt{x}} = \left(-\frac{1}{3ax} + \frac{3b}{a^2}\right) \frac{2}{X^3 \sqrt{x}} + \frac{21b^2}{a^2} \int \frac{\partial x}{X^4 \sqrt{x}}$$

$$\int \frac{\partial x}{X^4 x^3 \sqrt{x}} = \left(-\frac{1}{5ax^2} + \frac{11b}{15a^2x} - \frac{33b^2}{5a^3}\right) \frac{2}{X^3 \sqrt{x}} - \frac{231b^3}{5a^3} \int \frac{\partial x}{X^4 \sqrt{x}}$$

$$\int \frac{\partial x}{X^4 x^4 \sqrt{x}} = \left(-\frac{1}{7ax^3} + \frac{13b}{35a^2x^2} - \frac{143b^2}{105a^3x} + \frac{429b^3}{35a^4}\right) \frac{2}{X^3 \sqrt{x}} + \frac{429b^4}{5a^4} \int \frac{\partial x}{X^4 \sqrt{x}}$$

$$\int \frac{\partial x}{X^4 x^5 \sqrt{x}} = \left(-\frac{1}{9ax^4} + \frac{5b}{21a^2x^3} - \frac{13b^2}{21a^3x^2} + \frac{143b^3}{63a^4x} - \frac{143b^4}{7a^5}\right) \frac{2}{X^3 \sqrt{x}} - \frac{143b^5}{a^5} \int \frac{\partial x}{X^4 \sqrt{x}}$$

Taf. XC.

$$\int \frac{\partial x}{(f+gx)^n \sqrt{a+bx}}$$

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$$\text{VZ. } f+gx=X, a+bx=X', bf-ag=k$$


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$$\int \frac{\partial x}{X \sqrt{X'}} = \left\{ \begin{array}{l} \pm \frac{2}{\sqrt{gk}} \text{ArcTang } \sqrt{\frac{gX'}{k}} \\ \text{oder} \\ \frac{1}{\sqrt{-gk}} \log \frac{bf-ag-bgx+2\sqrt{-gk} \cdot \sqrt{X'}}{X} \end{array} \right\}^*)$$

$$\int \frac{\partial x}{X^2 \sqrt{X'}} = \frac{\sqrt{X'}}{kX} + \frac{b}{2k} \int \frac{\partial x}{X \sqrt{X'}}$$

$$\int \frac{\partial x}{X^3 \sqrt{X'}} = \left( \frac{1}{2kX^2} + \frac{3b}{4k^2X} \right) \sqrt{X'} + \frac{3b^2}{8k^2} \int \frac{\partial x}{X \sqrt{X'}}$$

$$\int \frac{\partial x}{X^4 \sqrt{X'}} = \left( \frac{1}{3kX^3} + \frac{5b}{12k^2X^2} + \frac{5b^2}{8k^3X} \right) \sqrt{X'} + \frac{5b^3}{16k^3} \int \frac{\partial x}{X \sqrt{X'}}$$

$$\int \frac{\partial x}{X^5 \sqrt{X'}} = \left( \frac{1}{4kX^4} + \frac{7b}{24k^2X^3} + \frac{35b^2}{96k^3X^2} + \frac{35b^3}{64k^4X} \right) \sqrt{X'} + \frac{35b^4}{128k^4} \int \frac{\partial x}{X \sqrt{X'}}$$

$$\int \frac{\partial x}{X^6 \sqrt{X'}} = \left( \frac{1}{5kX^5} + \frac{9b}{40k^2X^4} + \frac{21b^2}{80k^3X^3} + \frac{21b^3}{64k^4X^2} + \frac{63b^4}{128k^5X} \right) \sqrt{X'} + \frac{63b^5}{256k^5} \int \frac{\partial x}{X \sqrt{X'}}$$

$$\int \frac{\partial x}{X^7 \sqrt{X'}} = \left( \frac{1}{6kX^6} + \frac{11b}{60k^2X^5} + \frac{33b^2}{160k^3X^4} + \frac{77b^3}{320k^4X^3} + \frac{77b^4}{256k^5X^2} + \frac{231b^5}{512k^6X} \right) \sqrt{X'} + \frac{231b^6}{1024k^6} \int \frac{\partial x}{X \sqrt{X'}}$$

\*) Der erste Ausdruck mit dem Vorzeichen + wird genommen, wenn  $g$  und  $k$  zugleich positiv sind, und mit dem Vorzeichen —, wenn  $g$  und  $k$  zugleich negativ sind. Der zweite Ausdruck wird genommen, wenn  $g$  und  $k$  verschiedene Vorzeichen haben. Wenn  $k = 0$  wird, so geht

$$\int \frac{\partial x}{X \sqrt{X'}} \text{ in } \frac{b}{g} \int \frac{\partial x}{(a+bx)^{\frac{3}{2}}} = - \frac{2}{g \sqrt{a+bx}} \text{ über.}$$

$$\int \frac{x^m dx}{(f+gx)V(a+bx)}, \int \frac{x^m dx}{(f+gx)^2 V(a+bx)} \quad \text{Taf. XCI.}$$

$$\text{VZ. } f+gx=X, \quad a+bx=X'$$

$$\begin{aligned} \int \frac{x dx}{X V X'} &= \frac{1}{g} \int \frac{dx}{V X'} - \frac{f}{g} \int \frac{dx}{X V X'} \\ \int \frac{x^2 dx}{X V X'} &= \frac{1}{g} \int \frac{x dx}{V X'} - \frac{f}{g^2} \int \frac{dx}{V X'} + \frac{f^2}{g^2} \int \frac{dx}{X V X'} \\ \int \frac{x^3 dx}{X V X'} &= \frac{1}{g} \int \frac{x^2 dx}{V X'} - \frac{f}{g^2} \int \frac{x dx}{V X'} + \frac{f^2}{g^3} \int \frac{dx}{V X'} - \frac{f^3}{g^3} \int \frac{dx}{X V X'} \\ \int \frac{x^4 dx}{X V X'} &= \frac{1}{g} \int \frac{x^3 dx}{V X'} - \frac{f}{g^2} \int \frac{x^2 dx}{V X'} + \frac{f^2}{g^3} \int \frac{x dx}{V X'} - \frac{f^3}{g^4} \int \frac{dx}{V X'} \\ &\quad + \frac{f^4}{g^4} \int \frac{dx}{X V X'} \\ \int \frac{x^5 dx}{X V X'} &= \frac{1}{g} \int \frac{x^4 dx}{V X'} - \frac{f}{g^2} \int \frac{x^3 dx}{V X'} + \frac{f^2}{g^3} \int \frac{x^2 dx}{V X'} - \frac{f^3}{g^4} \int \frac{x dx}{V X'} \\ &\quad + \frac{f^4}{g^5} \int \frac{dx}{V X'} - \frac{f^5}{g^5} \int \frac{dx}{X V X'} \end{aligned}$$

$$\begin{aligned} \int \frac{x dx}{X^2 V X'} &= \frac{1}{g} \int \frac{dx}{X V X'} - \frac{f}{g} \int \frac{dx}{X^2 V X'} \\ \int \frac{x^2 dx}{X^2 V X'} &= \frac{1}{g^2} \int \frac{dx}{V X'} - \frac{2f}{g^2} \int \frac{dx}{X V X'} + \frac{f^2}{g^2} \int \frac{dx}{X^2 V X'} \\ \int \frac{x^3 dx}{X^2 V X'} &= \frac{1}{g^2} \int \frac{x dx}{V X'} - \frac{2f}{g^3} \int \frac{dx}{V X'} + \frac{3f^2}{g^3} \int \frac{dx}{X V X'} - \frac{f^3}{g^3} \int \frac{dx}{X^2 V X'} \\ \int \frac{x^4 dx}{X^2 V X'} &= \frac{1}{g^2} \int \frac{x^2 dx}{V X'} - \frac{2f}{g^3} \int \frac{x dx}{V X'} + \frac{3f^2}{g^4} \int \frac{dx}{V X'} - \frac{4f^3}{g^4} \int \frac{dx}{X V X'} \\ &\quad + \frac{f^4}{g^4} \int \frac{dx}{X^2 V X'} \\ \int \frac{x^5 dx}{X^2 V X'} &= \frac{1}{g^2} \int \frac{x^3 dx}{V X'} - \frac{2f}{g^3} \int \frac{x^2 dx}{V X'} + \frac{3f^2}{g^4} \int \frac{x dx}{V X'} - \frac{4f^3}{g^5} \int \frac{dx}{V X'} \\ &\quad + \frac{5f^4}{g^5} \int \frac{dx}{X V X'} - \frac{f^5}{g^5} \int \frac{dx}{X^2 V X'} \\ \int \frac{x^6 dx}{X^2 V X'} &= \frac{1}{g^2} \int \frac{x^4 dx}{V X'} - \frac{2f}{g^3} \int \frac{x^3 dx}{V X'} + \frac{3f^2}{g^4} \int \frac{x^2 dx}{V X'} - \frac{4f^3}{g^5} \int \frac{x dx}{V X'} \\ &\quad + \frac{5f^4}{g^6} \int \frac{dx}{V X'} - \frac{6f^5}{g^6} \int \frac{dx}{X V X'} + \frac{f^6}{g^6} \int \frac{dx}{X^2 V X'} \end{aligned}$$

Taf. XCII.  $\int \frac{x^m dx}{(f+gx)^3 V(a+bx)}, \int \frac{x^m dx}{(f+gx)^4 V(a+bx)}$

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VZ.  $f+gx=X, a+bx=X'$

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$$\begin{aligned} \int \frac{x dx}{X^3 V X'} &= \frac{1}{g} \int \frac{dx}{X^2 V X'} - \frac{f}{g} \int \frac{dx}{X^3 V X'} \\ \int \frac{x^2 dx}{X^3 V X'} &= \frac{1}{g^2} \int \frac{dx}{X V X'} - \frac{2f}{g^2} \int \frac{dx}{X^2 V X'} + \frac{f^2}{g^2} \int \frac{dx}{X^3 V X'} \\ \int \frac{x^3 dx}{X^3 V X'} &= \frac{1}{g^3} \int \frac{dx}{V X'} - \frac{3f}{g^3} \int \frac{dx}{X V X'} + \frac{3f^2}{g^3} \int \frac{dx}{X^2 V X'} - \frac{f^3}{g^3} \int \frac{dx}{X^3 V X'} \\ \int \frac{x^4 dx}{X^3 V X'} &= \frac{1}{g^3} \int \frac{x dx}{V X'} - \frac{3f}{g^4} \int \frac{dx}{V X'} + \frac{6f^2}{g^4} \int \frac{dx}{X V X'} - \frac{4f^3}{g^4} \int \frac{dx}{X^2 V X'} \\ &\quad + \frac{f^4}{g^4} \int \frac{dx}{X^3 V X'} \\ \int \frac{x^5 dx}{X^3 V X'} &= \frac{1}{g^3} \int \frac{x^3 dx}{V X'} - \frac{3f}{g^4} \int \frac{x dx}{V X'} + \frac{6f^2}{g^5} \int \frac{dx}{V X'} - \frac{10f^3}{g^5} \int \frac{dx}{X V X'} \\ &\quad + \frac{5f^4}{g^5} \int \frac{dx}{X^2 V X'} - \frac{f^5}{g^5} \int \frac{dx}{X^3 V X'} \end{aligned}$$

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$$\begin{aligned} \int \frac{x dx}{X^4 V X'} &= \frac{1}{g} \int \frac{dx}{X^3 V X'} - \frac{f}{g} \int \frac{dx}{X^4 V X'} \\ \int \frac{x^2 dx}{X^4 V X'} &= \frac{1}{g^2} \int \frac{dx}{X^2 V X'} - \frac{2f}{g^2} \int \frac{dx}{X^3 V X'} + \frac{f^2}{g^2} \int \frac{dx}{X^4 V X'} \\ \int \frac{x^3 dx}{X^4 V X'} &= \frac{1}{g^3} \int \frac{dx}{X V X'} - \frac{3f}{g^3} \int \frac{dx}{X^2 V X'} + \frac{3f^2}{g^3} \int \frac{dx}{X^3 V X'} \\ &\quad - \frac{f^3}{g^3} \int \frac{dx}{X^4 V X'} \\ \int \frac{x^4 dx}{X^4 V X'} &= \frac{1}{g^4} \int \frac{dx}{V X'} - \frac{4f}{g^4} \int \frac{dx}{X V X'} + \frac{6f^2}{g^4} \int \frac{dx}{X^2 V X'} - \frac{4f^3}{g^4} \int \frac{dx}{X^3 V X'} \\ &\quad + \frac{f^4}{g^4} \int \frac{dx}{X^4 V X'} \\ \int \frac{x^5 dx}{X^4 V X'} &= \frac{1}{g^4} \int \frac{x dx}{V X'} - \frac{4f}{g^5} \int \frac{dx}{V X'} + \frac{10f^2}{g^5} \int \frac{dx}{X V X'} - \frac{10f^3}{g^5} \int \frac{dx}{X^2 V X'} \\ &\quad + \frac{5f^4}{g^5} \int \frac{dx}{X^3 V X'} - \frac{f^5}{g^5} \int \frac{dx}{X^4 V X'} \end{aligned}$$

$$\int \frac{x^m dx}{(f+gx)\sqrt{a+bx^2}}$$

Taf. XCIII.

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VZ.  $a+bx^2=X$ ,  $f+gx=X'$ ,  $ag^2+bf^2=k$

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$$\int \frac{dx}{X'\sqrt{X}} = \left\{ \begin{array}{l} \pm \frac{1}{\sqrt{k}} \log \frac{ag-bfx \mp \sqrt{k} \cdot \sqrt{X}}{X'} \\ \text{oder} \\ \frac{1}{\sqrt{-k}} \text{Arc Tang} \frac{ag-bfx}{\sqrt{-k} \cdot \sqrt{X}} \end{array} \right\}^*)$$

$$\int \frac{x dx}{X'\sqrt{X}} = \frac{1}{g} \int \frac{dx}{\sqrt{X}} - \frac{f}{g} \int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^2 dx}{X'\sqrt{X}} = \frac{1}{g} \int \frac{x dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{dx}{\sqrt{X}} + \frac{f^2}{g^2} \int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^3 dx}{X'\sqrt{X}} = \frac{1}{g} \int \frac{x^2 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{dx}{\sqrt{X}} - \frac{f^3}{g^3} \int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^4 dx}{X'\sqrt{X}} = \frac{1}{g} \int \frac{x^3 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x^2 dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{x dx}{\sqrt{X}} - \frac{f^3}{g^4} \int \frac{dx}{\sqrt{X}} + \frac{f^4}{g^4} \int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^5 dx}{X'\sqrt{X}} = \frac{1}{g} \int \frac{x^4 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x^3 dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{x^2 dx}{\sqrt{X}} - \frac{f^3}{g^4} \int \frac{x dx}{\sqrt{X}} + \frac{f^4}{g^5} \int \frac{dx}{\sqrt{X}} - \frac{f^5}{g^5} \int \frac{dx}{X'\sqrt{X}}$$

\*; Der erste Ausdruck wird reell, wenn  $k$  positiv, der zweite, wenn  $k$  negativ ist. Von den Vorzeichen  $\pm$  und  $\mp$ , welche in dem ersten Ausdruck vorkommen, gehören die oberen zusammen, und eben so die unteren; sonst ist es gleichgültig, welche man braucht. Uebrigens ist

$$\begin{aligned} \text{Arc Tang} \frac{ag-bfx}{\sqrt{-k} \cdot \sqrt{X}} &= \text{Arc Sin} \frac{ag-bfx}{(f+gx)\sqrt{-ab}} \\ &= \text{Arc Cos} \frac{\sqrt{-k} \cdot \sqrt{X}}{(f+gx)\sqrt{-ab}} = \text{etc.} \end{aligned}$$

Der Factor  $\sqrt{-ab}$ , welcher hier im Sinus und Cosinus vorkommt, wird nothwendig reell, weil  $a$  und  $b$  weder zugleich positiv, noch zugleich negativ seyn können; denn im ersten Falle würde  $k$  gewiß positiv werden, und also der logarithmische Ausdruck gelten, im zweiten Falle würde  $\sqrt{a+bx^2}$  nothwendig imaginär werden.

Taf. XCIV.  $\int \frac{x^m dx}{(f+gx^2)V(a+bx^2)}, \int \frac{x^m dx V(a+bx^2)}{f+gx^2}$

VZ.  $a+bx^2=X, f+gx^2=X'$

$$\left. \begin{aligned} \int \frac{dx}{X'VX} &= \int \frac{dx}{X'VX} \\ \int \frac{x dx}{X'VX} &= \int \frac{x dx}{X'VX} \end{aligned} \right\} \text{ (Man s. die folgende Seite.)}$$

$$\int \frac{x^2 dx}{X'VX} = \frac{1}{g} \int \frac{dx}{VX} - \frac{f}{g} \int \frac{dx}{X'VX}$$

$$\int \frac{x^3 dx}{X'VX} = \frac{1}{g} \int \frac{x dx}{VX} - \frac{f}{g} \int \frac{x dx}{X'VX}$$

$$\int \frac{x^4 dx}{X'VX} = \frac{1}{g} \int \frac{x^2 dx}{VX} - \frac{f}{g^2} \int \frac{dx}{VX} + \frac{f^2}{g^2} \int \frac{dx}{X'VX}$$

$$\int \frac{x^5 dx}{X'VX} = \frac{1}{g} \int \frac{x^3 dx}{VX} - \frac{f}{g^2} \int \frac{x dx}{VX} + \frac{f^2}{g^2} \int \frac{x dx}{X'VX}$$

$$\int \frac{x^6 dx}{X'VX} = \frac{1}{g} \int \frac{x^4 dx}{VX} - \frac{f}{g^2} \int \frac{x^2 dx}{VX} + \frac{f^2}{g^3} \int \frac{dx}{VX} - \frac{f^3}{g^3} \int \frac{dx}{X'VX}$$

$$\int \frac{x^7 dx}{X'VX} = \frac{1}{g} \int \frac{x^5 dx}{VX} - \frac{f}{g^2} \int \frac{x^3 dx}{VX} + \frac{f^2}{g^3} \int \frac{x dx}{VX} + \frac{f^3}{g^3} \int \frac{x dx}{X'VX}$$

$$\int \frac{dx V X}{X'} = \frac{b}{g} \int \frac{dx}{VX} + \left(a - \frac{bf}{g}\right) \int \frac{dx}{X'VX}$$

$$\int \frac{x dx V X}{X'} = \frac{b}{g} \int \frac{x dx}{VX} + \left(a - \frac{bf}{g}\right) \int \frac{x dx}{X'VX}$$

$$\int \frac{x^2 dx V X}{X'} = \frac{b}{g} \int \frac{x^2 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{dx}{VX} - \left(\frac{af}{g} - \frac{bf^2}{g^2}\right) \int \frac{dx}{X'VX}$$

$$\int \frac{x^3 dx V X}{X'} = \frac{b}{g} \int \frac{x^3 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{x dx}{VX} - \left(\frac{af}{g} - \frac{bf^2}{g^2}\right) \int \frac{x dx}{X'VX}$$

$$\int \frac{x^4 dx V X}{X'} = \frac{b}{g} \int \frac{x^4 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{x^2 dx}{VX} - \left(\frac{af}{g^2} - \frac{bf^2}{g^3}\right) \int \frac{dx}{VX}$$

$$+ \left(\frac{af^2}{g^2} - \frac{bf^3}{g^3}\right) \int \frac{dx}{X'VX}$$

$$\int \frac{x^5 dx V X}{X'} = \frac{b}{g} \int \frac{x^5 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{x^3 dx}{VX} - \left(\frac{af}{g^2} - \frac{bf^2}{g^3}\right) \int \frac{x dx}{VX}$$

$$+ \left(\frac{af^2}{g^2} - \frac{bf^3}{g^3}\right) \int \frac{x dx}{X'VX}$$

*Anmerkung zur vorhergehenden Tafel.*

$$I. \int \frac{\partial x}{X' \sqrt{X}}$$

Es ist im Allgemeinen, was auch  $a, b, f, g$  für Vorzeichen haben mögen,

$$\int \frac{\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(bf^2 - afg)}} \log \frac{f\sqrt{(a + bx^2)} + x\sqrt{(bf^2 - afg)}}{\sqrt{(f + gx^2)}}$$

$$\text{oder } \int \frac{\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(afg - bf^2)}} \text{Arc Tang} \frac{x\sqrt{(afg - bf^2)}}{f\sqrt{(a + bx^2)}}.$$

Die erste Form wird reell, wenn  $bf^2 - afg$  eine positive, die zweite, wenn  $bf^2 - afg$  eine negative GröÙe ist. Für  $\sqrt{(f + gx^2)}$  in der ersten Form kann man unbeschadet auch  $\sqrt{-(f + gx^2)}$  setzen, wenn  $f$  und  $g$  negativ seyn sollten. Uebrigens ist

$$\begin{aligned} \text{Arc Tang} \frac{x\sqrt{(afg - bf^2)}}{f\sqrt{(a + bx^2)}} &= \text{Arc Cos} \sqrt{\frac{af + bfx^2}{af + agx^2}} \\ &= \text{Arc Sin} x\sqrt{\frac{ag - bf}{af + agx^2}} = \text{etc.} \end{aligned}$$

$$II. \int \frac{x \partial x}{X' \sqrt{X}}$$

Für jedes  $a, b, f, g$ , ist entweder

$$\int \frac{x \partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(ag^2 - bfg)}} \log \frac{g\sqrt{(a + bx^2)} - \sqrt{(ag^2 - bfg)}}{\sqrt{(f + gx^2)}}$$

$$\text{oder } \int \frac{x \partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(bfg - ag^2)}} \text{Arc Tang} \frac{g\sqrt{(a + bx^2)}}{\sqrt{(bfg - ag^2)}}.$$

Die erste Form wird reell, wenn  $ag^2 - bfg$  eine positive, die zweite, wenn  $ag^2 - bfg$  eine negative GröÙe ist. Wegen  $\sqrt{(f + gx^2)}$  in der ersten Form die nämliche Bemerkung wie oben. Uebrigens ist

$$\begin{aligned} \text{Arc Tang} \frac{g\sqrt{(a + bx^2)}}{\sqrt{(bfg - ag^2)}} &= \text{Arc Cos} \sqrt{\frac{bf - ag}{bf + bgx^2}} \\ &= \text{Arc Sin} \sqrt{\frac{ag + bgx^2}{bf + bgx^2}} = \text{etc.} \end{aligned}$$



Taf. XCV.

$$\int \frac{\partial x}{(fx^m + gx^{m+1})V(a+bx)}, \int \frac{\partial x}{(fx^m + gx^{m+1})V(a+bx^2)}$$

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$$\text{VL: } a+bx=X, a+bx^2=X', f+gx=Z$$


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$$\begin{aligned} \int \frac{\partial x}{xZVX} &= \frac{1}{f} \int \frac{\partial x}{xVX} - \frac{g}{f} \int \frac{\partial x}{ZVX} \\ \int \frac{\partial x}{x^2ZVX} &= \frac{1}{f} \int \frac{\partial x}{x^2VX} - \frac{g}{f^2} \int \frac{\partial x}{xVX} + \frac{g^2}{f^2} \int \frac{\partial x}{ZVX} \\ \int \frac{\partial x}{x^3ZVX} &= \frac{1}{f} \int \frac{\partial x}{x^3VX} - \frac{g}{f^2} \int \frac{\partial x}{x^2VX} + \frac{g^2}{f^3} \int \frac{\partial x}{xVX} - \frac{g^3}{f^3} \int \frac{\partial x}{ZVX} \\ \int \frac{\partial x}{x^4ZVX} &= \frac{1}{f} \int \frac{\partial x}{x^4VX} - \frac{g}{f^2} \int \frac{\partial x}{x^3VX} + \frac{g^2}{f^3} \int \frac{\partial x}{x^2VX} - \frac{g^3}{f^4} \int \frac{\partial x}{xVX} \\ &\quad + \frac{g^4}{f^4} \int \frac{\partial x}{ZVX} \end{aligned}$$

.....

$$\begin{aligned} \int \frac{\partial x}{x^mZVX} &= \frac{1}{f} \int \frac{\partial x}{x^mVX} - \frac{g}{f^2} \int \frac{\partial x}{x^{m-1}VX} + \frac{g^2}{f^3} \int \frac{\partial x}{x^{m-2}VX} - \text{etc.} \\ &\quad \pm \frac{g^{m-1}}{f^m} \int \frac{\partial x}{xVX} \mp \frac{g^m}{f^m} \int \frac{\partial x}{ZVX} \end{aligned}$$


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$$\begin{aligned} \int \frac{\partial x}{xZVX'} &= \frac{1}{f} \int \frac{\partial x}{xVX'} - \frac{g}{f} \int \frac{\partial x}{ZVX'} \\ \int \frac{\partial x}{x^2ZVX'} &= \frac{1}{f} \int \frac{\partial x}{x^2VX'} - \frac{g}{f^2} \int \frac{\partial x}{xVX'} + \frac{g^2}{f^2} \int \frac{\partial x}{ZVX'} \\ \int \frac{\partial x}{x^3ZVX'} &= \frac{1}{f} \int \frac{\partial x}{x^3VX'} - \frac{g}{f^2} \int \frac{\partial x}{x^2VX'} + \frac{g^2}{f^3} \int \frac{\partial x}{xVX'} - \frac{g^3}{f^3} \int \frac{\partial x}{ZVX'} \\ \int \frac{\partial x}{x^4ZVX'} &= \frac{1}{f} \int \frac{\partial x}{x^4VX'} - \frac{g}{f^2} \int \frac{\partial x}{x^3VX'} + \frac{g^2}{f^3} \int \frac{\partial x}{x^2VX'} - \frac{g^3}{f^4} \int \frac{\partial x}{xVX'} \\ &\quad + \frac{g^4}{f^4} \int \frac{\partial x}{ZVX'} \end{aligned}$$

.....

$$\begin{aligned} \int \frac{\partial x}{x^mZVX'} &= \frac{1}{f} \int \frac{\partial x}{x^mVX'} - \frac{g}{f^2} \int \frac{\partial x}{x^{m-1}VX'} + \frac{g^2}{f^3} \int \frac{\partial x}{x^{m-2}VX'} - \text{etc.} \\ &\quad \pm \frac{g^{m-1}}{f^m} \int \frac{\partial x}{xVX'} \mp \frac{g^m}{f^m} \int \frac{\partial x}{ZVX'} \end{aligned}$$

$$\int \frac{x^m dx}{(f+gx)\sqrt{(a+bx+cx^2)}} \quad \text{Taf. XCVI.}$$

$$\text{VZ. } a+bx+cx^2=X, \quad f+gx=Z$$

$$ag^2-bfg+cf^2=k$$

$$\int \frac{dx}{Z\sqrt{X}} = \left\{ \begin{array}{l} \pm \frac{1}{\sqrt{k}} \log \frac{2ag-bf+(bg-2cf)x+2\sqrt{k}\cdot\sqrt{X}}{f+gx} \\ \text{oder} \\ \frac{1}{\sqrt{-k}} \text{Arc Tang} \frac{2ag-bf+(bg-2cf)x}{2\sqrt{-k}\cdot\sqrt{X}} \end{array} \right\}^*)$$

$$\int \frac{x dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{dx}{\sqrt{X}} - \frac{f}{g} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^2 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{dx}{\sqrt{X}} + \frac{f^2}{g^2} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^3 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x^2 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{dx}{\sqrt{X}} - \frac{f^3}{g^3} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^4 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x^3 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x^2 dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{x dx}{\sqrt{X}} - \frac{f^3}{g^4} \int \frac{dx}{\sqrt{X}} + \frac{f^4}{g^4} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^5 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x^4 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x^3 dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{x^2 dx}{\sqrt{X}} - \frac{f^3}{g^4} \int \frac{x dx}{\sqrt{X}} + \frac{f^4}{g^5} \int \frac{dx}{\sqrt{X}} - \frac{f^5}{g^5} \int \frac{dx}{Z\sqrt{X}}$$

\*) Der erste Ausdruck wird reell, wenn  $k$  positiv, der zweite, wenn  $k$  negativ ist. Von den Zeichen  $\pm$  in dem ersten Ausdrucke gehören die oberen zusammen, und eben so die unteren; sonst ist es gleichgültig, welche man braucht. Uebrigens ist

$$\text{Arc Tang} \frac{2ag-bf+(bg-2cf)x}{2\sqrt{-k}\cdot\sqrt{X}} = \text{Arc Cos} \frac{2\sqrt{-k}\cdot\sqrt{X}}{(f+gx)\sqrt{(b^2-4ac)}}$$

$$= \text{Arc Sin} \frac{2ag-bf+(bg-2cf)x}{(f+gx)\sqrt{(b^2-4ac)}} = \text{etc.}$$

Die Wurzelgröße  $\sqrt{(b^2-4ac)}$ , welche hier im Sinus und Cosinus vorkommt, wird gewiss reell, wenn  $ag^2-bfg+cf^2$  eine negative GröÙe ist, weil sonst  $\sqrt{(a+bx+cx^2)}$  nicht reell seyn könnte.

T a f e l  
einiger allgemeineren Formeln.

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VZ.  $a + bx^p = X$

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$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+1} - \frac{pnb}{m+1} \int x^{m+p} dx X^{p-1}$$

$$\int \frac{x^m dx}{X^p} = -\frac{x^{m-p+1}}{(p-1)nb X^{p-1}} + \frac{m-n+1}{(p-1)nb} \int \frac{x^{m-p} dx}{X^{p-1}}$$

$$\int x^m dx X^p = \frac{x^{m-p+1} X^{p+1}}{(m+np+1)b} - \frac{(m-n+1)a}{(m+np+1)b} \int x^{m-p} dx X^p$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-p+1}}{(m-np+1)b X^{p-1}} - \frac{(m-n+1)a}{(m-np+1)b} \int \frac{x^{m-p} dx}{X^p}$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+np+1} + \frac{pna}{m+np+1} \int x^m dx X^{p-1}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^p}{(m-np-1)x^{m-1}} - \frac{pna}{m-np-1} \int \frac{dx X^{p-1}}{x^m}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^{p+1}}{(m-1)ax^{m-1}} - \frac{(m-n-np-1)b}{(m-1)a} \int \frac{dx X^p}{x^{m-p}}$$

$$\int \frac{dx}{x^m X^p} = -\frac{1}{(m-1)ax^{m-1} X^{p-1}} - \frac{(m-n+np-1)b}{(m-1)a} \int \frac{dx}{x^{m-p} X^p}$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m+1}}{(p-1)na X^{p-1}} - \frac{m+n-np+1}{(p-1)na} \int \frac{x^m dx}{X^{p-1}}$$

$$\int \frac{dx}{x^m X^p} = \frac{1}{(p-1)nax^{m-1} X^{p-1}} + \frac{m-n+np-1}{(p-1)na} \int \frac{dx}{x^m X^{p-1}}$$

$$\int \frac{dx}{X^p} = \frac{x}{(p-1)na X^{p-1}} + \frac{np-n-1}{(p-1)na} \int \frac{dx}{X^{p-1}}$$

$$\int dx X^p = \frac{x X^p}{np+1} + \frac{pna}{np+1} \int dx X^{p-1}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } ax^k + bx^{k+1} = X$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+pk+1} - \frac{pnb}{m+pk+1} \int x^{m+k+1} dx X^{p-1}$$

$$\int \frac{x^m dx}{X^p} = -\frac{x^{m-k-n+1}}{(p-1)nb X^{p-1}} + \frac{m-pk-n+1}{(p-1)nb} \int \frac{x^{m-k-n} dx}{X^{p-1}}$$

$$\int x^m dx X^p = \frac{x^{m-k-n+1} X^{p+1}}{(m+pk+np+1)b} - \frac{(m+pk-n+1)a}{(m+pk+np+1)b} \int x^{m-n} dx X^p$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-k-n+1}}{(m-pk-np+1)b X^{p-1}} - \frac{(m-pk-n+1)a}{(m-pk-np+1)b} \int \frac{x^{m-n} dx}{X^p}$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+pk+np+1} + \frac{pna}{m+pk+np+1} \int x^{m+k} dx X^{p-1}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^p}{(m-pk-np-1)x^{m-1}} - \frac{pna}{m-pk-np-1} \int \frac{dx X^{p-1}}{x^{m-k}}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^{p+1}}{(m-pk-1)ax^{m+k-1}} - \frac{(m-n-pk-np-1)b}{(m-pk-1)a} \int \frac{dx X^p}{x^{m-n}}$$

$$\int \frac{dx}{x^m X^p} = -\frac{1}{(m+pk-1)ax^{m+k-1} X^{p-1}} - \frac{(m-n+pk+np-1)b}{(m+pk-1)a} \int \frac{dx}{x^{m-n} X^p}$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-k+1}}{(p-1)na X^{p-1}} - \frac{m+n-pk-np+1}{(p-1)na} \int \frac{x^{m-k} dx}{X^{p-1}}$$

$$\int \frac{dx}{x^m X^p} = \frac{1}{(p-1)na x^{m+k-1} X^{p-1}} + \frac{m-n+pk+np-1}{(p-1)na} \int \frac{dx}{x^{m+k} X^{p-1}}$$

$$\int \frac{dx}{X^p} = \frac{1}{(p-1)na x^{k-1} X^{p-1}} + \frac{pk+np-n-1}{(p-1)na} \int \frac{dx}{x^k X^{p-1}}$$

$$\int dx X^p = \frac{x X^p}{pk+np+1} + \frac{pna}{pk+np+1} \int x^k dx X^{p-1}$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VZ. } a + bx = X$$


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$$\int \frac{x^m dx}{\sqrt{X}} = \left( \frac{X^m}{2m+1} - \frac{{}^m\mathcal{A}X^{m-1}}{2m-1} + \frac{{}^m\mathcal{B}X^{m-2}}{2m-3} - \frac{{}^m\mathcal{C}X^{m-3}}{2m-5} + \dots \right. \\ \left. \dots \pm \frac{{}^{m-2}\mathcal{M}X^{m-2}}{5} + \frac{{}^{m-1}\mathcal{M}X^{m-1}}{3} \pm \frac{{}^m\mathcal{M}X^m}{1} \right) \frac{2\sqrt{X}}{b^{m+1}}$$

$$\int \frac{x^m dx}{X^{\frac{1}{2}}} = \left( \frac{X^m}{2m-1} - \frac{{}^m\mathcal{A}X^{m-1}}{2m-3} + \frac{{}^m\mathcal{B}X^{m-2}}{2m-5} - \frac{{}^m\mathcal{C}X^{m-3}}{2m-7} + \dots \right. \\ \left. \dots \pm \frac{{}^{m-2}\mathcal{M}X^{m-2}}{3} + \frac{{}^{m-1}\mathcal{M}X^{m-1}}{1} \pm \frac{{}^m\mathcal{M}X^m}{-1} \right) \frac{2}{b^{m+1}\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{3}{2}}} = \left( \frac{X^m}{2m-3} - \frac{{}^m\mathcal{A}X^{m-1}}{2m-5} + \frac{{}^m\mathcal{B}X^{m-2}}{2m-7} - \frac{{}^m\mathcal{C}X^{m-3}}{2m-9} + \dots \right. \\ \left. \dots \pm \frac{{}^{m-2}\mathcal{M}X^{m-2}}{1} + \frac{{}^{m-1}\mathcal{M}X^{m-1}}{-1} \pm \frac{{}^m\mathcal{M}X^m}{-3} \right) \frac{2}{b^{m+1}X\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{5}{2}}} = \left( \frac{X^m}{2m-5} - \frac{{}^m\mathcal{A}X^{m-1}}{2m-7} + \frac{{}^m\mathcal{B}X^{m-2}}{2m-9} - \frac{{}^m\mathcal{C}X^{m-3}}{2m-11} + \dots \right. \\ \left. \dots \pm \frac{{}^{m-2}\mathcal{M}X^{m-2}}{-1} + \frac{{}^{m-1}\mathcal{M}X^{m-1}}{-3} \pm \frac{{}^m\mathcal{M}X^m}{-5} \right) \frac{2}{b^{m+1}X^2\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \left( \frac{X^m}{2m-n+2} - \frac{{}^m\mathcal{A}X^{m-1}}{2m-n} + \frac{{}^m\mathcal{B}X^{m-2}}{2m-n-2} - \dots \right. \\ \left. \dots \pm \frac{{}^{m-2}\mathcal{M}X^{m-2}}{-(n-6)} + \frac{{}^{m-1}\mathcal{M}X^{m-1}}{-(n-4)} + \frac{{}^m\mathcal{M}X^m}{-(n-2)} \right) \frac{2}{b^{m+1}X^{\frac{n-2}{2}}}$$

$$\int x^m dx \sqrt{X} = \left( \frac{X^m}{2m+3} - \frac{{}^m\mathcal{A}X^{m-1}}{2m+1} + \frac{{}^m\mathcal{B}X^{m-2}}{2m-1} - \frac{{}^m\mathcal{C}X^{m-3}}{2m-3} + \dots \right. \\ \left. \dots \pm \frac{{}^{m-2}\mathcal{M}X^{m-2}}{7} + \frac{{}^{m-1}\mathcal{M}X^{m-1}}{5} \pm \frac{{}^m\mathcal{M}X^m}{3} \right) \frac{2X\sqrt{X}}{b^{m+1}}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a + bx = X$$

$$\int x^m dx X^{\frac{1}{2}} = \left( \frac{X^m}{2m+5} - \frac{2aX^{m-1}}{2m+3} + \frac{3a^2X^{m-2}}{2m+1} - \frac{4a^3X^{m-3}}{2m-1} + \dots \right. \\ \left. \dots \dots \dots + \frac{2^m a^{m-2} X^2}{9} + \frac{2^{m-1} a^{m-1} X}{7} + \frac{2^m a^m}{5} \right) \frac{X^{\frac{1}{2}}}{b^{m+1}}$$

$$\int x^m dx X^{\frac{1}{3}} = \left( \frac{X^m}{2m+7} - \frac{2aX^{m-1}}{2m+5} + \frac{3a^2X^{m-2}}{2m+3} - \frac{4a^3X^{m-3}}{2m+1} + \dots \right. \\ \left. \dots \dots \dots + \frac{2^m a^{m-2} X^2}{11} + \frac{2^{m-1} a^{m-1} X}{9} + \frac{2^m a^m}{7} \right) \frac{X^{\frac{1}{3}}}{b^{m+1}}$$

$$\int x^m dx X^{\frac{2}{3}} = \left( \frac{X^m}{2m+n+2} - \frac{2aX^{m-1}}{2m+n} + \frac{3a^2X^{m-2}}{2m+n-2} - \frac{4a^3X^{m-3}}{2m+n-4} + \dots \right. \\ \left. \dots \dots \dots + \frac{2^m a^{m-2} X^2}{n+6} + \frac{2^{m-1} a^{m-1} X}{n+4} + \frac{2^m a^m}{n+2} \right) \frac{X^{\frac{2}{3}}}{b^{m+1}}$$

$$\int \frac{x^m dx}{X^{\frac{1}{2}}} = \left( \frac{X^m}{qm-p+q} - \frac{2aX^{m-1}}{qm-p} + \frac{3a^2X^{m-2}}{qm-p-q} - \frac{4a^3X^{m-3}}{qm-p-2q} \right. \\ \left. + \frac{2^m a^{m-2} X^2}{qm-p-3q} \dots \dots \dots + \frac{2^m a^{m-2} X^2}{-(p-3q)} + \frac{2^{m-1} a^{m-1} X}{-(p-2q)} + \frac{2^m a^m}{-(p-q)} \right) \frac{q}{b^{m+1} X^{\frac{1}{2}-1}}$$

$$\int x^m dx X^{\frac{1}{4}} = \left( \frac{X^m}{qm+p+q} - \frac{2aX^{m-1}}{qm+p} + \frac{3a^2X^{m-2}}{qm+p-q} - \frac{4a^3X^{m-3}}{qm+p-2q} \right. \\ \left. + \frac{2^m a^{m-2} X^2}{qm+p-3q} \dots \dots \dots + \frac{2^m a^{m-2} X^2}{p+3q} + \frac{2^{m-1} a^{m-1} X}{p+2q} + \frac{2^m a^m}{p+q} \right) \frac{q X^{\frac{1}{4}+1}}{b^{m+1}}$$

Taf. XCIV.  $\int \frac{x^n dx}{(f+gx^n)V(a+bx^2)}, \int \frac{x^n dx V(a+bx^2)}{f+gx^n}$

VZ.  $a+bx^2 = X, f+gx^2 = X'$

$$\left. \begin{aligned} \int \frac{dx}{X'VX} &= \int \frac{dx}{X'VX} \\ \int \frac{x dx}{X'VX} &= \int \frac{x dx}{X'VX} \end{aligned} \right\} \text{ (Man s. die folgende Seite.)}$$

$$\int \frac{x^2 dx}{X'VX} = \frac{1}{g} \int \frac{dx}{VX} - \frac{f}{g} \int \frac{dx}{X'VX}$$

$$\int \frac{x^3 dx}{X'VX} = \frac{1}{g} \int \frac{x dx}{VX} - \frac{f}{g^2} \int \frac{x dx}{X'VX}$$

$$\int \frac{x^4 dx}{X'VX} = \frac{1}{g} \int \frac{x^2 dx}{VX} - \frac{f}{g^2} \int \frac{dx}{VX} + \frac{f^2}{g^2} \int \frac{dx}{X'VX}$$

$$\int \frac{x^5 dx}{X'VX} = \frac{1}{g} \int \frac{x^3 dx}{VX} - \frac{f}{g^2} \int \frac{x dx}{VX} + \frac{f^2}{g^2} \int \frac{x dx}{X'VX}$$

$$\int \frac{x^6 dx}{X'VX} = \frac{1}{g} \int \frac{x^4 dx}{VX} - \frac{f}{g^2} \int \frac{x^2 dx}{VX} + \frac{f^2}{g^3} \int \frac{dx}{VX} - \frac{f^3}{g^3} \int \frac{dx}{X'VX}$$

$$\int \frac{x^7 dx}{X'VX} = \frac{1}{g} \int \frac{x^5 dx}{VX} - \frac{f}{g^2} \int \frac{x^3 dx}{VX} + \frac{f^2}{g^3} \int \frac{x dx}{VX} + \frac{f^3}{g^3} \int \frac{x dx}{X'VX}$$

$$\int \frac{dx V X}{X'} = \frac{b}{g} \int \frac{dx}{VX} + \left(a - \frac{bf}{g}\right) \int \frac{dx}{X'VX}$$

$$\int \frac{x dx V X}{X'} = \frac{b}{g} \int \frac{x dx}{VX} + \left(a - \frac{bf}{g}\right) \int \frac{x dx}{X'VX}$$

$$\int \frac{x^2 dx V X}{X'} = \frac{b}{g} \int \frac{x^2 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{dx}{VX} - \left(\frac{af}{g} - \frac{bf^2}{g^2}\right) \int \frac{dx}{X'VX}$$

$$\int \frac{x^3 dx V X}{X'} = \frac{b}{g} \int \frac{x^3 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{x dx}{VX} - \left(\frac{af}{g} - \frac{bf^2}{g^2}\right) \int \frac{x dx}{X'VX}$$

$$\int \frac{x^4 dx V X}{X'} = \frac{b}{g} \int \frac{x^4 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{x^2 dx}{VX} - \left(\frac{af}{g^2} - \frac{bf^2}{g^3}\right) \int \frac{dx}{VX} + \left(\frac{af^2}{g^2} - \frac{bf^3}{g^3}\right) \int \frac{dx}{X'VX}$$

$$\int \frac{x^5 dx V X}{X'} = \frac{b}{g} \int \frac{x^5 dx}{VX} + \left(\frac{a}{g} - \frac{bf}{g^2}\right) \int \frac{x^3 dx}{VX} - \left(\frac{af}{g^2} - \frac{bf^2}{g^3}\right) \int \frac{x dx}{VX} + \left(\frac{af^2}{g^2} - \frac{bf^3}{g^3}\right) \int \frac{x dx}{X'VX}$$

*Anmerkung zur vorhergehenden Tafel.*

$$I. \int \frac{\partial x}{X' \sqrt{X}}$$

Es ist im Allgemeinen, was auch  $a, b, f, g$  für Vorzeichen haben mögen,

$$\int \frac{\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(bf^2 - afg)}} \log \frac{\sqrt{V(a + bx^2)} + x \sqrt{(bf^2 - afg)}}{\sqrt{(f + gx^2)}}$$

$$\text{oder } \int \frac{\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(afg - bf^2)}} \text{Arc Tang} \frac{x \sqrt{(afg - bf^2)}}{\sqrt{V(a + bx^2)}}.$$

Die erste Form wird reell, wenn  $bf^2 - afg$  eine positive, die zweite, wenn  $bf^2 - afg$  eine negative Gröfse ist. Für  $\sqrt{(f + gx^2)}$  in der ersten Form kann man unbeschadet auch  $\sqrt{-(f + gx^2)}$  setzen, wenn  $f$  und  $g$  negativ seyn sollten. Uebrigens ist

$$\text{Arc Tang} \frac{x \sqrt{(afg - bf^2)}}{\sqrt{V(a + bx^2)}} = \text{Arc Cos} \sqrt{\frac{af + bfx^2}{af + agx^2}}$$

$$= \text{Arc Sin} x \sqrt{\frac{ag - bf}{af + agx^2}} = \text{etc.}$$

$$II. \int \frac{x \partial x}{X' \sqrt{X}}$$

Für jedes  $a, b, f, g$ , ist entweder

$$\int \frac{x \partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(ag^2 - bfg)}} \log \frac{g \sqrt{V(a + bx^2)} - \sqrt{(ag^2 - bfg)}}{\sqrt{(f + gx^2)}}$$

$$\text{oder } \int \frac{x \partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(bfg - ag^2)}} \text{Arc Tang} \frac{g \sqrt{V(a + bx^2)}}{\sqrt{(bfg - ag^2)}}.$$

Die erste Form wird reell, wenn  $ag^2 - bfg$  eine positive, die zweite, wenn  $ag^2 - bfg$  eine negative Gröfse ist. Wegen  $\sqrt{(f + gx^2)}$  in der ersten Form die nämliche Bemerkung wie oben. Uebrigens ist

$$\text{Arc Tang} \frac{g \sqrt{V(a + bx^2)}}{\sqrt{(bfg - ag^2)}} = \text{Arc Cos} \sqrt{\frac{bf - ag}{bf + bgx^2}}$$

$$= \text{Arc Sin} \sqrt{\frac{ag + bgx^2}{bf + bgx^2}} = \text{etc.}$$



Taf. XCV.

$$\int \frac{\partial x}{(fx^m + gx^{m+1})V(a+bx)}, \int \frac{\partial x}{(fx^m + gx^{m+1})V(a+bx^2)}$$

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$$VZ: a+bx=X, a+bx^2=X', f+gx=Z$$


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$$\int \frac{\partial x}{xZVX} = \frac{1}{f} \int \frac{\partial x}{xVX} - \frac{g}{f} \int \frac{\partial x}{ZVX}$$

$$\int \frac{\partial x}{x^2ZVX} = \frac{1}{f} \int \frac{\partial x}{x^2VX} - \frac{g}{f^2} \int \frac{\partial x}{xVX} + \frac{g^2}{f^2} \int \frac{\partial x}{ZVX}$$

$$\int \frac{\partial x}{x^3ZVX} = \frac{1}{f} \int \frac{\partial x}{x^3VX} - \frac{g}{f^2} \int \frac{\partial x}{x^2VX} + \frac{g^2}{f^3} \int \frac{\partial x}{xVX} - \frac{g^3}{f^3} \int \frac{\partial x}{ZVX}$$

$$\int \frac{\partial x}{x^4ZVX} = \frac{1}{f} \int \frac{\partial x}{x^4VX} - \frac{g}{f^2} \int \frac{\partial x}{x^3VX} + \frac{g^2}{f^3} \int \frac{\partial x}{x^2VX} - \frac{g^3}{f^4} \int \frac{\partial x}{xVX} + \frac{g^4}{f^4} \int \frac{\partial x}{ZVX}$$

.....

$$\int \frac{\partial x}{x^mZVX} = \frac{1}{f} \int \frac{\partial x}{x^mVX} - \frac{g}{f^2} \int \frac{\partial x}{x^{m-1}VX} + \frac{g^2}{f^3} \int \frac{\partial x}{x^{m-2}VX} - \text{etc.} \\ \pm \frac{g^{m-1}}{f^m} \int \frac{\partial x}{xVX} \mp \frac{g^m}{f^m} \int \frac{\partial x}{ZVX}$$


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$$\int \frac{\partial x}{xZVX'} = \frac{1}{f} \int \frac{\partial x}{xVX'} - \frac{g}{f} \int \frac{\partial x}{ZVX'}$$

$$\int \frac{\partial x}{x^2ZVX'} = \frac{1}{f} \int \frac{\partial x}{x^2VX'} - \frac{g}{f^2} \int \frac{\partial x}{xVX'} + \frac{g^2}{f^2} \int \frac{\partial x}{ZVX'}$$

$$\int \frac{\partial x}{x^3ZVX'} = \frac{1}{f} \int \frac{\partial x}{x^3VX'} - \frac{g}{f^2} \int \frac{\partial x}{x^2VX'} + \frac{g^2}{f^3} \int \frac{\partial x}{xVX'} - \frac{g^3}{f^3} \int \frac{\partial x}{ZVX'}$$

$$\int \frac{\partial x}{x^4ZVX'} = \frac{1}{f} \int \frac{\partial x}{x^4VX'} - \frac{g}{f^2} \int \frac{\partial x}{x^3VX'} + \frac{g^2}{f^3} \int \frac{\partial x}{x^2VX'} - \frac{g^3}{f^4} \int \frac{\partial x}{xVX'} + \frac{g^4}{f^4} \int \frac{\partial x}{ZVX'}$$

.....

$$\int \frac{\partial x}{x^mZVX'} = \frac{1}{f} \int \frac{\partial x}{x^mVX'} - \frac{g}{f^2} \int \frac{\partial x}{x^{m-1}VX'} + \frac{g^2}{f^3} \int \frac{\partial x}{x^{m-2}VX'} - \text{etc.} \\ \pm \frac{g^{m-1}}{f^m} \int \frac{\partial x}{xVX'} \mp \frac{g^m}{f^m} \int \frac{\partial x}{ZVX'}$$

$$\int \frac{x^m dx}{(f+gx)\sqrt{(a+bx+cx^2)}}$$

Taf. XCVI.

$$\text{VZ. } a+bx+cx^2=X, \quad f+gx=Z$$

$$ag^2-bfg+cf^2=k$$

$$\int \frac{dx}{Z\sqrt{X}} = \left\{ \begin{array}{l} \pm \frac{1}{\sqrt{k}} \log \frac{2ag-bf+(bg-2cf)x \pm 2\sqrt{k} \cdot \sqrt{X}}{f+gx} \\ \text{oder} \\ \frac{1}{\sqrt{-k}} \text{Arc Tang} \frac{2ag-bf+(bg-2cf)x}{2\sqrt{-k} \cdot \sqrt{X}} \end{array} \right\}^*)$$

$$\int \frac{x dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{dx}{\sqrt{X}} - \frac{f}{g} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^2 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{dx}{\sqrt{X}} + \frac{f^2}{g^2} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^3 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x^2 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{dx}{\sqrt{X}} - \frac{f^3}{g^3} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^4 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x^3 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x^2 dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{x dx}{\sqrt{X}} - \frac{f^3}{g^4} \int \frac{dx}{\sqrt{X}} + \frac{f^4}{g^4} \int \frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^5 dx}{Z\sqrt{X}} = \frac{1}{g} \int \frac{x^4 dx}{\sqrt{X}} - \frac{f}{g^2} \int \frac{x^3 dx}{\sqrt{X}} + \frac{f^2}{g^3} \int \frac{x^2 dx}{\sqrt{X}} - \frac{f^3}{g^4} \int \frac{x dx}{\sqrt{X}} + \frac{f^4}{g^5} \int \frac{dx}{\sqrt{X}} - \frac{f^5}{g^5} \int \frac{dx}{Z\sqrt{X}}$$

\*) Der erste Ausdruck wird reell, wenn  $k$  positiv, der zweite, wenn  $k$  negativ ist. Von den Zeichen  $\pm$  in dem ersten Ausdrucke gehören die oberen zusammen, und eben so die unteren; sonst ist es gleichgültig, welche man braucht. Uebrigens ist

$$\text{Arc Tang} \frac{2ag-bf+(bg-2cf)x}{2\sqrt{-k} \cdot \sqrt{X}} = \text{Arc Cos} \frac{2\sqrt{-k} \cdot \sqrt{X}}{(f+gx)\sqrt{(b^2-4ac)}}$$

$$= \text{Arc Sin} \frac{2ag-bf+(bg-2cf)x}{(f+gx)\sqrt{(b^2-4ac)}} = \text{etc.}$$

Die Wurzelgröße  $\sqrt{(b^2-4ac)}$ , welche hier im Sinus und Cosinus vorkommt, wird gewiß reell, wenn  $ag^2-bfg+cf^2$  eine negative Größe ist, weil sonst  $\sqrt{(a+bx+cx^2)}$  nicht reell seyn könnte.

T a f e l  
einiger allgemeineren Formeln.

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VZ.  $a + bx^p = X$

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$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+1} - \frac{pnb}{m+1} \int x^{m+1} dx X^{p-1}$$

$$\int \frac{x^m dx}{X^p} = -\frac{x^{m-n+1}}{(p-1)nb X^{p-1}} + \frac{m-n+1}{(p-1)nb} \int \frac{x^{m-n} dx}{X^{p-1}}$$

$$\int x^m dx X^p = \frac{x^{m-n+1} X^{p+1}}{(m+np+1)b} - \frac{(m-n+1)a}{(m+np+1)b} \int x^{m-n} dx X^p$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-n+1}}{(m-np+1)b X^{p-1}} - \frac{(m-n+1)a}{(m-np+1)b} \int \frac{x^{m-n} dx}{X^p}$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+np+1} + \frac{pna}{m+np+1} \int x^m dx X^{p-1}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^p}{(m-np-1)x^{m-1}} - \frac{pna}{m-np-1} \int \frac{dx X^{p-1}}{x^m}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^{p+1}}{(m-1)ax^{m-1}} - \frac{(m-n-np-1)b}{(m-1)a} \int \frac{dx X^p}{x^{m-n}}$$

$$\int \frac{dx}{x^m X^p} = -\frac{1}{(m-1)ax^{m-1} X^{p-1}} - \frac{(m-n+np-1)b}{(m-1)a} \int \frac{dx}{x^{m-n} X^p}$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m+1}}{(p-1)na X^{p-1}} - \frac{m+n-np+1}{(p-1)na} \int \frac{x^m dx}{X^{p-1}}$$

$$\int \frac{dx}{x^m X^p} = \frac{1}{(p-1)nax^{m-1} X^{p-1}} + \frac{m-n+np-1}{(p-1)na} \int \frac{dx}{x^m X^{p-1}}$$

$$\int \frac{dx}{X^p} = \frac{x}{(p-1)na X^{p-1}} + \frac{np-n-1}{(p-1)na} \int \frac{dx}{X^{p-1}}$$

$$\int dx X^p = \frac{x X^p}{np+1} + \frac{pna}{np+1} \int dx X^{p-1}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } ax^k + bx^{k+1} = X$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+pk+1} - \frac{pnb}{m+pk+1} \int x^{m+k+1} dx X^{p-1}$$

$$\int \frac{x^m dx}{X^p} = -\frac{x^{m-k-n+1}}{(p-1)nb X^{p-1}} + \frac{m-pk-n+1}{(p-1)nb} \int \frac{x^{m-k-n} dx}{X^{p-1}}$$

$$\int x^m dx X^p = \frac{x^{m-k-n+1} X^{p+1}}{(m+pk+np+1)b} - \frac{(m+pk-n+1)a}{(m+pk+np+1)b} \int x^{m-n} dx X^p$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-k-n+1}}{(m-pk-np+1)b X^{p-1}} - \frac{(m-pk-n+1)a}{(m-pk-np+1)b} \int \frac{x^{m-n} dx}{X^p}$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+pk+np+1} + \frac{pna}{m+pk+np+1} \int x^{m+k} dx X^{p-1}$$

$$\int \frac{dx X^p}{x^p} = -\frac{X^p}{(m-pk-np-1)x^{m-1}} - \frac{pna}{m-pk-np-1} \int \frac{dx X^{p-1}}{x^{m-k}}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^{p+1}}{(m-pk-1)ax^{m+k-1}} - \frac{(m-n-pk-np-1)b}{(m-pk-1)a} \int \frac{dx X^p}{x^{m-n}}$$

$$\int \frac{dx}{x^m X^p} = -\frac{1}{(m+pk-1)ax^{m+k-1} X^{p-1}} - \frac{(m-n+pk+np-1)b}{(m+pk-1)a} \int \frac{dx}{x^{m-n} X^p}$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-k+1}}{(p-1)na X^{p-1}} - \frac{m+n-pk-np+1}{(p-1)na} \int \frac{x^{m-k} dx}{X^{p-1}}$$

$$\int \frac{dx}{x^m X^p} = \frac{1}{(p-1)na x^{m+k-1} X^{p-1}} + \frac{m-n+pk+np-1}{(p-1)na} \int \frac{dx}{x^{m+k} X^{p-1}}$$

$$\int \frac{dx}{X^p} = \frac{1}{(p-1)na x^{k-1} X^{p-1}} + \frac{pk+np-n-1}{(p-1)na} \int \frac{dx}{x^k X^{p-1}}$$

$$\int dx X^p = \frac{x X^p}{pk+np+1} + \frac{pna}{pk+np+1} \int x^k dx X^{p-1}$$

T a f e l  
einiger allgemeineren Formeln.

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VZ.  $a + bx = X$

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$$\int \frac{x^m dx}{\sqrt{X}} = \left( \frac{X^m}{2m+1} - \frac{2aX^{m-1}}{2m-1} + \frac{2a^2X^{m-2}}{2m-3} - \frac{2a^3X^{m-3}}{2m-5} + \dots \right. \\ \left. \dots \pm \frac{2a^{m-2}X^2}{5} + \frac{2a^{m-1}X}{3} + \frac{2a^m}{1} \right) \frac{2}{b^{m+1}}$$

$$\int \frac{x^m dx}{X^{\frac{1}{2}}} = \left( \frac{X^m}{2m-1} - \frac{2aX^{m-1}}{2m-3} + \frac{2a^2X^{m-2}}{2m-5} - \frac{2a^3X^{m-3}}{2m-7} + \dots \right. \\ \left. \dots \pm \frac{2a^{m-2}X^2}{3} + \frac{2a^{m-1}X}{1} + \frac{2a^m}{-1} \right) \frac{2}{b^{m+1}\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{3}{2}}} = \left( \frac{X^m}{2m-3} - \frac{2aX^{m-1}}{2m-5} + \frac{2a^2X^{m-2}}{2m-7} - \frac{2a^3X^{m-3}}{2m-9} + \dots \right. \\ \left. \dots \pm \frac{2a^{m-2}X^2}{1} + \frac{2a^{m-1}X}{-1} + \frac{2a^m}{-3} \right) \frac{2}{b^{m+1}X\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{5}{2}}} = \left( \frac{X^m}{2m-5} - \frac{2aX^{m-1}}{2m-7} + \frac{2a^2X^{m-2}}{2m-9} - \frac{2a^3X^{m-3}}{2m-11} + \dots \right. \\ \left. \dots \pm \frac{2a^{m-2}X^2}{-1} + \frac{2a^{m-1}X}{-3} + \frac{2a^m}{-5} \right) \frac{2}{b^{m+1}X^2\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \left( \frac{X^m}{2m-n+2} - \frac{2aX^{m-1}}{2m-n} + \frac{2a^2X^{m-2}}{2m-n-2} - \dots \right. \\ \left. \dots \pm \frac{2a^{m-2}X^2}{-(n-6)} + \frac{2a^{m-1}X}{-(n-4)} + \frac{2a^m}{-(n-2)} \right) \frac{2}{b^{m+1}X^{\frac{n-2}{2}}}$$

$$\int x^m dx \sqrt{X} = \left( \frac{X^m}{2m+3} - \frac{2aX^{m-1}}{2m+1} + \frac{2a^2X^{m-2}}{2m-1} - \frac{2a^3X^{m-3}}{2m-3} + \dots \right. \\ \left. \dots \pm \frac{2a^{m-2}X^2}{7} + \frac{2a^{m-1}X}{5} + \frac{2a^m}{3} \right) \frac{2}{b^{m+1}}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a + bx = X$$

$$\int x^m dx X^{\frac{1}{2}} = \left( \frac{X^m}{2m+5} - \frac{2aX^{m-1}}{2m+3} + \frac{3a^2X^{m-2}}{2m+1} - \frac{4a^3X^{m-3}}{2m-1} + \dots \right. \\ \left. \dots \dots \dots \pm \frac{2^{\frac{m-1}{2}} a^{\frac{m-1}{2}} X^2}{9} + \frac{2^{\frac{m-1}{2}} a^{\frac{m-1}{2}} X}{7} \pm \frac{2^{\frac{m-1}{2}} a^{\frac{m-1}{2}}}{5} \right) \frac{2X^{\frac{1}{2}} \sqrt{X}}{b^{m+1}}$$

$$\int x^m dx X^{\frac{1}{3}} = \left( \frac{X^m}{2m+7} - \frac{2aX^{m-1}}{2m+5} + \frac{3a^2X^{m-2}}{2m+3} - \frac{4a^3X^{m-3}}{2m+1} + \dots \right. \\ \left. \dots \dots \dots \pm \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}} X^2}{11} + \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}} X}{9} \pm \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}}}{7} \right) \frac{2X^{\frac{1}{3}} \sqrt{X}}{b^{m+1}}$$

$$\int x^m dx X^{\frac{2}{3}} = \left( \frac{X^m}{2m+n+2} - \frac{2aX^{m-1}}{2m+n} + \frac{3a^2X^{m-2}}{2m+n-2} - \frac{4a^3X^{m-3}}{2m+n-4} + \dots \right. \\ \left. \dots \dots \dots \pm \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}} X^2}{n+6} + \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}} X}{n+4} \pm \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}}}{n+2} \right) \frac{2X^{\frac{2}{3}} \sqrt{X}}{b^{m+1}}$$

$$\int \frac{x^m dx}{X^{\frac{1}{2}}} = \left( \frac{X^m}{qm-p+q} - \frac{2aX^{m-1}}{qm-p} + \frac{3a^2X^{m-2}}{qm-p-q} - \frac{4a^3X^{m-3}}{qm-p-2q} \right. \\ \left. + \frac{5a^4X^{m-4}}{qm-p-3q} \dots \dots \dots \right. \\ \left. \dots \dots \dots \pm \frac{2^{\frac{m-1}{2}} a^{\frac{m-1}{2}} X^2}{-(p-3q)} + \frac{2^{\frac{m-1}{2}} a^{\frac{m-1}{2}} X}{-(p-2q)} + \frac{2^{\frac{m-1}{2}} a^{\frac{m-1}{2}}}{-(p-q)} \right) \frac{q}{b^{m+1} X^{\frac{1}{2}-1}}$$

$$\int x^m dx X^{\frac{1}{3}} = \left( \frac{X^m}{qm+p+q} - \frac{2aX^{m-1}}{qm+p} + \frac{3a^2X^{m-2}}{qm+p-q} - \frac{4a^3X^{m-3}}{qm+p-2q} \right. \\ \left. + \frac{5a^4X^{m-4}}{qm+p-3q} \dots \dots \dots \right. \\ \left. \dots \dots \dots \pm \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}} X^2}{p+3q} + \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}} X}{p+2q} + \frac{2^{\frac{m-1}{3}} a^{\frac{m-1}{3}}}{p+q} \right) \frac{qX^{\frac{1}{3}+1}}{b^{m+1}}$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x}{x^m \sqrt{X}} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \sqrt{X} + \frac{Lb}{2} \int \frac{\partial x}{x \sqrt{X}}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(2m-3)b}{(2m-4)a} A, \quad C = \frac{(2m-5)b}{(2m-6)a} B,$$

$$D = \frac{(2m-7)b}{(2m-8)a} C, \quad E = \frac{(2m-9)b}{(2m-10)a} D, \quad \dots \quad L = \frac{3b}{2a} K.$$

$$\int \frac{\partial x}{x^m X^{\frac{1}{2}}} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \frac{1}{\sqrt{X}} + \frac{3Lb}{2} \int \frac{\partial x}{x X^{\frac{1}{2}}}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(2m-1)b}{(2m-4)a} A, \quad C = \frac{(2m-3)b}{(2m-6)a} B,$$

$$D = \frac{(2m-5)b}{(2m-8)a} C, \quad E = \frac{(2m-7)b}{(2m-10)a} D, \quad \dots \quad L = \frac{5b}{2a} K.$$

$$\int \frac{\partial x}{x^m X^{\frac{3}{2}}} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \frac{1}{X \sqrt{X}} + \frac{5bL}{2} \int \frac{\partial x}{x X^{\frac{3}{2}}}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(2m+1)b}{(2m-4)a} A, \quad C = \frac{(2m-1)b}{(2m-6)a} B,$$

$$D = \frac{(2m-3)b}{(2m-8)a} C, \quad E = \frac{(2m-5)b}{(2m-10)a} D, \quad \dots \quad L = \frac{7b}{2a} K.$$

$$\int \frac{\partial x}{x^m X^{\frac{5}{2}}} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \frac{1}{X^{\frac{3}{2}}} + \frac{nbL}{2} \int \frac{\partial x}{x X^{\frac{5}{2}}}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VL. } a + bx = X$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(2m+n-4)b}{(2m-4)a} A, C = \frac{(2m+n-6)b}{(2m-6)a} B, \\ D = \frac{(2m+n-8)b}{(2m-8)a} C, E = \frac{(2m+n-10)b}{(2m-10)a} D, \dots L = \frac{(n+2)b}{2a} K.$$

$$\int \frac{\partial x}{x^m X^{\frac{1}{2}}} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \frac{1}{X^{\frac{1}{2}}} + \frac{pbL}{q} \int \frac{\partial x}{x X^{\frac{1}{2}}}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(qm+p-2q)b}{(m-2)qa} A, C = \frac{(qm+p-3q)b}{(m-3)qa} B, \\ D = \frac{(qm+p-4q)b}{(m-4)qa} C, E = \frac{(qm+p-5q)b}{(m-5)qa} D, \dots L = \frac{(p+q)b}{qa} K.$$

$$\int \frac{\partial x \sqrt{X}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) X \sqrt{X} \pm \frac{bL}{2} \int \frac{\partial x \sqrt{X}}{x}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(2m-5)b}{(2m-4)a} A, C = \frac{(2m-7)b}{(2m-6)a} B, \\ D = \frac{(2m-9)b}{(2m-8)a} C, E = \frac{(2m-11)b}{(2m-10)a} D, \dots L = \frac{b}{2a} K.$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) X^{\frac{3}{2}} \sqrt{X} \pm \frac{3bL}{2} \int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(2m-7)b}{(2m-4)a} A, C = \frac{(2m-9)b}{(2m-6)a} B, \\ D = \frac{(2m-11)b}{(2m-8)a} C, E = \frac{(2m-13)b}{(2m-10)a} D, \dots L = \frac{-b}{2a} K.$$



T a f e l  
einiger allgemeineren Formeln.

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$$\text{VL. } a + bx = X$$


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$$\int \frac{\partial x X^{\frac{1}{2}}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) X^{\frac{1}{2}} \sqrt{X} \pm \frac{5bL}{2} \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(2m-9)b}{(2m-4)a} A, C = \frac{(2m-11)b}{(2m-6)a} B,$$

$$D = \frac{(2m-13)b}{(2m-8)a} C, E = \frac{(2m-15)b}{(2m-10)a} D, \dots L = \frac{-3b}{2a} K.$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) X^{\frac{3}{2}} \pm \frac{nbL}{2} \int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(2m-n-4)b}{(2m-4)a} A, C = \frac{(2m-n-6)b}{(2m-6)a} B,$$

$$D = \frac{(2m-n-8)b}{(2m-8)a} C, \dots L = \frac{-(n-2)b}{2a} K.$$

$$\int \frac{\partial x X^{\frac{p}{q}}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) X^{\frac{p}{q}} \pm \frac{pbL}{q} \int \frac{\partial x X^{\frac{p}{q}}}{x}$$

$$A = -\frac{1}{(m-1)a}, B = \frac{(qm-p-2q)b}{(m-2)qa} A, C = \frac{(qm-p-3q)b}{(m-3)qa} B,$$

$$D = \frac{(qm-p-4q)b}{(m-4)qa} C, E = \frac{(qm-p-5q)b}{(m-5)qa} D, \dots L = \frac{(q-p)b}{qa} K.$$

## T a f e l

einiger allgemeineren Formeln.

$$VZ. \quad a + bx = X$$

$$\int \frac{dx X^{\frac{2n+1}{2}}}{x} = \left( \frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \dots \right. \\ \left. \dots \dots \dots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1} \right) 2\sqrt{X} + a^{n+1} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{p}{q}}}{x} = \frac{qX^{\frac{p}{q}}}{p} + \frac{qaX^{\frac{p}{q}-1}}{p-q} + \frac{qa^2X^{\frac{p}{q}-2}}{p-2q} + \frac{qa^3X^{\frac{p}{q}-3}}{p-3q} + \dots \\ \dots \dots \dots + \frac{qa^{i-1}X^{\frac{p}{q}-i+1}}{p-(i-1)q} + a^i \int \frac{dx X^{\frac{p}{q}-i}}{x}$$

$$\int \frac{dx}{x X^{\frac{2n+1}{2}}} = \left[ \frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^2X^{n-2}} + \frac{1}{(2n-5)a^3X^{n-3}} + \dots \right. \\ \left. \dots \dots \dots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^2} \right] \frac{2}{\sqrt{X}} + \frac{1}{a^2} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x X^{\frac{p}{q}}} = \frac{q}{(p-q)aX^{\frac{p}{q}-1}} + \frac{q}{(p-2q)a^2X^{\frac{p}{q}-2}} + \frac{q}{(p-3q)a^3X^{\frac{p}{q}-3}} + \dots \\ \dots \dots \dots + \frac{q}{(p-iq)a^iX^{\frac{p}{q}-i}} + \frac{1}{a^i} \int \frac{dx}{x X^{\frac{p}{q}-i}}$$

$$\int \frac{x^m dx \sqrt{x}}{X^n} = \frac{2x^m \sqrt{x}}{(2m-2n+3)bX^{n-1}} - \frac{(2m+1)a}{(2m-2n+3)b} \int \frac{x^{m-1} dx \sqrt{x}}{X^n}$$

$$\int \frac{x^m dx \sqrt{x}}{X^n} = \left( Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + \dots \right. \\ \left. \dots \dots \dots + Kx^2 + Lx \right) \frac{2\sqrt{x}}{X^{n-1}} + \frac{3aL}{2} \int \frac{dx \sqrt{x}}{X^n}$$

$$A = \frac{1}{(2m-2n+3)b}, \quad B = \frac{(2m+1)a}{(2m-2n+1)b}, \quad C = \frac{(2m-1)a}{(2m-2n-1)b}, \quad D = \frac{(2m-3)a}{(2m-2n-3)b}, \quad \dots \dots \dots L = \frac{5a}{(5-2n)b} K.$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VZ. } a + bx = X, ad - bc = k$$


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$$\int \frac{\partial x}{X^p V(c+dx)} = \frac{V(c+dx)}{(p-1)kX^{p-1}} + \frac{(2p-3)d}{(2p-2)k} \int \frac{\partial x}{X^{p-1} V(c+dx)}$$

$$\int \frac{\partial x}{X^p V(c+dx)} = \left( \frac{A}{X^{p-1}} + \frac{B}{X^{p-2}} + \frac{C}{X^{p-3}} + \frac{D}{X^{p-4}} + \frac{E}{X^{p-5}} + \dots \dots \dots \right. \\ \left. \dots \dots \dots + \frac{K}{X^2} + \frac{L}{X} \right) V(c+dx) + \frac{dL}{2} \int \frac{\partial x}{X V(c+dx)}$$

$$A = \frac{1}{(p-1)k}, B = \frac{(2p-3)d}{(2p-4)k} A, C = \frac{(2p-5)d}{(2p-6)k} B,$$

$$D = \frac{(2p-7)d}{(2p-8)k} C, E = \frac{(2p-9)d}{(2p-10)k} D, \dots \dots L = \frac{3d}{2k} K.$$


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$$\text{VZ. } a + bx^2 = X$$


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$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m+1} X^{\frac{n}{2}}}{m+1} - \frac{nb}{m+1} \int x^{m+2} dx X^{\frac{n}{2}-1}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = - \frac{x^{m+1}}{(n-2)b X^{\frac{n}{2}-1}} + \frac{m-1}{(n-2)b} \int \frac{x^{m-2} dx}{X^{\frac{n}{2}-1}}$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m+1} X^{\frac{n}{2}+1}}{(m+n+1)b} - \frac{(m-1)a}{(m+n+1)b} \int x^{m-2} dx X^{\frac{n}{2}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \frac{x^{m+1}}{(m-n+1)b X^{\frac{n}{2}-1}} - \frac{(m-1)a}{(m-n+1)b} \int \frac{x^{m-2} dx}{X^{\frac{n}{2}}}$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m+1} X^{\frac{n}{2}}}{m+n+1} + \frac{na}{m+n+1} \int x^m dx X^{\frac{n}{2}-1}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = - \frac{X^{\frac{n}{2}}}{(m-n-1)x^{m-1}} - \frac{na}{m-n-1} \int \frac{dx X^{\frac{n}{2}-1}}{x^m}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a + bx^2 = X$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = -\frac{X^{\frac{n}{2}+1}}{(m-1)ax^{m-1}} - \frac{(m-n-3)b}{(m-1)a} \int \frac{\partial x X^{\frac{n}{2}}}{x^{m-2}}$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = -\frac{1}{(m-1)ax^{m-1}X^{\frac{n}{2}-1}} - \frac{(m+n-3)b}{(m-1)a} \int \frac{\partial x}{x^{m-2}X^{\frac{n}{2}}}$$

$$\int \frac{x^m \partial x}{X^{\frac{n}{2}}} = \frac{x^{m+1}}{(n-2)aX^{\frac{n}{2}-1}} - \frac{m-n+3}{(n-2)a} \int \frac{x^m \partial x}{X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = \frac{1}{(n-2)ax^{m-1}X^{\frac{n}{2}-1}} + \frac{m+n-3}{(n-2)a} \int \frac{\partial x}{x^m X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \frac{x}{(n-2)aX^{\frac{n}{2}-1}} + \frac{n-3}{(n-2)a} \int \frac{\partial x}{X^{\frac{n}{2}-1}}$$

$$\int \partial x X^{\frac{n}{2}} = \frac{x X^{\frac{n}{2}}}{n+1} + \frac{na}{n+1} \int \partial x X^{\frac{n}{2}-1}$$

$$\int \frac{\partial x}{x X^{\frac{n}{2}}} = \frac{1}{(n-2)aX^{\frac{n}{2}-1}} + \frac{1}{a} \int \frac{\partial x}{x X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x} = \frac{X^{\frac{n}{2}}}{n} + a \int \frac{\partial x X^{\frac{n}{2}-1}}{x}$$

$$\int \frac{\partial x}{X^{\frac{2n+1}{2}}} = \left( \frac{A}{X^{n-1}} + \frac{B}{X^{n-3}} + \frac{C}{X^{n-5}} + \dots + \frac{K}{X} + L \right) \frac{x}{VX}$$

$$A = \frac{1}{(2n-1)a}, \quad B = \frac{2n-2}{(2n-3)a} A, \quad C = \frac{2n-4}{(2n-5)a} B,$$

$$D = \frac{2n-6}{(2n-7)a} C, \quad E = \frac{2n-8}{(2n-9)a} D, \quad \dots \dots L = \frac{2}{a} K.$$

$$\int \frac{x \partial x}{X^n} = -\frac{1}{(n-1)2bX^{n-1}}$$

**T a f e l**  
 einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$\int dx X^{\frac{2n+1}{2}} = \left( AX^n + BX^{n-1} + CX^{n-2} + DX^{n-3} + \dots \dots \dots + KX + L \right) \sqrt{X} + La \int \frac{dx}{\sqrt{X}}$$

$$A = \frac{1}{2n+2}, B = \frac{(2n+1)a}{2n} A, C = \frac{(2n-1)a}{2n-2} B,$$

$$D = \frac{(2n-3)a}{2n-4} C, E = \frac{(2n-5)a}{2n-6} D, \dots \dots L = \frac{3a}{2} K.$$

$$\int x dx X^n = \frac{X^{n+1}}{(n+1)2b}$$

$$\int \frac{dx X^{\frac{2n+1}{2}}}{x} = \left( \frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \dots \dots \dots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1} \right) \sqrt{X} + a^{n+1} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x X^{\frac{2n+1}{2}}} = \left[ \frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^2X^{n-2}} + \frac{1}{(2n-5)a^3X^{n-3}} + \dots \dots \dots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n} \right] \frac{1}{\sqrt{X}} + \frac{1}{a^n} \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{x^n dx}{X^{\frac{1}{2}}} = \left( Ax^{n-1} - Bx^{n-3} + Cx^{n-5} - Dx^{n-7} + Ex^{n-9} - \dots \dots \dots + Kx^{n-2i+3} + Lx^{n-2i+1} \right) X^{-\frac{1}{2}+1} + (m-2i+1)aL \int \frac{x^{n-2i} dx}{X^{\frac{1}{2}}}$$

$$A = \frac{1}{(m-n+1)b}, B = \frac{(m-1)a}{(m-n-1)b} A, C = \frac{(m-5)a}{(m-n-3)b} B,$$

$$D = \frac{(m-9)a}{(m-n-5)b} C, E = \frac{(m-13)a}{(m-n-7)b} D, \dots \dots L = \frac{(m-2i+5)a}{(m-n-2i+3)b} K.$$

$$\int \frac{x^{2n+1} dx}{\sqrt{X}} = \left( Ax^{2n} - Bx^{2n-2} + Cx^{2n-4} - Dx^{2n-6} + \dots \dots \dots + Kx^4 + Lx^2 \right) \sqrt{X} + 2aL \int \frac{x dx}{\sqrt{X}}$$

T a f e l  
einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$A = \frac{1}{2m+1}, \quad B = \frac{2ma}{(2m-1)b} A, \quad C = \frac{(2m-2)a}{(2m-3)b} B,$$

$$D = \frac{(2m-4)a}{(2m-5)b} C, \quad E = \frac{(2m-6)a}{(2m-7)b} D, \quad \dots \quad L = \frac{4a}{3b} K.$$

$$\int \frac{x^{2m} dx}{\sqrt{X}} = \left( Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \dots \right. \\ \left. \dots \dots \dots \pm Kx^3 \mp Lx \right) \sqrt{X} \pm aL \int \frac{dx}{\sqrt{X}}$$

$$A = \frac{1}{2mb}, \quad B = \frac{(2m-1)a}{(2m-2)b} A, \quad C = \frac{(2m-3)a}{(2m-4)b} B,$$

$$D = \frac{(2m-5)a}{(2m-6)b} C, \quad E = \frac{(2m-7)a}{(2m-8)b} D, \quad \dots \quad L = \frac{3a}{2b} K.$$

$$\int \frac{x^{2m+1} dx}{X^{\frac{1}{2}}} = \left( Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \dots \right. \\ \left. \dots \dots \dots \pm Kx^4 \mp Lx^2 \right) \frac{1}{\sqrt{X}} \pm 2aL \int \frac{x dx}{X^{\frac{1}{2}}}$$

$$A = \frac{1}{(2m-1)b}, \quad B = \frac{2ma}{(2m-3)b} A, \quad C = \frac{(2m-2)a}{(2m-5)b} B,$$

$$D = \frac{(2m-4)a}{(2m-7)b} C, \quad E = \frac{(2m-6)a}{(2m-9)b} D, \quad \dots \quad L = \frac{4a}{b} K.$$

$$\int \frac{x^{2m} dx}{X^{\frac{1}{2}}} = \left( Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \dots \right. \\ \left. \dots \dots \dots \pm Kx^5 \mp Lx^3 \right) \frac{1}{\sqrt{X}} \pm 3aL \int \frac{x^2 dx}{X^{\frac{1}{2}}}$$

$$A = \frac{1}{(2m-2)b}, \quad B = \frac{(2m-1)a}{(2m-4)b} A, \quad C = \frac{(2m-3)a}{(2m-6)b} B,$$

$$D = \frac{(2m-5)a}{(2m-8)b} C, \quad E = \frac{(2m-7)a}{(2m-10)b} D, \quad \dots \quad L = \frac{5a}{2b} K.$$

$$\int \frac{dx}{x^n X^{\frac{1}{2}}} = \left( \frac{A}{x^{n-1}} - \frac{B}{x^{n-3}} + \frac{C}{x^{n-5}} - \frac{D}{x^{n-7}} + \frac{E}{x^{n-9}} - \dots \right. \\ \left. \dots \dots \dots \pm \frac{K}{x^{n-2i+3}} \mp \frac{L}{x^{n-2i+1}} \right) X^{-\frac{n}{2}+1} \mp (m+n-2i-1)bL \int \frac{dx}{x^{n-2i} X^{\frac{1}{2}}}$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VZ. } a + bx^2 = X$$


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$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+n-3)b}{(m-3)a} A, \quad C = \frac{(m+n-5)b}{(m-5)a} B, \\ D = \frac{(m+n-7)b}{(m-7)a} C, \quad E = \frac{(m+n-9)b}{(m-9)a} D, \quad \dots L = \frac{(m+n-2i+1)b}{(m-2i+1)a} K.$$

$$\int \frac{\partial x}{x^{2m+1} \sqrt{X}} = \left( \frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^4} \mp \frac{L}{x^2} \right) \sqrt{X} \mp bL \int \frac{\partial x}{x \sqrt{X}}$$

$$A = -\frac{1}{2ma}, \quad B = \frac{(2m-1)b}{(2m-2)a} A, \quad C = \frac{(2m-3)b}{(2m-4)a} B, \\ D = \frac{(2m-5)b}{(2m-6)a} C, \quad E = \frac{(2m-7)b}{(2m-8)a} D, \quad \dots L = \frac{3b}{2a} K.$$

$$\int \frac{\partial x}{x^{2m} \sqrt{X}} = \left( \frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^3} \mp \frac{L}{x} \right) \sqrt{X}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{(2m-2)b}{(2m-3)a} A, \quad C = \frac{(2m-4)b}{(2m-5)a} B, \\ D = \frac{(2m-6)b}{(2m-7)a} C, \quad E = \frac{(2m-8)b}{(2m-9)a} D, \quad \dots L = \frac{2b}{a} K.$$

$$\int \frac{\partial x}{x^{2m+1} X^{\frac{1}{2}}} = \left( \frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^4} \mp \frac{L}{x^2} \right) \frac{1}{\sqrt{X}} \mp 5bL \int \frac{\partial x}{x X^{\frac{1}{2}}}$$

$$A = -\frac{1}{2ma}, \quad B = \frac{(2m+1)b}{(2m-2)a} A, \quad C = \frac{(2m-1)b}{(2m-4)a} B, \\ D = \frac{(2m-3)b}{(2m-6)a} C, \quad E = \frac{(2m-5)b}{(2m-8)a} D, \quad \dots L = \frac{5b}{2a} K.$$

T a f e l  
einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$\int \frac{\partial x}{x^{2m} X^{\frac{1}{2}}} = \left( \frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^3} + \frac{L}{x} \right) \frac{1}{\sqrt{X}} + 2bL \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{2mb}{(2m-3)a}, \quad C = \frac{(2m-2)b}{(2m-5)a},$$

$$D = \frac{(2m-4)b}{(2m-7)a}, \quad E = \frac{(2m-6)b}{(2m-9)a}, \quad \dots \quad L = \frac{4b}{a} K.$$

$$\int x^m \partial x X^{\frac{n}{2}} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \dots \\ \dots \pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) X^{\frac{n}{2}+1} \pm (m-2i+1)aL \int x^{m-2i} \partial x X^{\frac{n}{2}}$$

$$A = \frac{1}{(m+n+1)b}, \quad B = \frac{(m-1)a}{(m+n-1)b}, \quad C = \frac{(m-3)a}{(m+n-3)b},$$

$$D = \frac{(m-5)a}{(m+n-5)b}, \quad E = \frac{(m-7)a}{(m+n-7)b}, \quad \dots \quad L = \frac{(m-2i+3)a}{(m+n-2i+3)b} K.$$

$$\int x^{2m+1} \partial x \sqrt{X} = (Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \dots \\ \dots \pm Kx^4 \mp Lx^2) \sqrt{X} \pm 2aL \int x \partial x \sqrt{X}$$

$$A = \frac{1}{(2m+3)b}, \quad B = \frac{2ma}{(2m+1)b}, \quad C = \frac{(2m-2)a}{(2m-1)b},$$

$$D = \frac{(2m-4)a}{(2m-3)b}, \quad E = \frac{(2m-6)a}{(2m-5)b}, \quad \dots \quad L = \frac{4a}{5b} K.$$

$$\int x^{2m} \partial x \sqrt{X} = (Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \dots \\ \dots \pm Kx^3 \mp Lx) \sqrt{X} \pm aL \int \partial x \sqrt{X}$$

$$A = \frac{1}{(2m+2)b}, \quad B = \frac{(2m-1)a}{2mb}, \quad C = \frac{(2m-3)a}{(2m-2)b},$$

$$D = \frac{(2m-5)a}{(2m-4)b}, \quad E = \frac{(2m-7)a}{(2m-6)b}, \quad \dots \quad L = \frac{3a}{4b} K.$$



## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$\int x^{2m+1} X^{\frac{1}{2}} = (Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \dots \\ \dots \pm Kx^4 \mp Lx^2) X^{\frac{1}{2}} \sqrt{X} \pm 2aL \int x dx X^{\frac{1}{2}}$$

$$A = \frac{1}{(2m+5)a}, \quad B = \frac{2ma}{(2m+3)b}, \quad C = \frac{(2m-2)a}{(2m+1)b} B, \\ D = \frac{(2m-4)a}{(2m-1)b} C, \quad E = \frac{(2m-6)a}{(2m-3)b} D, \quad \dots \quad L = \frac{4a}{7b} K.$$

$$\int x^{2m} X^{\frac{1}{2}} = (Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \dots \\ \dots \pm Kx^3 \mp Lx) X^{\frac{1}{2}} \sqrt{X} \pm aL \int dx X^{\frac{1}{2}}$$

$$A = \frac{1}{(2m+4)b}, \quad B = \frac{(2m-1)a}{(2m+2)b} A, \quad C = \frac{(2m-3)a}{2mb} B, \\ D = \frac{(2m-5)a}{(2m-2)b} C, \quad E = \frac{(2m-7)a}{(2m-4)b} D, \quad \dots \quad L = \frac{3a}{6b} K.$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^{m-2i+5}} \mp \frac{L}{x^{m-2i+1}} \right) X^{\frac{n}{2}+1} \mp (m-n-2i-1)bL \int \frac{\partial x X^{\frac{n}{2}}}{x^{m-2i}}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m-n-3)b}{(m-3)a} A, \quad C = \frac{(m-n-5)b}{(m-5)a} B, \\ D = \frac{(m-n-7)b}{(m-7)a} C, \quad E = \frac{(m-n-9)b}{(m-9)a} D, \quad \dots \quad L = \frac{(m-n-2i+1)b}{(m-2i+1)a} K.$$

$$\int \frac{\partial x \sqrt{X}}{x^{2m+1}} = \left( \frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \dots \right. \\ \left. \dots \pm \frac{K}{x^4} \mp \frac{L}{x^2} \right) X \sqrt{X} \pm bL \int \frac{\partial x \sqrt{X}}{x}$$

T a f e l  
einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$A = -\frac{1}{2ma}, \quad B = \frac{(2m-3)b}{(2m-2)a}A, \quad C = \frac{(2m-5)b}{(2m-4)a}B,$$

$$D = \frac{(2m-7)b}{(2m-6)a}C, \quad E = \frac{(2m-9)b}{(2m-8)a}D, \quad \dots\dots L = \frac{b}{2a}K.$$

$$\int \frac{\partial x \sqrt{X}}{x^{2m}} = \left( \frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots\dots \right. \\ \left. \dots\dots \pm \frac{K}{x} \mp \frac{L}{x^3} \right) X \sqrt{X}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{(2m-4)b}{(2m-3)a}A, \quad C = \frac{(2m-6)b}{(2m-5)a}B,$$

$$D = \frac{(2m-8)b}{(2m-7)a}C, \quad E = \frac{(2m-10)b}{(2m-9)a}D, \quad \dots\dots L = \frac{2b}{3a}K.$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{2m+1}} = \left( \frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \dots\dots \right. \\ \left. \dots\dots \pm \frac{K}{x^4} \mp \frac{L}{x^2} \right) X^{\frac{1}{2}} \sqrt{X} \pm 3bL \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$A = -\frac{1}{2ma}, \quad B = \frac{(2m-5)b}{(2m-2)a}A, \quad C = \frac{(2m-7)b}{(2m-4)a}B,$$

$$D = \frac{(2m-9)b}{(2m-6)a}C, \quad E = \frac{(2m-11)b}{(2m-8)a}D, \quad \dots\dots L = \frac{-b}{2a}K.$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{2m}} = \left( \frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots\dots \right. \\ \left. \dots\dots \pm \frac{K}{x^3} \mp \frac{L}{x} \right) X^{\frac{3}{2}} \sqrt{X} \pm 4bL \int \partial x X^{\frac{1}{2}}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{(2m-6)b}{(2m-3)a}A, \quad C = \frac{(2m-8)b}{(2m-5)a}B,$$

$$D = \frac{(2m-10)b}{(2m-7)a}C, \quad E = \frac{(2m-12)b}{(2m-9)a}D, \quad \dots\dots L = \frac{-2b}{a}K.$$

## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } a + bx^2 = X$$


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$$\int x^{2m+1} X^{\frac{1}{2}} = (Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \dots \\ \dots \pm Kx^4 \mp Lx^2) X^{\frac{1}{2}} \sqrt{X} \pm 2aL \int x dx X^{\frac{1}{2}}$$

$$A = \frac{1}{(2m+5)a}, \quad B = \frac{2ma}{(2m+3)b} A, \quad C = \frac{(2m-2)a}{(2m+1)b} B,$$

$$D = \frac{(2m-4)a}{(2m-1)b} C, \quad E = \frac{(2m-6)a}{(2m-3)b} D, \quad \dots \dots L = \frac{4a}{7b} K.$$

$$\int x^{2m} X^{\frac{1}{2}} = (Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \dots \\ \dots \pm Kx^3 \mp Lx) X^{\frac{1}{2}} \sqrt{X} \pm aL \int dx X^{\frac{1}{2}}$$

$$A = \frac{1}{(2m+4)b}, \quad B = \frac{(2m-1)a}{(2m+2)b} A, \quad C = \frac{(2m-3)a}{2mb} B,$$

$$D = \frac{(2m-5)a}{(2m-2)b} C, \quad E = \frac{(2m-7)a}{(2m-4)b} D, \quad \dots \dots L = \frac{3a}{6b} K.$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = \left( \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \dots \dots \right. \\ \left. \dots \dots \pm \frac{K}{x^{m-2i+5}} \mp \frac{L}{x^{m-2i+1}} \right) X^{\frac{n}{2}+1} \mp (m-n-2i-1)bL \int \frac{dx X^{\frac{n}{2}}}{x^{m-2i}}$$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m-n-3)b}{(m-3)a} A, \quad C = \frac{(m-n-5)b}{(m-5)a} B,$$

$$D = \frac{(m-n-7)b}{(m-7)a} C, \quad E = \frac{(m-n-9)b}{(m-9)a} D, \quad \dots \dots L = \frac{(m-n-2i+1)b}{(m-2i+1)a} K.$$

$$\int \frac{dx \sqrt{X}}{x^{2m+1}} = \left( \frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \dots \dots \right. \\ \left. \dots \dots \pm \frac{K}{x^4} \mp \frac{L}{x^2} \right) X \sqrt{X} \pm bL \int \frac{dx \sqrt{X}}{x}$$

**T a f e l**  
 einiger allgemeineren Formeln.

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$$\text{VZ. } a + bx^2 = X$$


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$$A = -\frac{1}{2ma}, \quad B = \frac{(2m-3)b}{(2m-2)a}A, \quad C = \frac{(2m-5)b}{(2m-4)a}B,$$

$$D = \frac{(2m-7)b}{(2m-6)a}C, \quad E = \frac{(2m-9)b}{(2m-8)a}D, \quad \dots L = \frac{b}{2a}K.$$

$$\int \frac{\partial x \sqrt{X}}{x^{2m}} = \left( \frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots \right. \\
 \left. \dots \pm \frac{K}{x^3} \mp \frac{L}{x} \right) X \sqrt{X}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{(2m-4)b}{(2m-3)a}A, \quad C = \frac{(2m-6)b}{(2m-5)a}B,$$

$$D = \frac{(2m-8)b}{(2m-7)a}C, \quad E = \frac{(2m-10)b}{(2m-9)a}D, \quad \dots L = \frac{2b}{3a}K.$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^{2m+1}} = \left( \frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \dots \right. \\
 \left. \dots \pm \frac{K}{x^4} \mp \frac{L}{x^2} \right) X^{\frac{1}{2}} \sqrt{X} \pm 3bL \int \frac{\partial x X^{\frac{1}{2}}}{x}$$

$$A = -\frac{1}{2ma}, \quad B = \frac{(2m-5)b}{(2m-2)a}A, \quad C = \frac{(2m-7)b}{(2m-4)a}B,$$

$$D = \frac{(2m-9)b}{(2m-6)a}C, \quad E = \frac{(2m-11)b}{(2m-8)a}D, \quad \dots L = \frac{-b}{2a}K.$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{2m}} = \left( \frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots \right. \\
 \left. \dots \pm \frac{K}{x^3} \mp \frac{L}{x} \right) X^{\frac{3}{2}} \sqrt{X} \pm 4bL \int \partial x X^{\frac{1}{2}}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{(2m-6)b}{(2m-3)a}A, \quad C = \frac{(2m-8)b}{(2m-5)a}B,$$

$$D = \frac{(2m-10)b}{(2m-7)a}C, \quad E = \frac{(2m-12)b}{(2m-9)a}D, \quad \dots L = \frac{-2b}{a}K.$$

## T a f e l

einiger allgemeineren Formeln.

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 VZ.  $ax + bx^2 = X$ 


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$$\int x^m dx X^{\frac{n}{2}} = \frac{2x^{m+1} X^{\frac{n}{2}}}{2m+n+2} - \frac{nb}{2m+n+2} \int x^{m+1} dx X^{\frac{n}{2}-1}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = -\frac{2x^{m-1}}{(n-2)b X^{\frac{n}{2}-1}} + \frac{2m-n}{(n-2)b} \int \frac{x^{m-2} dx}{X^{\frac{n}{2}-1}}$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m+1} X^{\frac{n}{2}-1}}{(m+n+1)b} - \frac{(2m+n)a}{(m+n+1)2b} \int x^{m-1} dx X^{\frac{n}{2}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \frac{x^{m-1}}{(m-n+1)b X^{\frac{n}{2}}} - \frac{(2m-n)a}{(m-n+1)2b} \int \frac{x^{m-2} dx}{X^{\frac{n}{2}}}$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m+1} X^{\frac{n}{2}}}{m+n+1} + \frac{na}{2(m+n+1)} \int x^{m+1} dx X^{\frac{n}{2}-1}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = -\frac{X^{\frac{n}{2}}}{(m-n-1)x^{m-1}} - \frac{na}{2(m-n-1)} \int \frac{dx X^{\frac{n}{2}-1}}{x^{m-1}}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = -\frac{2X^{\frac{n}{2}+1}}{(2m-n-2)ax^m} - \frac{(m-n-2)2b}{(2m-n-2)a} \int \frac{dx X^{\frac{n}{2}}}{x^{m-1}}$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = -\frac{2}{(2m+n-2)ax^m X^{\frac{n}{2}-1}} - \frac{(m+n-2)2b}{(2m+n-2)a} \int \frac{dx}{x^{m-1} X^{\frac{n}{2}}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \frac{2x^m}{(n-2)a X^{\frac{n}{2}-1}} - \frac{2(m-n+2)}{(n-2)a} \int \frac{x^{m-1} dx}{X^{\frac{n}{2}-1}}$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = \frac{2}{(n-2)ax^m X^{\frac{n}{2}-1}} + \frac{2(m+n-2)}{(n-2)a} \int \frac{dx}{x^{m+1} X^{\frac{n}{2}-1}}$$

$$\int \frac{dx}{X^{\frac{n}{2}}} = -\frac{2(2bx+a)}{(n-2)a^2 X^{\frac{n}{2}-1}} - \frac{(n-3)4b}{(n-2)a^2} \int \frac{dx}{X^{\frac{n}{2}-1}}$$

$$\int dx X^{\frac{n}{2}} = \frac{(2bx+a)X^{\frac{n}{2}}}{(n+1)2b} - \frac{na^2}{(n+1)4b} \int dx X^{\frac{n}{2}-1}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VL. } ax + bx^2 = X$$

$$\int x^m dx X^{\frac{n}{2}} = (Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots \dots \dots \pm Kx^{m-i+1} \mp Lx^{m-i}) X^{\frac{n}{2}+1} \pm (m + \frac{n}{2} - i + 1)aL \int x^{m-i} dx X^{\frac{n}{2}}$$

$$A = \frac{1}{(m+n+1)b}, B = \frac{(2m+n)a}{(m+n)2b} A, C = \frac{(2m+n-2)a}{(m+n-1)2b} B,$$

$$D = \frac{(2m+n-4)a}{(m+n-2)2b} C, E = \frac{(2m+n-6)a}{(m+n-3)2b} D, \dots \dots \dots$$

$$\dots \dots \dots L = \frac{(2m+n-2i+4)a}{(m+n-i+2)2b} K.$$

$$\int x^m dx \sqrt{X} = (Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots \dots \dots \pm Kx \mp L) \sqrt{X} \pm \frac{3aL}{2} \int dx \sqrt{X}$$

$$A = \frac{1}{(m+2)b}, B = \frac{(2m+1)a}{(m+1)2b} A, C = \frac{(2m-1)a}{2mb} B,$$

$$D = \frac{(2m-3)a}{(m-1)2b} C, E = \frac{(2m-5)a}{(m-2)2b} D, \dots \dots \dots L = \frac{5a}{6b} K.$$

$$\int x^m dx X^{\frac{3}{2}} = (Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots \dots \dots \pm Kx \mp L) X^{\frac{3}{2}} \pm \frac{5aL}{2} \int dx X^{\frac{3}{2}}$$

$$A = \frac{1}{(m+4)b}, B = \frac{(2m+3)a}{(m+3)2b} A, C = \frac{(2m+1)a}{(m+2)2b} B,$$

$$D = \frac{(2m-1)a}{(m+1)2b} C, E = \frac{(2m-3)a}{2mb} D, \dots \dots \dots L = \frac{7a}{10b} K.$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = \left( \frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots \dots \dots \right.$$

$$\left. \dots \dots \dots \pm \frac{K}{x^{m-i+2}} \mp \frac{L}{x^{m-i+1}} \right) X^{\frac{n}{2}+1} \mp (m-n-i-1)bL \int \frac{dx X^{\frac{n}{2}}}{x^{m-i}}$$

**T a f e l**  
 einiger allgemeineren Formeln.

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$$\text{VL. } ax + bx^2 = X$$


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$$A = -\frac{2}{(2m-n-2)a}, B = \frac{(m-n-2)2b}{(2m-n-4)a}, C = \frac{(m-n-3)2b}{(2m-n-6)a},$$

$$D = \frac{(m-n-4)2b}{(2m-n-8)a}, E = \frac{(m-n-5)2b}{(2m-n-10)a}, \dots\dots\dots$$

$$\dots\dots\dots L = \frac{(m-n-i)2b}{(2m-n-2i)a} K.$$

$$\int \frac{\partial x \sqrt{X}}{x^m} = \left( \frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots\dots\dots \right. \\ \left. \dots\dots\dots \pm \frac{K}{x^5} + \frac{L}{x^4} \right) X \sqrt{X} + bL \int \frac{\partial x \sqrt{X}}{x^3}$$

$$A = -\frac{2}{(2m-3)a}, B = \frac{(m-3)2b}{(2m-5)a}, C = \frac{(m-4)2b}{(2m-7)a},$$

$$D = \frac{(m-5)2b}{(2m-9)a}, E = \frac{(m-6)2b}{(2m-11)a}, \dots\dots\dots L = \frac{4b}{5a} K.$$

$$\int \frac{\partial x X^{\frac{1}{2}}}{x^m} = \left( \frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots\dots\dots \right. \\ \left. \dots\dots\dots \pm \frac{K}{x^7} + \frac{L}{x^6} \right) X^2 \sqrt{X} + bL \int \frac{\partial x X^{\frac{1}{2}}}{x^5}$$

$$A = -\frac{2}{(2m-5)a}, B = \frac{(m-5)2b}{(2m-7)a}, C = \frac{(m-6)2b}{(2m-9)a},$$

$$D = \frac{(m-7)2b}{(2m-11)a}, E = \frac{(m-8)2b}{(2m-13)a}, \dots\dots\dots L = \frac{4b}{7a} K.$$

$$\int \frac{x^m \partial x}{X^{\frac{3}{2}}} = \left( Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots\dots\dots \right. \\ \left. \dots\dots\dots \pm Kx^{m-i+1} + Lx^{m-i} \right) X^{-\frac{m}{2}+1} \pm (m-\frac{n}{2}-i+1)aL \int \frac{x^{m-i} \partial x}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(m-n+1)b}, B = \frac{(2m-n)a}{(m-n)2b}, C = \frac{(2m-n-2)a}{(m-n-1)2b},$$

$$D = \frac{(2m-n-4)a}{(m-n-2)2b}, E = \frac{(2m-n-6)a}{(m-n-3)2b}, \dots\dots\dots$$

$$\dots\dots\dots L = \frac{(2m-n-2i+4)a}{(m-n-i+2)2b} K.$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VL. } ax + bx^2 = X$$

$$\int \frac{x^m dx}{\sqrt{X}} = (Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots \dots \dots \pm Kx \mp L) \sqrt{X} \pm \frac{aL}{2} \int \frac{dx}{\sqrt{X}}$$

$$A = \frac{1}{mb}, B = \frac{(2m-1)a}{(m-1)2b} A, C = \frac{(2m-3)a}{(m-2)2b} B,$$

$$D = \frac{(2m-5)a}{(m-3)2b} C, E = \frac{(2m-7)a}{(m-4)2b} D, \dots \dots L = \frac{3a}{2b} K.$$

$$\int \frac{x^m dx}{X^{\frac{1}{2}}} = (Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots \dots \dots \pm Kx^2 \mp Lx^2) \frac{1}{\sqrt{X}} \pm \frac{3aL}{2} \int \frac{x^2 dx}{X^{\frac{1}{2}}}$$

$$A = \frac{1}{(m-2)b}, B = \frac{(2m-3)a}{(m-3)2b} A, C = \frac{(2m-5)a}{(m-4)2b} B,$$

$$D = \frac{(2m-7)a}{(m-5)2b} C, E = \frac{(2m-9)a}{(m-6)2b} D, \dots \dots L = \frac{5a}{2b} K.$$

$$\int \frac{dx}{x^m X^{\frac{1}{2}}} = \left( \frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots \dots \dots \pm \frac{K}{x^{m-i+2}} \mp \frac{L}{x^{m-i+1}} \right) X^{-\frac{1}{2}+1} \mp (m+n-i-1)bL \int \frac{dx}{x^{m-i} X^{\frac{1}{2}}}$$

$$A = -\frac{2}{(2m+n-2)a}, B = \frac{(m+n-2)2b}{(2m+n-4)a} A, C = \frac{(m+n-3)2b}{(2m+n-6)a} B,$$

$$D = \frac{(m+n-4)2b}{(2m+n-8)a} C, E = \frac{(m+n-5)2b}{(2m+n-10)a} D, \dots \dots \dots$$

$$\dots \dots L = \frac{(m+n-i)2b}{(2m+n-2i)a} K.$$



## T a f e l

einiger allgemeineren Formeln.

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$$\text{VL. } ax + bx^2 = X$$


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$$\int \frac{\partial x}{x^m \sqrt{X}} = \left( \frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots \dots \dots \right. \\ \left. \dots \dots \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \sqrt{X}$$

$$A = -\frac{2}{(2m-1)a}, \quad B = \frac{(m-1)2b}{(2m-3)a} A, \quad C = \frac{(m-2)2b}{(2m-5)a} B,$$

$$D = \frac{(m-3)2b}{(2m-7)a} C, \quad E = \frac{(m-4)2b}{(2m-9)a} D, \quad \dots \dots L = \frac{2b}{a} K.$$

$$\int \frac{\partial x}{x^m X^{\frac{1}{2}}} = \left( \frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots \dots \dots \right. \\ \left. \dots \dots \dots \pm \frac{K}{x^2} + \frac{L}{x} \right) \frac{1}{\sqrt{X}} + 2bL \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$A = -\frac{2}{(2m+1)a}, \quad B = \frac{(m+1)2b}{(2m-1)a} A, \quad C = \frac{2mb}{(2m-3)a} B,$$

$$D = \frac{(m-1)2b}{(2m-5)a} C, \quad E = \frac{(m-2)2b}{(2m-7)a} D, \quad \dots \dots L = \frac{6b}{3a} K.$$

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$$\text{VL. } ax + bx^2 = X, \quad 2bx + a = U$$


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$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left( \frac{A}{X^{\frac{n-5}{2}}} - \frac{B}{X^{\frac{n-3}{2}}} + \frac{C}{X^{\frac{n-1}{2}}} - \frac{D}{X^{\frac{n-2}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} - \dots \dots \dots \right. \\ \left. \dots \dots \dots \pm \frac{K}{X^{\frac{n-2i+1}{2}}} + \frac{L}{X^{\frac{n-2i-1}{2}}} \right) \frac{2U}{\sqrt{X}} + (n-2i-1)4bL \int \frac{\partial x}{X^{\frac{n}{2}-i}}$$

$$A = -\frac{1}{(n-2)a^2}, \quad B = \frac{(n-3)4b}{(n-4)a^2} A, \quad C = \frac{(n-5)4b}{(n-6)a^2} B,$$

$$D = \frac{(n-7)4b}{(n-8)a^2} C, \quad E = \frac{(n-9)4b}{(n-10)a^2} D, \quad \dots \dots L = \frac{(n-2i+1)4b}{(n-2i)a^2} K.$$

T a f e l  
einiger allgemeineren Formeln.

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VZ.  $ax + bx^2 = X$ ,  $2bx + a = U$

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$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left( \frac{A}{X^{\frac{n-3}{2}}} - \frac{B}{X^{\frac{n-5}{2}}} + \frac{C}{X^{\frac{n-7}{2}}} - \frac{D}{X^{\frac{n-9}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} - \dots \right. \\ \left. \dots \pm \frac{K}{X^2} + \frac{L}{X} \right) \frac{2U}{\sqrt{X}} + 8bL \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$A = -\frac{1}{(n-2)a^2}, \quad B = \frac{(n-3)4b}{(n-4)a^2} A, \quad C = \frac{(n-5)4b}{(n-6)a^2} B, \\ D = \frac{(n-7)4b}{(n-8)a^2} C, \quad E = \frac{(n-9)4b}{(n-10)a^2} D, \quad \dots \dots L = \frac{4 \cdot 4b}{3a^2} K.$$


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VZ.  $a + bx + cx^2 = X$

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$$\int x^m \partial x X^p = \frac{x^{m+1} X^p}{m+1} - \frac{pb}{m+1} \int x^{m+1} \partial x X^{p-1} - \frac{2pc}{m+1} \int x^{m+2} \partial x X^{p-1}$$

$$\int x^m \partial x X^p = \frac{x^{m+1} X^{p+1}}{(m+2p+1)c} - \frac{(m-1)a}{(m+2p+1)c} \int x^{m-2} \partial x X^p \\ - \frac{(m+p)b}{(m+2p+1)c} \int x^{m-1} \partial x X^p$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m-1}}{(m-2p+1)c X^{p-1}} - \frac{(m-1)a}{(m-2p+1)c} \int \frac{x^{m-2} \partial x}{X^p} \\ - \frac{(m-p)b}{(m-2p+1)c} \int \frac{x^{m-1} \partial x}{X^p}$$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^p}{(m-1)x^{m-1}} + \frac{pb}{m-1} \int \frac{\partial x X^{p-1}}{x^{m-1}} + \frac{2pc}{m-1} \int \frac{\partial x X^{p-1}}{x^{m-2}}$$

$$\int x^m \partial x X^p = \frac{x^{m+1} X^p}{m+2p+1} + \frac{2pa}{m+2p+1} \int x^m \partial x X^{p-1} \\ + \frac{pb}{m+2p+1} \int x^{m+1} \partial x X^{p-1}$$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^p}{(m-2p-1)x^{m-1}} - \frac{2pa}{m-2p-1} \int \frac{\partial x X^{p-1}}{x^m} \\ - \frac{pb}{m-2p-1} \int \frac{\partial x X^{p-1}}{x^{m-1}}$$

## T a f e l

einiger allgemeineren Formeln.

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 VZ.  $a + bx + cx^2 = X$ ,  $4ac - b^2 = k$ 


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$$\int \frac{\partial x X^p}{x^m} = - \frac{X^{p+1}}{(m-1)ax^{m-1}} - \frac{(m+p-2)b}{(m-1)a} \int \frac{\partial x X^p}{x^{m-1}} \\ - \frac{(m+2p-3)c}{(m-1)a} \int \frac{\partial x X^p}{x^{m-2}}$$

$$\int \frac{\partial x}{x^m X^p} = - \frac{1}{(m-1)ax^{m-1}X^{p-1}} - \frac{(m+p-2)b}{(m-1)a} \int \frac{\partial x}{x^{m-1}X^p} \\ - \frac{(m+2p-3)c}{(m-1)a} \int \frac{\partial x}{x^{m-2}X^p}$$

$$\int \frac{\partial x}{X^p} = \frac{2cx+b}{(p-1)kX^{p-1}} + \frac{(2p-3)2c}{(p-1)k} \int \frac{\partial x}{X^{p-1}}$$

$$\int \partial x X^p = \frac{(2cx+b)X^p}{(2p+1)2c} + \frac{pk}{(2p+1)2c} \int \partial x X^{p-1}$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left( \frac{A}{X^{\frac{n-5}{2}}} + \frac{B}{X^{\frac{n-7}{2}}} + \frac{C}{X^{\frac{n-9}{2}}} + \frac{D}{X^{\frac{n-11}{2}}} + \dots \right) \\ + \frac{K}{X^{\frac{n-2i+1}{2}}} + \frac{L}{X^{\frac{n-2i-1}{2}}} \Bigg) \frac{2(2cx+b)}{VX} \\ + (n-2i-1)4cL \int \frac{\partial x}{X^{\frac{n}{2}-1}}$$

$$A = \frac{1}{(n-2)k}, \quad B = \frac{(n-3)4c}{(n-4)k} A, \quad C = \frac{(n-5)4c}{(n-6)k} B,$$

$$D = \frac{(n-7)4c}{(n-8)k} C, \quad E = \frac{(n-9)4c}{(n-10)k} D, \quad \dots \quad L = \frac{(n-2i+1)4c}{(n-2i)k} K.$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left( \frac{A}{X^{\frac{n-5}{2}}} + \frac{B}{X^{\frac{n-7}{2}}} + \frac{C}{X^{\frac{n-9}{2}}} + \frac{D}{X^{\frac{n-11}{2}}} + \dots \right) \\ + \frac{K}{X^{\frac{n}{2}}} + \frac{L}{X} \Bigg) \frac{2(2cx+b)}{VX} + 8cL \int \frac{\partial x}{X^{\frac{n}{2}}}$$

## T a f e l

einiger allgemeineren Formeln.

$$\text{VZ. } a+bx+cx^2=X, 4ac-b^2=k$$

$$A=\frac{1}{(n-2)k}, B=\frac{(n-3)4c}{(n-4)k}A, C=\frac{(n-5)4c}{(n-6)k}B, \\ D=\frac{(n-7)4c}{(n-8)k}C, E=\frac{(n-9)4c}{(n-10)k}D, \dots\dots L=\frac{4\cdot 4c}{3k}K.$$

$$\int dx X^{\frac{n}{2}} = \left( AX^{\frac{n-1}{2}} + BX^{\frac{n-3}{2}} + CX^{\frac{n-5}{2}} + DX^{\frac{n-7}{2}} + EX^{\frac{n-9}{2}} + \dots\dots \right. \\ \left. \dots\dots + KX^{\frac{n-2i+3}{2}} + LX^{\frac{n-2i+1}{2}} \right) (2cx+b) \sqrt{X} \\ + \frac{n-2i+2}{2} kL \int dx X^{\frac{n}{2}-1}$$

$$A=\frac{1}{(n+1)2c}, B=\frac{nk}{(n-1)4c}A, C=\frac{(n-2)k}{(n-3)4c}B, \\ D=\frac{(n-4)k}{(n-5)4c}C, E=\frac{(n-6)k}{(n-7)4c}D, \dots\dots\dots \\ \dots\dots\dots L=\frac{(n-2i+4)k}{(n-2i+3)4c}K.$$

$$\int dx X^{\frac{n}{2}} = \left( AX^{\frac{n-1}{2}} + BX^{\frac{n-3}{2}} + CX^{\frac{n-5}{2}} + DX^{\frac{n-7}{2}} + EX^{\frac{n-9}{2}} + \dots\dots \right. \\ \left. \dots\dots + KX^2 + LX \right) (2cx+b) \sqrt{X} + \frac{3kL}{2} \int dx \sqrt{X}$$

$$A=\frac{1}{(n+1)2c}, B=\frac{nk}{(n-1)4c}A, C=\frac{(n-2)k}{(n-3)4c}B, \\ D=\frac{(n-4)k}{(n-5)4c}C, E=\frac{(n-6)k}{(n-7)4c}D, \dots\dots\dots L=\frac{5k}{4\cdot 4c}K.$$

$$\int \frac{x dx}{X^{\frac{n}{2}}} = -\frac{1}{(n-2)cX^{\frac{n}{2}-1}} - \frac{b}{2c} \int \frac{dx}{X^{\frac{n}{2}}}$$

$$\int x dx X^{\frac{n}{2}} = \frac{X^{\frac{n}{2}+1}}{(n+2)c} - \frac{b}{2c} \int dx X^{\frac{n}{2}}$$

T a f e l  
einiger allgemeineren Formeln.

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VZ.  $a + bx + cx^2 = X$

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$$\int \frac{dx}{xX^{\frac{n}{2}}} = \frac{1}{(n-2)aX^{\frac{n-2}{2}}} + \frac{1}{a} \int \frac{dx}{xX^{\frac{n-2}{2}}} - \frac{b}{2a} \int \frac{dx}{X^{\frac{n}{2}}}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x} = \frac{X^{\frac{n}{2}}}{n} + a \int \frac{dx X^{\frac{n-2}{2}}}{x} + \frac{b}{2} \int dx X^{\frac{n-2}{2}}$$

$$\begin{aligned} \int \frac{dx}{xX^{\frac{2n+1}{2}}} &= \left[ \frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^3X^{n-3}} + \frac{1}{(2n-5)a^5X^{n-5}} + \dots \right. \\ &\quad \left. \dots \dots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n} \right] \frac{1}{\sqrt{X}} \\ &\quad - \frac{b}{2a} \int \frac{dx}{X^{\frac{2n+1}{2}}} - \frac{b}{2a^2} \int \frac{dx}{X^{\frac{2n-1}{2}}} - \frac{b}{2a^3} \int \frac{dx}{X^{\frac{2n-3}{2}}} - \dots \dots \dots \\ &\quad \dots \dots \dots - \frac{b}{2a^{n-1}} \int \frac{dx}{X^{\frac{3}{2}}} - \frac{b}{2a^n} \int \frac{dx}{X^{\frac{1}{2}}} + \frac{1}{a^n} \int \frac{dx}{x\sqrt{X}} \end{aligned}$$

$$\begin{aligned} \int \frac{dx X^{\frac{2n+1}{2}}}{x} &= \left( \frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \dots \dots \dots \right. \\ &\quad \left. \dots \dots \dots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1} \right) \sqrt{X} \\ &\quad + \frac{b}{2} \int dx X^{\frac{2n-1}{2}} + \frac{ab}{2} \int dx X^{\frac{2n-3}{2}} + \frac{a^2b}{2} \int dx X^{\frac{2n-5}{2}} + \dots \dots \dots \\ &\quad \dots \dots \dots + \frac{a^{n-2}b}{2} \int dx X^{\frac{3}{2}} + \frac{a^{n-1}b}{2} \int dx \sqrt{X} \\ &\quad + \frac{a^n b}{2} \int \frac{dx}{\sqrt{X}} + a^{n+1} \int \frac{dx}{x\sqrt{X}} \end{aligned}$$

## Werthe der bestimmten Integrale

$$\int \frac{x^m dx}{V(a^2 - x^2)}, \quad \int x^m dx V(a^2 - x^2),$$

 von  $x = 0$  bis  $x = a$ .

$$VZ. a^2 - x^2 = X, \quad \pi = 3,14159 \dots$$

$$\int \frac{dx}{VX} = \frac{\pi}{2}$$

$$\int \frac{x dx}{VX} = a$$

$$\int \frac{x^2 dx}{VX} = \frac{1}{2} \cdot \frac{\pi a^2}{2}$$

$$\int \frac{x^3 dx}{VX} = \frac{2}{3} \cdot a^3$$

$$\int \frac{x^4 dx}{VX} = \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{\pi a^4}{2}$$

$$\int \frac{x^5 dx}{VX} = \frac{2 \cdot 4}{3 \cdot 5} \cdot a^5$$

$$\int \frac{x^6 dx}{VX} = \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{\pi a^6}{2}$$

$$\int \frac{x^7 dx}{VX} = \frac{2 \cdot 4 \cdot 6}{3 \cdot 5 \cdot 7} \cdot a^7$$

$$\int \frac{x^8 dx}{VX} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{\pi a^8}{2}$$

$$\int \frac{x^9 dx}{VX} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{3 \cdot 5 \cdot 7 \cdot 9} \cdot a^9$$

$$\int \frac{x^{10} dx}{VX} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{\pi a^{10}}{2}$$

$$\int \frac{x^{11} dx}{VX} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11} \cdot a^{11}$$

$$\int \frac{x^{2r} dx}{VX} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2r-3)(2r-1)}{2 \cdot 4 \cdot 6 \cdot 8 \dots (2r-2) 2r} \cdot \frac{\pi a^{2r}}{2}$$

$$\int \frac{x^{2r+1} dx}{VX} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \dots (2r-2) 2r}{3 \cdot 5 \cdot 7 \cdot 9 \dots (2r-1)(2r+1)} \cdot a^{2r+1}$$

$$\int' dx V X = \frac{\pi a^2}{4}$$

$$\int' x dx V X = \frac{a^3}{3}$$

$$\int' x^2 dx V X = \frac{1}{4} \cdot \frac{\pi a^4}{4}$$

$$\int' x^3 dx V X = \frac{2}{5} \cdot \frac{a^5}{3}$$

$$\int' x^4 dx V X = \frac{1 \cdot 3}{4 \cdot 6} \cdot \frac{\pi a^6}{4}$$

$$\int' x^5 dx V X = \frac{2 \cdot 4}{5 \cdot 7} \cdot \frac{a^7}{3}$$

$$\int' x^6 dx V X = \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} \cdot \frac{\pi a^8}{4}$$

$$\int' x^7 dx V X = \frac{2 \cdot 4 \cdot 6}{5 \cdot 7 \cdot 9} \cdot \frac{a^9}{3}$$

$$\int' x^8 dx V X = \frac{1 \cdot 3 \cdot 5 \cdot 7}{4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{\pi a^{10}}{4}$$

$$\int' x^9 dx V X = \frac{2 \cdot 4 \cdot 6 \cdot 8}{5 \cdot 7 \cdot 9 \cdot 11} \cdot \frac{a^{11}}{3}$$

$$\int' x^{2r} dx V X = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2r-3)(2r-1)}{4 \cdot 6 \cdot 8 \cdot 10 \dots 2r(2r+2)} \cdot \frac{\pi a^{2r+2}}{4}$$

$$\int' x^{2r+1} dx V X = \frac{2 \cdot 4 \cdot 6 \cdot 8 \dots (2r-2) 2r}{5 \cdot 7 \cdot 9 \cdot 11 \dots (2r+1)(2r+3)} \cdot \frac{a^{2r+3}}{3}$$

## Werthe der bestimmten Integrale

$$\int' x^m dx (a^2 - x^2)^{\frac{1}{2}}, \int' x^m dx (a^2 + x^2)^{\frac{1}{2}},$$

von  $x = 0$  bis  $x = a$ .

$$\text{VZ. } a^2 - x^2 = X, \pi = 3,14159 \dots$$

$$\int' dx X^{\frac{1}{2}} = \frac{3\pi a^4}{16}$$

$$\int' x dx X^{\frac{1}{2}} = \frac{a^5}{5}$$

$$\int' x^2 dx X^{\frac{1}{2}} = \frac{1}{6} \cdot \frac{3\pi a^6}{16}$$

$$\int' x^3 dx X^{\frac{1}{2}} = \frac{2}{7} \cdot \frac{a^7}{5}$$

$$\int' x^4 dx X^{\frac{1}{2}} = \frac{1 \cdot 3}{6 \cdot 8} \cdot \frac{3\pi a^8}{16}$$

$$\int' x^5 dx X^{\frac{1}{2}} = \frac{2 \cdot 4}{7 \cdot 9} \cdot \frac{a^9}{5}$$

$$\int' x^6 dx X^{\frac{1}{2}} = \frac{1 \cdot 3 \cdot 5}{6 \cdot 8 \cdot 10} \cdot \frac{3\pi a^{10}}{16}$$

$$\int' x^7 dx X^{\frac{1}{2}} = \frac{2 \cdot 4 \cdot 6}{7 \cdot 9 \cdot 11} \cdot \frac{a^{11}}{5}$$

$$\int' x^8 dx X^{\frac{1}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{6 \cdot 8 \cdot 10 \cdot 12} \cdot \frac{3\pi a^{12}}{16}$$

$$\int' x^9 dx X^{\frac{1}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{7 \cdot 9 \cdot 11 \cdot 13} \cdot \frac{a^{13}}{5}$$

$$\int' x^{2r} dx X^{\frac{1}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2r-3)(2r-1)}{6 \cdot 8 \cdot 10 \cdot 12 \dots (2r+2)(2r+4)} \cdot \frac{3\pi a^{2r+4}}{16}$$

$$\int' x^{2r+1} dx X^{\frac{1}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \dots (2r-2) 2r}{7 \cdot 9 \cdot 11 \cdot 13 \dots (2r+3)(2r+5)} \cdot \frac{a^{2r+5}}{5}$$

$$\int' dx X^{\frac{1}{2}} = \frac{5\pi a^6}{32}$$

$$\int' x dx X^{\frac{1}{2}} = \frac{a^7}{7}$$

$$\int' x^2 dx X^{\frac{1}{2}} = \frac{1}{8} \cdot \frac{5\pi a^8}{32}$$

$$\int' x^3 dx X^{\frac{1}{2}} = \frac{2}{9} \cdot \frac{a^9}{7}$$

$$\int' x^4 dx X^{\frac{1}{2}} = \frac{1 \cdot 3}{8 \cdot 10} \cdot \frac{5\pi a^{10}}{32}$$

$$\int' x^5 dx X^{\frac{1}{2}} = \frac{2 \cdot 4}{9 \cdot 11} \cdot \frac{a^{11}}{7}$$

$$\int' x^6 dx X^{\frac{1}{2}} = \frac{1 \cdot 3 \cdot 5}{8 \cdot 10 \cdot 12} \cdot \frac{5\pi a^{12}}{32}$$

$$\int' x^7 dx X^{\frac{1}{2}} = \frac{2 \cdot 4 \cdot 6}{9 \cdot 11 \cdot 13} \cdot \frac{a^{13}}{7}$$

$$\int' x^8 dx X^{\frac{1}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{8 \cdot 10 \cdot 12 \cdot 14} \cdot \frac{5\pi a^{14}}{32}$$

$$\int' x^9 dx X^{\frac{1}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{9 \cdot 11 \cdot 13 \cdot 15} \cdot \frac{a^{15}}{7}$$

$$\int' x^{2r} dx X^{\frac{1}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2r-3)(2r-1)}{8 \cdot 10 \cdot 12 \cdot 14 \dots (2r+4)(2r+6)} \cdot \frac{5\pi a^{2r+6}}{32}$$

$$\int' x^{2r+1} dx X^{\frac{1}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \dots (2r-2) 2r}{9 \cdot 11 \cdot 13 \cdot 15 \dots (2r+5)(2r+7)} \cdot \frac{a^{2r+7}}{7}$$

Werthe der bestimmten Integrale

$$\int' \partial x (a^2 - x^2)^{\frac{n}{2}}, \quad \int' x^n \partial x (a^2 - x^2)^{\frac{n}{2}}$$

von  $x = 0$  bis  $x = a$ .

---


$$\text{VZ. } a^2 - x^2 = X, \quad \pi = 3,14159\ldots$$


---

$$\int' \partial x \sqrt{X} = \frac{1}{2} \cdot \frac{\pi a^2}{2}$$

$$\int' \partial x X^{\frac{3}{2}} = \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{\pi a^4}{2}$$

$$\int' \partial x X^{\frac{5}{2}} = \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{\pi a^6}{2}$$

$$\int' \partial x X^{\frac{7}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{\pi a^8}{2}$$

$$\int' \partial x X^{\frac{9}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{\pi a^{10}}{2}$$

$$\dots\dots\dots$$

$$\int' \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (n-2) \cdot n}{2 \cdot 4 \cdot 6 \cdot 8 \dots (n-1)(n+1)} \cdot \frac{\pi a^{n+2}}{2}$$


---

$$\int' x^2 \partial x X^{\frac{n}{2}} = \frac{1}{n+3} \cdot a^2 \int' \partial x X^{\frac{n}{2}}$$

$$\int' x^4 \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3}{(n+3)(n+5)} \cdot a^4 \int' \partial x X^{\frac{n}{2}}$$

$$\int' x^6 \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3 \cdot 5}{(n+3)(n+5)(n+7)} \cdot a^6 \int' \partial x X^{\frac{n}{2}}$$

$$\dots\dots\dots$$

$$\int' x \partial x X^{\frac{n}{2}} = \frac{a^{n+2}}{n+2}$$

$$\int' x^3 \partial x X^{\frac{n}{2}} = \frac{2}{n+4} \cdot \frac{a^{n+4}}{n+2}$$

$$\int' x^5 \partial x X^{\frac{n}{2}} = \frac{2 \cdot 4}{(n+4)(n+6)} \cdot \frac{a^{n+6}}{n+2}$$

$$\dots\dots\dots$$

$$\int' x^{2r} \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2r-1)}{(n+3)(n+5)(n+7) \dots (n+2r+1)} \cdot a^{2r} \int' \partial x X^{\frac{n}{2}}$$

$$\int' x^{2r+1} \partial x X^{\frac{n}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \dots 2r}{(n+4)(n+6)(n+8) \dots (n+2r+2)} \cdot \frac{a^{2r+2}}{n+2}$$



Werthe der bestimmten Integrale

$$\int \frac{x^m dx}{V(a^4 - x^4)}, \quad \int x^m dx (a^4 - x^4)^{\frac{n}{2}},$$

von  $x=0$  bis  $x=a$ .

---


$$\text{VZ. } a^2 - x^2 = X, \quad \pi = 3,14159 \dots$$


---

$$\begin{aligned} \int \frac{dx}{V(a^4 - x^4)} &= \int \frac{dx(a^2 + x^2)^{-\frac{1}{2}}}{VX} = \frac{1}{a} \int \frac{dx}{VX} + \frac{-\frac{1}{2}\mathfrak{A}}{a^3} \int \frac{x^2 dx}{VX} \\ &+ \frac{-\frac{1}{2}\mathfrak{B}}{a^5} \int \frac{x^4 dx}{VX} + \frac{-\frac{1}{2}\mathfrak{C}}{a^7} \int \frac{x^6 dx}{VX} + \frac{-\frac{1}{2}\mathfrak{D}}{a^9} \int \frac{x^8 dx}{VX} + \text{etc.} \\ &= \frac{\pi}{2a} \left[ 1 - \left(\frac{1}{2}\right)^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 + \left(\frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\right)^2 \right. \\ &\quad \left. - \left(\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10}\right)^2 + \left(\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12}\right)^2 - \text{etc.} \right] \end{aligned}$$

$$\begin{aligned} \int dx V(a^4 - x^4) &= \int dx (a^2 + x^2)^{\frac{1}{2}} VX = a \int dx VX + \frac{\frac{1}{2}\mathfrak{A}}{a} \int x^2 dx VX \\ &+ \frac{\frac{1}{2}\mathfrak{B}}{a^3} \int x^4 dx VX + \frac{\frac{1}{2}\mathfrak{C}}{a^5} \int x^6 dx VX + \text{etc.} \\ &= \frac{\pi a^3}{4} \left[ 1 + \frac{1}{2} \cdot \frac{1}{4} - \frac{1 \cdot 1}{2 \cdot 4} \cdot \frac{1 \cdot 3}{4 \cdot 6} + \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6} \cdot \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} - \frac{1 \cdot 1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1 \cdot 3 \cdot 5 \cdot 7}{4 \cdot 6 \cdot 8 \cdot 10} \right. \\ &\quad \left. + \frac{1 \cdot 1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{4 \cdot 6 \cdot 8 \cdot 10 \cdot 12} - \text{etc.} \right] \end{aligned}$$

$$\begin{aligned} \int \frac{x^m dx}{V(a^4 - x^4)} &= \frac{1}{a} \int \frac{x^m dx}{VX} + \frac{-\frac{1}{2}\mathfrak{A}}{a^3} \int \frac{x^{m+2} dx}{VX} + \frac{-\frac{1}{2}\mathfrak{B}}{a^5} \int \frac{x^{m+4} dx}{VX} \\ &+ \frac{-\frac{1}{2}\mathfrak{C}}{a^7} \int \frac{x^{m+6} dx}{VX} + \frac{-\frac{1}{2}\mathfrak{D}}{a^9} \int \frac{x^{m+8} dx}{VX} + \text{etc.} \end{aligned}$$

$$\begin{aligned} \int x^m dx (a^4 - x^4)^{\frac{n}{2}} &= a^n \int x^m dx X^{\frac{n}{2}} + \frac{\frac{n}{2}\mathfrak{A} a^{n-2}}{1} \int x^{m+2} dx X^{\frac{n}{2}} \\ &+ \frac{\frac{n}{2}\mathfrak{B} a^{n-4}}{1} \int x^{m+4} dx X^{\frac{n}{2}} + \frac{\frac{n}{2}\mathfrak{C} a^{n-6}}{1} \int x^{m+6} dx X^{\frac{n}{2}} + \text{etc.} \end{aligned}$$

Werthe der bestimmten Integrale

$$\int \frac{x^m dx (1+cx^h)^{\frac{p}{h}}}{V(a^2-x^2)^{\frac{q}{2}}}, \quad \int x^m dx (1+cx^h)^{\frac{p}{h}} (a^2-x^2)^{\frac{q}{2}}$$

von  $x = 0$  bis  $x = a$ .

---


$$\text{VZ. } a^2 - x^2 = X$$


---

$$\int \frac{dx (1+cx^h)^{\frac{p}{h}}}{VX} = \int \frac{dx}{VX} + \frac{p}{h} \mathfrak{A}_c \int \frac{x^h dx}{VX} + \frac{p}{h} \mathfrak{B}_c^2 \int \frac{x^{2h} dx}{VX} \\ + \frac{p}{h} \mathfrak{C}_c^3 \int \frac{x^{3h} dx}{VX} + \frac{p}{h} \mathfrak{D}_c^4 \int \frac{x^{4h} dx}{VX} + \text{etc.}$$

$$\int (1+cx^h)^{\frac{p}{h}} X^{\frac{q}{2}} = \int dx X^{\frac{q}{2}} + \frac{p}{h} \mathfrak{A}_c \int x^h dx X^{\frac{q}{2}} + \frac{p}{h} \mathfrak{B}_c^2 \int x^{2h} dx X^{\frac{q}{2}} \\ + \frac{p}{h} \mathfrak{C}_c^3 \int x^{3h} dx X^{\frac{q}{2}} + \frac{p}{h} \mathfrak{D}_c^4 \int x^{4h} dx X^{\frac{q}{2}} + \text{etc.}$$

$$\int \frac{dx}{(1+cx^h)^{\frac{p}{h}} VX} = \int \frac{dx}{VX} + \frac{-p}{h} \mathfrak{A}_c \int \frac{x^h dx}{VX} + \frac{-p}{h} \mathfrak{B}_c^2 \int \frac{x^{2h} dx}{VX} \\ + \frac{-p}{h} \mathfrak{C}_c^3 \int \frac{x^{3h} dx}{VX} + \frac{-p}{h} \mathfrak{D}_c^4 \int \frac{x^{4h} dx}{VX} + \text{etc.}$$

$$\int \frac{dx X^{\frac{q}{2}}}{(1+cx^h)^{\frac{p}{h}}} = \int dx X^{\frac{q}{2}} + \frac{-p}{h} \mathfrak{A}_c \int x^h dx X^{\frac{q}{2}} + \frac{-p}{h} \mathfrak{B}_c^2 \int x^{2h} dx X^{\frac{q}{2}} \\ + \frac{-p}{h} \mathfrak{C}_c^3 \int x^{3h} dx X^{\frac{q}{2}} + \frac{-p}{h} \mathfrak{D}_c^4 \int x^{4h} dx X^{\frac{q}{2}} + \text{etc.}$$

$$\int \frac{x^m dx (1+cx^h)^{\frac{p}{h}}}{VX} = \int \frac{x^m dx}{VX} + \frac{p}{h} \mathfrak{A}_c \int \frac{x^{m+h} dx}{VX} + \frac{p}{h} \mathfrak{B}_c^2 \int \frac{x^{m+2h} dx}{VX} \\ + \frac{p}{h} \mathfrak{C}_c^3 \int \frac{x^{m+3h} dx}{VX} + \frac{p}{h} \mathfrak{D}_c^4 \int \frac{x^{m+4h} dx}{VX} + \text{etc.}$$

$$\int x^m dx (1+cx^h)^{\frac{p}{h}} X^{\frac{q}{2}} = \int x^m dx X^{\frac{q}{2}} + \frac{p}{h} \mathfrak{A}_c \int x^{m+h} dx X^{\frac{q}{2}} \\ + \frac{p}{h} \mathfrak{B}_c^2 \int x^{m+2h} dx X^{\frac{q}{2}} + \frac{p}{h} \mathfrak{C}_c^3 \int x^{m+3h} dx X^{\frac{q}{2}} + \text{etc.}$$

$$\int \frac{x^m dx}{(1+cx^h)^{\frac{p}{h}} VX} = \int \frac{x^m dx}{VX} + \frac{-p}{h} \mathfrak{A}_c \int \frac{x^{m+h} dx}{VX} + \frac{-p}{h} \mathfrak{B}_c^2 \int \frac{x^{m+2h} dx}{VX} \\ + \frac{-p}{h} \mathfrak{C}_c^3 \int \frac{x^{m+3h} dx}{VX} + \frac{-p}{h} \mathfrak{D}_c^4 \int \frac{x^{m+4h} dx}{VX} + \text{etc.}$$

$$\int \frac{x^m dx X^{\frac{q}{2}}}{(1+cx^h)^{\frac{p}{h}}} = \int x^m dx X^{\frac{q}{2}} + \frac{-p}{h} \mathfrak{A}_c \int x^{m+h} dx X^{\frac{q}{2}} \\ + \frac{-p}{h} \mathfrak{B}_c^2 \int x^{m+2h} dx X^{\frac{q}{2}} + \frac{-p}{h} \mathfrak{C}_c^3 \int x^{m+3h} dx X^{\frac{q}{2}} + \text{etc.}$$

Relationen zwischen den Werthen der  
bestimmten Integrale.

VZ.  $1 - x^2 = X$ ,  $\pi = 3,14159\ldots$

$m, n, p, r$ , beliebige positive Zahlen.

Die Integrale von  $x = 0$  bis  $x = 1$  genommen.

$$\int \frac{x^m dx}{V(x-x^2)} = 2 \int \frac{x^{2m} dx}{V(1-x^2)}$$

$$\int \frac{x^r dx}{V(1-x^2)} \times \int \frac{x^{r+1} dx}{V(1-x^2)} = \frac{1}{r+1} \cdot \frac{\pi}{2}$$

$$\int \frac{x^r dx}{V(1-x^4)} \times \int \frac{x^{r+2} dx}{V(1-x^4)} = \frac{1}{2(r+1)} \cdot \frac{\pi}{2}$$

$$\int \frac{x^r dx}{V(1-x^{2n})} \times \int \frac{x^{r+2} dx}{V(1-x^{2n})} = \frac{1}{n(r+1)} \cdot \frac{\pi}{2}$$

$$\int \frac{dx}{V(1-x^{2n})} \times \int \frac{x^n dx}{V(1-x^{2n})} = \frac{1}{n} \cdot \frac{\pi}{2}$$

$$\int x^{m-1} dx X^{\frac{r-n}{2}} = \int x^{r-1} dx X^{\frac{m-n}{2}}$$

$$\int x^{m-1} dx X^{\frac{r-n}{2}} = \frac{(m+p)(m+p+n)(m+p+2n)\cdots(m+p+in)}{m(m+n)(m+2n)\cdots(m+in)} \times \int x^{m+(i+1)n-1} dx X^{\frac{r-n}{2}}$$

$$\frac{\int x^{m-1} dx X^{\frac{r-n}{2}}}{\int x^{r-1} dx X^{\frac{r-n}{2}}} = \frac{(m+p)(m+p+n)(m+p+2n) \text{ in inf.}}{m(m+n)(m+2n) \text{ in inf.}} \times \frac{r(r+n)(r+2n) \text{ in inf.}}{(r+p)(r+p+n)(r+p+2n) \text{ in inf.}}$$

$$\frac{\int x^{m-1} dx X^{\frac{r-n}{2}}}{\int x^{m+i-1} dx X^{\frac{r-n}{2}}} = \frac{\int x^{r-1} dx X^{\frac{r-n}{2}}}{\int x^{r+i-1} dx X^{\frac{r-n}{2}}}$$

$$\int \frac{x^{m-1} dx}{1+x^n} = \frac{\pi}{n \sin \frac{m\pi}{n}} \quad \left[ \begin{array}{l} \text{Dieser und der folgende Werth} \\ \text{gilt nur so lange als } m-1 < n. \end{array} \right]$$

$$\int \frac{x^{m-1} dx}{V X^n} = \int x^{m-m-1} dx X^{\frac{m-n}{2}} = \frac{\pi}{n \sin \frac{m\pi}{n}}$$

## Entwicklung der Integralformel.

 $\int x^m dx (a + bx^n)^p$  in Reihen.

$$\text{VL. } a + bx^n = X$$

$$\int x^m dx X^p = a^p x^{m+1} \left( A + Bx^n + Cx^{2n} + Dx^{3n} + Ex^{4n} + \text{etc.} \right)$$

$$A = \frac{1}{m+1}, B = \frac{pA}{m+n+1} \cdot \frac{b}{a}, C = \frac{pB}{m+2n+1} \cdot \frac{b^2}{a^2},$$

$$D = \frac{pC}{m+3n+1} \cdot \frac{b^3}{a^3}, E = \frac{pD}{m+4n+1} \cdot \frac{b^4}{a^4} + \text{etc.}$$

$$\int x^m dx X^p = b^p x^{m+np+1} \left( A + \frac{B}{x^n} + \frac{C}{x^{2n}} + \frac{D}{x^{3n}} + \frac{E}{x^{4n}} + \text{etc.} \right)$$

$$A = \frac{1}{m+np+1}, B = \frac{pA}{m+(p-1)n+1} \cdot \frac{a}{b},$$

$$C = \frac{pB}{m+(p-2)n+1} \cdot \frac{a^2}{b^2}, D = \frac{pC}{m+(p-3)n+1} \cdot \frac{a^3}{b^3},$$

$$E = \frac{pD}{m+(p-4)n+1} \cdot \frac{a^4}{b^4}, \text{ etc.}$$

$$\int x^m dx X^p = x^{m+1} X^{p+1} \left( \frac{A}{x^n} - \frac{B}{x^{2n}} + \frac{C}{x^{3n}} - \frac{D}{x^{4n}} + \frac{E}{x^{5n}} - \text{etc.} \right)$$

$$A = \frac{1}{(m+np+1)b}, B = \frac{(m-n+1)a}{(m-n+np+1)b} A,$$

$$C = \frac{(m-2n+1)a}{(m-2n+np+1)b} B, D = \frac{(m-3n+1)a}{(m-3n+np+1)b} C,$$

$$E = \frac{(m-4n+1)a}{(m-4n+np+1)b} D, \text{ etc.}$$

$$\int x^m dx X^p = x^{m+1} X^{p+1} \left( A - Bx^n + Cx^{2n} - Dx^{3n} + Ex^{4n} - \text{etc.} \right)$$

$$A = \frac{1}{(m+1)a}, B = \frac{(m+n+np+1)b}{(m+n+1)a} A, C = \frac{(m+2n+np+1)b}{(m+2n+1)a} B,$$

$$D = \frac{(m+3n+np+1)b}{(m+3n+1)a} C, E = \frac{(m+4n+np+1)b}{(m+4n+1)a} D, \text{ etc.}$$

## Entwicklung der Integralformel

 $\int x^m dx (a + bx^2)^p$  in Reihen.

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$$\text{VZ. } a + bx^2 = X$$


---

$$\int x^m dx X^p = -x^{m+1} X^{p+1} (A + BX + CX^2 + DX^3 + \text{etc.})$$

$$A = \frac{1}{(p+1)na}, B = \frac{m+n+np+1}{(p+2)na} A, C = \frac{m+2n+np+1}{(p+3)na} B,$$

$$D = \frac{m+3n+np+1}{(p+4)na} C, E = \frac{m+4n+np+1}{(p+5)na} D, \text{ etc.}$$

$$\int x^m dx X^p = x^{m+1} X^p \left( A + \frac{B}{X} + \frac{C}{X^2} + \frac{D}{X^3} + \frac{E}{X^4} + \text{etc.} \right)$$

$$A = \frac{1}{m+np+1}, B = \frac{pxa}{m-n+np+1} A, C = \frac{(p-1)na}{m-2n+np+1} B,$$

$$D = \frac{(p-3)na}{m-3n+np+1} C, E = \frac{(p-5)na}{m-4n+np+1} D, \text{ etc.}$$

$$\int x^m dx X^p = x^{m+1} X^p \left[ A - B \left( \frac{x^2}{X} \right) + C \left( \frac{x^2}{X} \right)^2 - D \left( \frac{x^2}{X} \right)^3 \right. \\ \left. + E \left( \frac{x^2}{X} \right)^4 - F \left( \frac{x^2}{X} \right)^5 + \text{etc.} \right]$$

$$A = \frac{1}{m+1}, B = \frac{mna}{m-n+1} A, C = \frac{p-1}{m-2n+1} B,$$

$$D = \frac{(p-3)na}{m-3n+1} C, E = \frac{(p-5)na}{m-4n+1} D, F = \frac{(p-7)na}{m-5n+1} E, \text{ etc.}$$

$$\int x^m dx X^p = x^{m+1} X^p \left[ A - E \frac{X'}{X^2} - C \frac{X'}{X^2} - D \frac{X'}{X^2} \right. \\ \left. + E \frac{X'}{X^2} - F \frac{X'}{X^2} + \text{etc.} \right]$$

$$A = \frac{1}{(p+1)na}, E = \frac{m-n+1}{(p+2)na} A, C = \frac{m-2n+1}{(p+3)na} E,$$

$$D = \frac{m-3n+1}{(p+4)na} C, F = \frac{m-4n+1}{(p+5)na} D, \text{ etc.}$$

# Integrations - Methoden. \*)

VZ.  $F: [x, y, z, t, \text{etc.}]$  eine rationale Function  
der Gröſsen  $x, y, z, t, \text{etc.}$

## I. Das Differential

$$\partial x F: [x, \sqrt[n]{x}, \sqrt[m]{x}, \sqrt[p]{x}, \sqrt[q]{x}, \text{etc.}]$$

wird rational, wenn  $x = y^{mn \dots}$  gesetzt wird; denn dadurch wird  
 $\sqrt[n]{x} = y^{m \dots}$ ,  $\sqrt[m]{x} = y^{n \dots}$ ,  $\sqrt[p]{x} = y^{mn \dots}$ ,  $\sqrt[q]{x} = y^{mp \dots}$ , etc.  
und  $\partial x = (mnpq \dots) y^{(mn \dots)-1} \partial y$ . Dahin gehört z. B. das Diffe-  
rential  $\frac{x^3 + 2\sqrt[3]{ax^2} + \sqrt[3]{x}}{bx + c\sqrt[4]{dx}}$   $\partial x$ ; es wird rational, wenn  $x = y^{60}$  ge-  
setzt wird.

## II. Das Differential

$$\partial x F: [x, \sqrt[n]{a+bx}]$$

wird rational, wenn  $a + bx = y^n$  gesetzt wird; denn alsdann ist  
 $\sqrt[n]{a+bx} = y$ ,  $x = \frac{y^n - a}{b}$ ,  $\partial x = \frac{ny^{n-1} \partial y}{b}$ . Dahin gehören z. B.

die Differentiale  $\frac{x^4 \partial x}{cx^5 + \sqrt[4]{a+bx}^3}$ ,  $\frac{x^2 \partial x \sqrt[5]{a+bx}^3}{cx + d\sqrt[5]{a+bx}^3}$ . Das er-  
stere wird rational, wenn  $a + bx = y^5$  gesetzt wird.

## III. Das Differential

$$\partial x F: \left[ x, \sqrt[n]{\frac{a+bx}{f+gx}} \right]$$

wird rational, wenn  $\frac{a+bx}{f+gx} = y^n$  gesetzt wird; denn hierdurch wird

\*) Ein Differential soll hier als integrirt angesehen werden, sobald man dasselbe durch irgend eine Verwandlung rational gemacht, oder auf solche irrationale Integrale zurückgeführt hat, welche sich rational machen lassen.

$$\sqrt[n]{\frac{a+bx}{f+gx}} = y, \quad x = \frac{a-fy^n}{gy^n-b}, \quad dx = \frac{n(bf-ag)y^{n-1}dy}{(gy^n-b)^2}. \quad \text{Dahin}$$

gehören z. B. die Differentiale  $x^n dx \left( \frac{a+bx}{f+gx} \right)^{\frac{1}{n}}, \frac{dx \sqrt[n]{a+bx}}{x^n \sqrt[n]{f+gx}},$

$\frac{dx \sqrt[n]{a+bx}}{\sqrt[n]{a+bx} + \sqrt[n]{f+gx}}.$  Es lassen sich nämlich diesen Differentialen folgende Formen geben:

$$x^n dx \left[ \left( \frac{a+bx}{f+gx} \right)^{\frac{1}{n}} \right]^{\frac{1}{n}}, \quad \frac{dx \sqrt[n]{a+bx}}{x^n \sqrt[n]{f+gx}}, \quad \frac{dx}{\sqrt[n]{a+bx} + \sqrt[n]{f+gx}} \cdot \sqrt[n]{a+bx}.$$

#### IV. Das Differential

$$\partial x F : [x, (a+bx)^{\frac{m}{n}}, (a+bx)^{\frac{p}{q}}, (a+bx)^{\frac{r}{s}}, \text{etc.}]$$

wird rational, wenn  $a+bx = y^{nq} \dots$  gesetzt wird; denn hierdurch wird  $(a+bx)^{\frac{m}{n}} = y^{mq} \dots, (a+bx)^{\frac{p}{q}} = y^{pq} \dots, (a+bx)^{\frac{r}{s}} = y^{rs} \dots, \text{etc.},$   
 $x = \frac{y^{nq} \dots - a}{b}, \quad dx = \frac{nqs \dots}{b} y^{(nq \dots)-1} dy.$

#### V. Auf eine ähnliche Weise wird das Differential

$$\partial x F : \left[ x, \left( \frac{a+bx}{f+gx} \right)^{\frac{m}{n}}, \left( \frac{a+bx}{f+gx} \right)^{\frac{p}{q}}, \left( \frac{a+bx}{f+gx} \right)^{\frac{r}{s}}, \text{etc.} \right]$$

rational, wenn man  $\frac{a+bx}{f+gx} = y^{nq} \dots$  setzt.

#### VI. Um das Differential

$$\partial x F : [x, V(a+bx+cx^2)]$$

rational zu machen, muß man die beiden Fälle unterscheiden, wo  $c$  positiv und wo  $c$  negativ ist.

**Erster Fall.** Das Differential  $\partial x F : [x, V(a+bx+cx^2)]$  wird rational, wenn  $a+bx+cx^2 = c(x+y)^2$  gesetzt wird; hieraus erhält man nämlich  $x = \frac{a-cy^2}{2cy-b}, \quad dx = -\frac{2c(cy^2-by+a)dy}{(2cy-b)^2},$   
 $V(a+bx+cx^2) = \frac{(cy^2-by+a)Vc}{2cy-b}.$

**Zweiter Fall.** Es bezeichnen  $r$  und  $r'$  die beiden Wurzeln der Gleichung  $a + bx - cx^2 = 0$ ; so ist  $V(a + bx - cx^2) = Vc(x-r)(r'-x)$ . Das Differential  $\partial xF: [x, V(a + bx - cx^2)]$  wird daher rational, wenn man  $Vc(x-r)(r'-x) = (x-r)cy$  setzt; denn hieraus erhält man  $x = \frac{cry^2 + r'}{cy^2 + 1}$ ,  $dx = \frac{(r-r')2cydy}{(cy^2 + 1)^2}$ ,  $V(a + bx - cx^2) = \frac{(r'-r)cy}{cy^2 + 1}$ .

Die Wurzeln der Gleichung  $a + bx - cx^2 = 0$  sind nothwendig reell,  $a$  und  $b$  mögen positiv oder negativ seyn, weil im entgegengesetzten Falle  $V(a + bx - cx^2)$  für jeden Werth des  $x$  imaginär seyn würde.

### VII. Die Differentiale

$$\partial xF: [x, V(a + cx^2)], \partial xF: [x, V(bx + cx^2)]$$

sind unter den vorigen begriffen; denn man erhält sie daraus, wenn man  $b = 0$ , oder  $a = 0$  setzt.

### VIII. Um das Differential

$$\partial xF: [x, V(a + bx), V(a' + b'x)]$$

rational zu machen, setze man zuerst  $a + bx = (a' + b'x)y^2$ ; dies

$$\text{giebt } x = \frac{a - a'y^2}{b'y^2 - b}, \partial x = \frac{(a'b - ab')2ydy}{(b'y^2 - b)^2}, V(a + bx) = \frac{yV(ab' - a'b)}{V(b'y^2 - b)},$$

$$V(a' + b'x) = \frac{V(ab' - a'b)}{V(b'y^2 - b)}.$$

Durch die Substitution dieser Werthe verwandelt sich das gegebene Differential in ein anderes von der Form  $\partial yF': [y, V(b'y^2 - b)]$ , wenn  $F'$  irgend eine andre rationale Function als  $F$  bezeichnet; und dieses Differential kann wieder nach dem Vorhergehenden rational gemacht werden.

### IX. Das Differential

$$x^{m-1} \partial x(a + bx^{\frac{p}{q}})$$

läßt sich in den beiden Fällen, wo  $\frac{m}{n}$  oder  $\frac{m}{n} + \frac{p}{q}$  eine ganze positive oder negative Zahl ist, rational machen.



Erster Fall. Man setze  $a+bx^m = y^r$ , so wird  $(a+bx^m)^{\frac{r}{m}} = y^r$ ,  
 $x^m = \frac{y^r - a}{b}$ ,  $x^m = \left(\frac{y^r - a}{b}\right)^{\frac{m}{m}}$ ,  $x^{m-1}dx = \frac{qy^{r-1}}{nb} \left(\frac{y^r - a}{b}\right)^{\frac{m-n}{m}}$ .  
 Durch die Substitution dieser Werthe verwandelt sich das obige  
 Differential in  $\frac{q}{nb} y^{r-1-n} dy \left(\frac{y^r - a}{b}\right)^{\frac{m-n}{m}}$ , und wird daher rational,  
 wenn  $\frac{m-n}{n}$ , also auch  $\frac{m}{n}$  eine ganze Zahl ist.

Zweiter Fall. Man setze  $a+bx^m = x^m y^r$ ; so ist  $x^m = \frac{a}{y^r - b}$ ,  
 $a+bx^m = \frac{ay^r}{y^r - b}$ ,  $(a+bx^m)^{\frac{r}{m}} = \frac{a^{\frac{r}{m}} y^r}{(y^r - b)^{\frac{r}{m}}}$ ,  $x^m = \frac{a^{\frac{m}{m}}}{(y^r - b)^{\frac{m}{m}}}$ ,  
 $x^{m-1}dx = -\frac{qa^{\frac{m}{m}} y^{r-1}}{n(y^r - b)^{\frac{m}{m}+1}}$ . Das gegebene Differential verwandelt sich  
 daher in  $-\frac{qa^{\frac{m}{m}+\frac{r}{m}} y^{r-1-n}}{n(y^r - b)^{\frac{m}{m}+\frac{r}{m}+1}}$ , und wird folglich rational, sobald  $\frac{m}{n} + \frac{p}{q}$   
 eine ganze Zahl ist,

X. In den nämlichen zwey Fällen und durch dieselben Substitutionen wird überhaupt das Differential

$$x^{m-1}dx(a+bx^m)^{\frac{r}{m}}F:[x^m]$$

rational. Hieher gehört z. B. das Differential  $x^{m-1}dx(a+bx^m)^{\frac{r}{m}}$ ,  
 und das noch allgemeinere  $\frac{Px^{m-1}dx}{Q}(a+bx^m)^{\frac{r}{m}}$ , wenn  $P = A + Bx^m + Cx^{2m} + Dx^{3m} + \text{etc.}$ ,  $Q = A' + B'x^m + C'x^{2m} + D'x^{3m} + \text{etc.}$

#### XI. Das Differential

$$x^{m-1}dx F:[x^m, x^n, \sqrt[m]{a+bx^m}],$$

welches das in IX und X in sich schließt, wird rational, wenn  $\frac{m}{n}$  eine ganze positive oder negative Zahl ist; denn man setze

$$\sqrt[n]{a+bx^m} = y, \text{ so wird } x^m = \frac{y^n - a}{b}, \quad x^m = \left(\frac{y^n - a}{b}\right)^{\frac{m}{n}},$$

$$x^{m-1}dx = \frac{ny^{n-1}(y^n - a)^{\frac{m}{n}-1}}{nb} dy$$

XII. Es bezeichnen  $X, X', X''$ , rationale Functionen von  $x$ , so lassen sich die Differentiale

$$\frac{Xdx}{X' + X''V(a+bx+cx^2)},$$

$$\frac{Xdx}{X'V(a+bx+cx^2) + X''V(a'+b'x+c'x^2)},$$

immer rational machen, wenn man ersteres mit  $X' - X''V(a+bx+cx^2)$ , und letzteres mit  $X'V(a+bx+cx^2) - X''V(a'+b'x+c'x^2)$  multiplicirt; denn hierdurch verwandelt sich das erstere in

$$\frac{XX'dx}{X'^2 - X''^2(a+bx+cx^2)} - \frac{XX''dxV(a+bx+cx^2)}{X'^2 - X''^2(a+bx+cx^2)}, \text{ worin}$$

man blofs noch den zweiten Theil rational zu machen hat, und

$$\text{das zweite in } \frac{XX'dxV(a+bx+cx^2)}{X'^2(a+bx+cx^2) - X''^2(a'+b'x+c'x^2)} -$$

$$\frac{XX''dxV(a'+b'x+c'x^2)}{X'^2(a+bx+cx^2) - X''^2(a'+b'x+c'x^2)}, \text{ worin sich jeder Theil besonders rational machen läfst. *)}$$

### XIII. Das Differential

$$x^m dx F : [x^m, V(a+bx^m+cx^{2m})]$$

verwandelt sich, wenn  $x^m = y$  gesetzt wird, in

$$\frac{1}{n} y^{\frac{m+1}{n}-1} dy F : [y, V(a+by+cy^2)],$$

und kann in dieser Gestalt nach der Methode in VI. rational ge-

\*) Es lassen sich überhaupt immer die Wurzelgrößen aus dem Nenner eines Bruches wegschaffen, (m. s. meine Samml. v. Aufg. a. d. Th. d. Gl. Seite 213 — 215) und man hat es alsdann blofs mit der Integration von Monomen zu thun.

macht werden, wenn  $\frac{m+1}{n}$  eine ganze Zahl ist. Hieher gehört das Differential  $x^n \partial x (a + bx^n + cx^{2n})^{\frac{1}{n}}$ .

XIV. Unter der nämlichen Bedingung, daß  $\frac{m+1}{n}$  eine ganze Zahl sey, läßt sich auch das Differential

$$x^n \partial x F : [x^n, hx^n + V(a + h^2 x^{2n})]$$

und das noch allgemeinere

$$x^n \partial x F : [x^n, V(a + h^2 x^{2n}), hx^n + V(a + h^2 x^{2n})]$$

rational machen; denn man setze  $hx^n + V(a + h^2 x^{2n}) = y$ , so ist

$$x^n = \frac{y^2 - a}{2hy}, \quad V(a + h^2 x^{2n}) = \frac{y^2 + a}{2y},$$

$$x^n \partial x = \frac{1}{n(2h)^{\frac{n+1}{2}}} \left( \frac{y^2 + a}{y} \right) \left( \frac{y^2 - a}{y} \right)^{\frac{n+1}{2} - 1} \partial y.$$

Hieher gehört das weniger umfassende Differential

$$\partial x F : [x, V(a + h^2 x^2), hx + V(a + h^2 x^2)]$$

wöhin die einzelnen Differentiale  $[x + V(1 + x^2)] \cdot \partial x$ ,  $[x + V(1 + x^2)] \cdot X \partial x$ ,  $[ax + bV(1 + x^2)][x + V(1 + x^2)] \cdot \partial x$  zu rechnen sind, welche Euler in dem Anhang zu § 126 des ersten Bandes seiner Institutionen anführt.

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**Integraltafeln**  
für  
**transcendente Differentiale.**

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T a f e l  
der Reductionsformeln für das Integral  
 $\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi$

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I.

$$\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi = \frac{\operatorname{Sin}^{m+1} \varphi \operatorname{Cos}^{n-1} \varphi}{m+1} + \frac{n-1}{m+1} \int \partial \varphi \operatorname{Sin}^{m+2} \varphi \operatorname{Cos}^{n-2} \varphi$$

II.

$$\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi = -\frac{\operatorname{Sin}^{m-1} \varphi \operatorname{Cos}^{n+1} \varphi}{n+1} + \frac{m-1}{n+1} \int \partial \varphi \operatorname{Sin}^{m-2} \varphi \operatorname{Cos}^{n+2} \varphi$$

III.

$$\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi = -\frac{\operatorname{Sin}^{m-1} \varphi \operatorname{Cos}^{n+1} \varphi}{m+n} + \frac{m-1}{m+n} \int \partial \varphi \operatorname{Sin}^{m-2} \varphi \operatorname{Cos}^n \varphi$$

IV.

$$\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi = \frac{\operatorname{Sin}^{m+1} \varphi \operatorname{Cos}^{n-1} \varphi}{m+n} + \frac{n-1}{m+n} \int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^{n-2} \varphi$$

V.

$$\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi = \frac{\operatorname{Sin}^{m+1} \varphi \operatorname{Cos}^{n+1} \varphi}{m+1} + \frac{m+n+2}{m+1} \int \partial \varphi \operatorname{Sin}^{m+2} \varphi \operatorname{Cos}^n \varphi$$

VI.

$$\int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi = -\frac{\operatorname{Sin}^{m+1} \varphi \operatorname{Cos}^{n+1} \varphi}{n+1} + \frac{m+n+2}{n+1} \int \partial \varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^{n+2} \varphi$$


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Diese Formeln gelten,  $m$  und  $n$  mögen positive oder negative, ganze oder gebrochene Zahlen, oder auch  $= 0$  seyn.

Taf. I.

 $\int \partial \phi \sin^m \phi$ 

$$\int \partial \phi \sin \phi = -\cos \phi$$

$$\int \partial \phi \sin^2 \phi = -\frac{1}{3} \sin \phi \cos \phi + \frac{1}{3} \phi$$

$$\int \partial \phi \sin^3 \phi = \left(-\frac{1}{5} \sin^2 \phi - \frac{2}{5}\right) \cos \phi$$

$$\int \partial \phi \sin^4 \phi = \left(-\frac{1}{4} \sin^3 \phi - \frac{5}{8} \sin \phi\right) \cos \phi + \frac{5}{8} \phi$$

$$\int \partial \phi \sin^5 \phi = \left(-\frac{1}{5} \sin^4 \phi - \frac{4}{15} \sin^2 \phi - \frac{8}{15}\right) \cos \phi$$

$$\int \partial \phi \sin^6 \phi = \left(-\frac{1}{6} \sin^5 \phi - \frac{5}{24} \sin^3 \phi - \frac{5}{16} \sin \phi\right) \cos \phi + \frac{5}{16} \phi$$

$$\int \partial \phi \sin^7 \phi = \left(-\frac{1}{7} \sin^6 \phi - \frac{6}{55} \sin^4 \phi - \frac{8}{55} \sin^2 \phi - \frac{16}{55}\right) \cos \phi$$

$$\int \partial \phi \sin^8 \phi = \left(-\frac{1}{8} \sin^7 \phi - \frac{7}{48} \sin^5 \phi - \frac{55}{192} \sin^3 \phi - \frac{55}{128} \sin \phi\right) \cos \phi + \frac{55}{128} \phi$$

$$\int \partial \phi \sin^9 \phi = \left(-\frac{1}{9} \sin^8 \phi - \frac{8}{63} \sin^6 \phi - \frac{16}{105} \sin^4 \phi - \frac{64}{515} \sin^2 \phi - \frac{128}{515}\right) \cos \phi$$

$$\int \partial \phi \sin \phi = -\cos \phi$$

$$\int \partial \phi \sin^2 \phi = -\frac{1}{4} \sin 2\phi + \frac{1}{2} \phi$$

$$\int \partial \phi \sin^3 \phi = \frac{1}{12} \cos 3\phi - \frac{5}{4} \cos \phi$$

$$\int \partial \phi \sin^4 \phi = \frac{1}{54} \sin 4\phi - \frac{1}{4} \sin 2\phi + \frac{5}{8} \phi$$

$$\int \partial \phi \sin^5 \phi = -\frac{1}{80} \cos 5\phi + \frac{5}{48} \cos 3\phi - \frac{5}{8} \cos \phi$$

$$\int \partial \phi \sin^6 \phi = -\frac{1}{192} \sin 6\phi + \frac{3}{64} \sin 4\phi - \frac{15}{64} \sin 2\phi + \frac{5}{16} \phi$$

$$\int \partial \phi \sin^7 \phi = \frac{1}{448} \cos 7\phi - \frac{7}{520} \cos 5\phi + \frac{7}{64} \cos 3\phi - \frac{55}{64} \cos \phi$$

$$\int \partial \phi \sin^8 \phi = \frac{1}{1024} \sin 8\phi - \frac{1}{96} \sin 6\phi + \frac{7}{128} \sin 4\phi - \frac{7}{32} \sin 2\phi + \frac{55}{128} \phi$$

$$\int \partial \phi \sin^9 \phi = -\frac{1}{2304} \cos 9\phi + \frac{9}{1792} \cos 7\phi - \frac{9}{320} \cos 5\phi + \frac{7}{64} \cos 3\phi - \frac{63}{128} \cos \phi$$

$$\int \partial \varphi \cos^m \varphi$$

Taf. II.

$$\int \partial \varphi \cos \varphi = \sin \varphi$$

$$\int \partial \varphi \cos^2 \varphi = \frac{1}{3} \sin \varphi \cos \varphi + \frac{1}{3} \varphi = \frac{1}{12} \sin 2\varphi + \frac{1}{3} \varphi$$

$$\int \partial \varphi \cos^3 \varphi = \left( \frac{1}{5} \cos^2 \varphi + \frac{2}{5} \right) \sin \varphi$$

$$\int \partial \varphi \cos^4 \varphi = \left( \frac{1}{4} \cos^3 \varphi + \frac{5}{8} \cos \varphi \right) \sin \varphi + \frac{5}{8} \varphi$$

$$\int \partial \varphi \cos^5 \varphi = \left( \frac{1}{5} \cos^4 \varphi + \frac{4}{15} \cos^2 \varphi + \frac{6}{15} \right) \sin \varphi$$

$$\int \partial \varphi \cos^6 \varphi = \left( \frac{1}{6} \cos^5 \varphi + \frac{5}{24} \cos^3 \varphi + \frac{5}{16} \cos \varphi \right) \sin \varphi + \frac{5}{16} \varphi$$

$$\int \partial \varphi \cos^7 \varphi = \left( \frac{1}{7} \cos^6 \varphi + \frac{6}{35} \cos^4 \varphi + \frac{8}{35} \cos^2 \varphi + \frac{16}{35} \right) \sin \varphi$$

$$\int \partial \varphi \cos^8 \varphi = \left( \frac{1}{8} \cos^7 \varphi + \frac{7}{48} \cos^5 \varphi + \frac{35}{192} \cos^3 \varphi + \frac{35}{128} \cos \varphi \right) \sin \varphi + \frac{35}{128} \varphi$$

$$\int \partial \varphi \cos^9 \varphi = \left( \frac{1}{9} \cos^8 \varphi + \frac{8}{63} \cos^6 \varphi + \frac{16}{105} \cos^4 \varphi + \frac{64}{515} \cos^2 \varphi + \frac{128}{315} \right) \sin \varphi$$

$$\int \partial \varphi \cos \varphi = \sin \varphi$$

$$\int \partial \varphi \cos^2 \varphi = \frac{1}{4} \sin 2\varphi + \frac{1}{2} \varphi$$

$$\int \partial \varphi \cos^3 \varphi = \frac{1}{12} \sin 3\varphi + \frac{3}{4} \sin \varphi$$

$$\int \partial \varphi \cos^4 \varphi = \frac{1}{32} \sin 4\varphi + \frac{1}{4} \sin 2\varphi + \frac{5}{8} \varphi$$

$$\int \partial \varphi \cos^5 \varphi = \frac{1}{80} \sin 5\varphi + \frac{5}{48} \sin 3\varphi + \frac{5}{8} \sin \varphi$$

$$\int \partial \varphi \cos^6 \varphi = \frac{1}{192} \sin 6\varphi + \frac{5}{64} \sin 4\varphi + \frac{15}{64} \sin 2\varphi + \frac{5}{16} \varphi$$

$$\int \partial \varphi \cos^7 \varphi = \frac{1}{448} \sin 7\varphi + \frac{7}{320} \sin 5\varphi + \frac{7}{64} \sin 3\varphi + \frac{35}{64} \sin \varphi$$

$$\int \partial \varphi \cos^8 \varphi = \frac{1}{1024} \sin 8\varphi + \frac{1}{96} \sin 6\varphi + \frac{7}{128} \sin 4\varphi + \frac{7}{32} \sin 2\varphi + \frac{35}{128} \varphi$$

$$\int \partial \varphi \cos^9 \varphi = \frac{1}{2304} \sin 9\varphi + \frac{9}{1792} \sin 7\varphi + \frac{9}{320} \sin 5\varphi + \frac{7}{64} \sin 3\varphi + \frac{63}{128} \sin \varphi$$



Auf. III.

$$\int \sin \varphi \cos^n \varphi$$

$$\int \sin \varphi \cos^n \varphi = -\frac{1}{n+1} \cos^{n+1} \varphi$$

$$\cos \varphi = \cos \varphi$$

$$\cos^2 \varphi = \frac{1}{2} \cos 2\varphi + \frac{1}{2}$$

$$\cos^3 \varphi = \frac{1}{4} \cos 3\varphi + \frac{3}{4} \cos \varphi$$

$$\cos^4 \varphi = \frac{1}{8} \cos 4\varphi + \frac{1}{2} \cos 2\varphi + \frac{5}{8}$$

$$\cos^5 \varphi = \frac{1}{16} \cos 5\varphi + \frac{5}{16} \cos 3\varphi + \frac{5}{8} \cos \varphi$$

$$\cos^6 \varphi = \frac{1}{32} \cos 6\varphi + \frac{3}{16} \cos 4\varphi + \frac{15}{32} \cos 2\varphi + \frac{5}{16}$$

$$\cos^7 \varphi = \frac{1}{64} \cos 7\varphi + \frac{7}{64} \cos 5\varphi + \frac{21}{64} \cos 3\varphi + \frac{35}{64} \cos \varphi$$

$$\cos^8 \varphi = \frac{1}{128} \cos 8\varphi + \frac{1}{16} \cos 6\varphi + \frac{7}{32} \cos 4\varphi + \frac{7}{16} \cos 2\varphi + \frac{35}{128}$$

$$\cos^9 \varphi = \frac{1}{256} \cos 9\varphi + \frac{9}{256} \cos 7\varphi + \frac{9}{64} \cos 5\varphi + \frac{21}{64} \cos 3\varphi + \frac{63}{256} \cos \varphi$$

$$\cos^{10} \varphi = \frac{1}{512} \cos 10\varphi + \frac{5}{256} \cos 8\varphi + \frac{35}{512} \cos 6\varphi + \frac{21}{64} \cos 4\varphi + \frac{105}{256} \cos 2\varphi + \frac{63}{512}$$

$$\cos^2 \varphi = \frac{1}{2} - \frac{1}{2} \cos 2\varphi = \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi$$

Die Reihe an den Werten an wird fortgesetzt, bis man zu  $\cos^2 \varphi = \frac{1}{2} \cos^2 \pi - \frac{1}{2} \cos^2 \pi = \frac{1}{2}$  kommt.

$$\int \partial \varphi \cos \varphi \sin^n \varphi$$

Taf. IV.

$$\int \partial \varphi \cos \varphi \sin^n \varphi = \frac{1}{n+1} \sin^{n+1} \varphi$$

$$\sin \varphi = \sin \varphi$$

$$\sin^2 \varphi = -\frac{1}{2} \cos 2\varphi + \frac{1}{2}$$

$$\sin^3 \varphi = -\frac{1}{4} \sin 3\varphi + \frac{3}{4} \sin \varphi$$

$$\sin^4 \varphi = \frac{1}{8} \cos 4\varphi - \frac{1}{2} \cos 2\varphi + \frac{5}{8}$$

$$\sin^5 \varphi = \frac{1}{16} \sin 5\varphi - \frac{5}{16} \sin 3\varphi + \frac{5}{8} \sin \varphi$$

$$\sin^6 \varphi = -\frac{1}{32} \cos 6\varphi + \frac{5}{16} \cos 4\varphi - \frac{15}{32} \cos 2\varphi + \frac{5}{16}$$

$$\sin^7 \varphi = -\frac{1}{64} \sin 7\varphi + \frac{7}{64} \sin 5\varphi - \frac{21}{64} \sin 3\varphi + \frac{35}{64} \sin \varphi$$

$$\sin^8 \varphi = \frac{1}{128} \cos 8\varphi - \frac{1}{16} \cos 6\varphi + \frac{7}{32} \cos 4\varphi - \frac{7}{16} \cos 2\varphi + \frac{35}{128}$$

$$\sin^9 \varphi = \frac{1}{256} \sin 9\varphi - \frac{9}{256} \sin 7\varphi + \frac{9}{64} \sin 5\varphi - \frac{21}{64} \sin 3\varphi + \frac{63}{128} \sin \varphi$$

$$\sin^{10} \varphi = -\frac{1}{512} \cos 10\varphi + \frac{5}{256} \cos 8\varphi - \frac{45}{512} \cos 6\varphi + \frac{15}{64} \cos 4\varphi - \frac{105}{256} \cos 2\varphi + \frac{63}{256}$$

$$\sin^n \varphi = \pm \frac{1}{2^{n-1}} \left[ \cos n\varphi - {}^{\text{II}} \cos(n-2)\varphi + {}^{\text{III}} \cos(n-4)\varphi - {}^{\text{IV}} \cos(n-6)\varphi + \text{etc.} \right]$$

$$\sin^n \varphi = \pm \frac{1}{2^{n-1}} \left[ \sin n\varphi - {}^{\text{II}} \sin(n-2)\varphi + {}^{\text{III}} \sin(n-4)\varphi - {}^{\text{IV}} \sin(n-6)\varphi + \text{etc.} \right]$$

Die erste Reihe für  $\sin^n \varphi$  mit dem Vorzeichen +, wenn  $n$  von der Form  $4k$ , und mit dem Vorzeichen -, wenn  $n$  von der Form  $4k+2$  ist; die zweite Reihe mit dem Vorzeichen +, wenn  $n$  von der Form  $4k+1$ , und mit dem Vorzeichen -, wenn  $n$  von der Form  $4k+3$  ist. Beide Reihen werden so weit fortgesetzt, bis man zu negativen Winkeln kommt, und anstatt  $\cos \varphi$  nur  $\frac{1}{2} \cos \varphi = \frac{1}{2}$  gesetzt.

Taf. V.

$$\int \partial \phi \sin^2 \phi \cos^2 \phi$$

$$\int \partial \phi \sin^2 \phi \cos \phi = \frac{1}{3} \sin^3 \phi$$

$$\int \partial \phi \sin^2 \phi \cos^2 \phi = \frac{1}{4} \sin^3 \phi \cos \phi - \frac{1}{8} \sin \phi \cos^3 \phi + \frac{1}{8} \phi$$

$$\int \partial \phi \sin^2 \phi \cos^3 \phi = \left( \frac{1}{5} \cos^2 \phi + \frac{8}{15} \right) \sin^3 \phi$$

$$\int \partial \phi \sin^2 \phi \cos^4 \phi = \frac{1}{6} \sin^3 \phi \cos^3 \phi + \frac{1}{2} \int \partial \phi \sin^2 \phi \cos^2 \phi$$

$$\int \partial \phi \sin^2 \phi \cos^5 \phi = \left( \frac{1}{7} \cos^4 \phi + \frac{4}{55} \cos^2 \phi + \frac{8}{105} \right) \sin^3 \phi$$

$$\int \partial \phi \sin^3 \phi \cos \phi = -\frac{1}{4} \left( \frac{1}{5} \sin 3\phi - \sin \phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^2 \phi = -\frac{1}{8} \left( \frac{1}{4} \sin 4\phi - \phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^3 \phi = -\frac{1}{16} \left( \frac{1}{5} \sin 5\phi + \frac{1}{5} \sin 3\phi - 2 \sin \phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^4 \phi = -\frac{1}{32} \left( \frac{1}{6} \sin 6\phi + \frac{1}{2} \sin 4\phi - \frac{1}{2} \sin 2\phi - 2\phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^5 \phi = -\frac{1}{64} \left( \frac{1}{7} \sin 7\phi + \frac{5}{5} \sin 5\phi + \frac{1}{5} \sin 3\phi - 5 \sin \phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^6 \phi = -\frac{1}{128} \left( \frac{1}{8} \sin 8\phi + \frac{2}{5} \sin 6\phi + \sin 4\phi - 2 \sin 2\phi - 5\phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^7 \phi = -\frac{1}{256} \left( \frac{1}{9} \sin 9\phi + \frac{5}{7} \sin 7\phi + \frac{8}{5} \sin 5\phi - 14 \sin \phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^8 \phi = -\frac{1}{512} \left( \frac{1}{10} \sin 10\phi + \frac{5}{4} \sin 8\phi + \frac{15}{6} \sin 6\phi + 2 \sin 4\phi - 7 \sin 2\phi - 14\phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^9 \phi = -\frac{1}{1024} \left( \frac{1}{11} \sin 11\phi + \frac{7}{9} \sin 9\phi + \frac{19}{7} \sin 7\phi + \frac{21}{5} \sin 5\phi - 2 \sin 3\phi - 12 \sin \phi \right)$$

$$\int \partial \phi \sin^3 \phi \cos^{10} \phi = -\frac{1}{2048} \left( \frac{1}{12} \sin 12\phi + \frac{4}{5} \sin 10\phi + \frac{15}{4} \sin 8\phi + \frac{20}{3} \sin 6\phi + \frac{15}{4} \sin 4\phi - 24 \sin 2\phi - 12\phi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi$$

Taf. VI.

$$\int \partial \varphi \sin^3 \varphi \cos \varphi = \frac{1}{4} \sin^4 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^2 \varphi = \left( \frac{1}{5} \sin^4 \varphi - \frac{1}{15} \sin^2 \varphi - \frac{2}{15} \right) \cos \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi = \left( \frac{1}{6} \cos^2 \varphi + \frac{1}{12} \right) \sin^4 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^4 \varphi = \frac{1}{7} \sin^4 \varphi \cos^3 \varphi - \frac{3}{7} \int \partial \varphi \sin^3 \varphi \cos^2 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos \varphi = \frac{1}{8} \left( \frac{1}{4} \cos 4\varphi - \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^2 \varphi = \frac{1}{16} \left( \frac{1}{5} \cos 5\varphi - \frac{1}{5} \cos 3\varphi - 2 \cos \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi = \frac{1}{32} \left( \frac{1}{6} \cos 6\varphi - \frac{5}{2} \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^4 \varphi = \frac{1}{64} \left( \frac{1}{7} \cos 7\varphi + \frac{1}{5} \cos 5\varphi - \cos 3\varphi - 3 \cos \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^5 \varphi = \frac{1}{128} \left( \frac{1}{8} \cos 8\varphi + \frac{1}{8} \cos 6\varphi - \frac{1}{2} \cos 4\varphi - 3 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^6 \varphi = \frac{1}{256} \left( \frac{1}{9} \cos 9\varphi + \frac{5}{7} \cos 7\varphi - \frac{8}{5} \cos 5\varphi - 6 \cos 3\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^7 \varphi = \frac{1}{512} \left( \frac{1}{10} \cos 10\varphi + \frac{1}{2} \cos 8\varphi + \frac{1}{2} \cos 6\varphi - 2 \cos 4\varphi - 7 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^8 \varphi = \frac{1}{1024} \left( \frac{1}{11} \cos 11\varphi + \frac{5}{9} \cos 9\varphi + \cos 7\varphi - \cos 5\varphi - \frac{22}{5} \cos 3\varphi - 14 \cos \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^9 \varphi = \frac{1}{2048} \left( \frac{1}{12} \cos 12\varphi + \frac{3}{5} \cos 10\varphi + \frac{5}{2} \cos 8\varphi + \frac{1}{5} \cos 6\varphi - \frac{27}{4} \cos 4\varphi - 18 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^{10} \varphi = \frac{1}{4096} \left( \frac{1}{13} \cos 13\varphi + \frac{7}{11} \cos 11\varphi + 2 \cos 9\varphi + 2 \cos 7\varphi - 5 \cos 5\varphi - 21 \cos 3\varphi - 36 \cos \varphi \right)$$

Taf. VII.

$$\int \partial \varphi \sin^4 \varphi \cos^n \varphi$$

$$\int \partial \varphi \sin^4 \varphi \cos \varphi = \frac{1}{5} \sin^5 \varphi$$

$$\int \partial \varphi \sin^4 \varphi \cos^2 \varphi = \left( \frac{1}{6} \sin^5 \varphi - \frac{1}{24} \sin^3 \varphi - \frac{1}{16} \sin \varphi \right) \cos \varphi + \frac{1}{16} \varphi$$

$$\int \partial \varphi \sin^4 \varphi \cos^3 \varphi = \left( \frac{1}{7} \cos^2 \varphi + \frac{2}{55} \right) \sin^5 \varphi$$

$$\int \partial \varphi \sin^4 \varphi \cos \varphi = \frac{1}{16} \left( \frac{1}{5} \sin 5\varphi - \sin 3\varphi + 2 \sin \varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^2 \varphi = \frac{1}{52} \left( \frac{1}{6} \sin 6\varphi - \frac{1}{2} \sin 4\varphi - \frac{1}{2} \sin 2\varphi + 2\varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^3 \varphi = \frac{1}{64} \left( \frac{1}{7} \sin 7\varphi - \frac{1}{5} \sin 5\varphi - \sin 3\varphi + 3 \sin \varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^4 \varphi = \frac{1}{128} \left( \frac{1}{8} \sin 8\varphi - \sin 4\varphi + 3\varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^5 \varphi = \frac{1}{256} \left( \frac{1}{9} \sin 9\varphi + \frac{1}{7} \sin 7\varphi - \frac{4}{5} \sin 5\varphi - \frac{4}{5} \sin 3\varphi + 6 \sin \varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^6 \varphi = \frac{1}{512} \left( \frac{1}{10} \sin 10\varphi + \frac{1}{4} \sin 8\varphi - \frac{1}{2} \sin 6\varphi - 2 \sin 4\varphi + \sin 2\varphi + 6\varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^7 \varphi = \frac{1}{1024} \left( \frac{1}{11} \sin 11\varphi + \frac{1}{3} \sin 9\varphi - \frac{1}{7} \sin 7\varphi - \frac{11}{5} \sin 5\varphi - 2 \sin 3\varphi + 14 \sin \varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^8 \varphi = \frac{1}{2048} \left( \frac{1}{12} \sin 12\varphi + \frac{2}{5} \sin 10\varphi + \frac{1}{4} \sin 8\varphi - 2 \sin 6\varphi - \frac{17}{4} \sin 4\varphi + 4 \sin 2\varphi + 14\varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^9 \varphi = \frac{1}{4096} \left( \frac{1}{13} \sin 13\varphi + \frac{5}{11} \sin 11\varphi + \frac{2}{5} \sin 9\varphi - \frac{10}{7} \sin 7\varphi - \frac{29}{5} \sin 5\varphi - 3 \sin 3\varphi + 36 \sin \varphi \right)$$

$$\int \partial \varphi \sin^4 \varphi \cos^{10} \varphi = \frac{1}{8192} \left( \frac{1}{14} \sin 14\varphi + \frac{1}{2} \sin 12\varphi + \frac{11}{10} \sin 10\varphi - \frac{1}{2} \sin 8\varphi - \frac{15}{2} \sin 6\varphi - \frac{19}{2} \sin 4\varphi + \frac{27}{2} \sin 2\varphi + 36\varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^2 \varphi$$

Taf. VIII.

$$\int d\varphi \sin^5 \varphi \cos \varphi = \frac{1}{6} \sin^6 \varphi$$

$$\int d\varphi \sin^5 \varphi \cos^2 \varphi = \frac{1}{7} \sin^6 \varphi \cos \varphi + \frac{1}{7} \int d\varphi \sin^5 \varphi$$

$$\int d\varphi \sin^5 \varphi \cos^2 \varphi = \left( \frac{1}{8} \cos^2 \varphi + \frac{1}{24} \right) \sin^6 \varphi$$

$$\int d\varphi \sin^5 \varphi \cos \varphi = -\frac{1}{32} \left( \frac{1}{6} \cos 6\varphi - \cos 4\varphi + \frac{5}{2} \cos 2\varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^2 \varphi = -\frac{1}{64} \left( \frac{1}{7} \cos 7\varphi - \frac{5}{5} \cos 5\varphi + \frac{1}{3} \cos 3\varphi + 5 \cos \varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^3 \varphi = -\frac{1}{128} \left( \frac{1}{8} \cos 8\varphi - \frac{1}{3} \cos 6\varphi - \frac{1}{2} \cos 4\varphi + 3 \cos 2\varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^4 \varphi = -\frac{1}{256} \left( \frac{1}{9} \cos 9\varphi - \frac{1}{7} \cos 7\varphi - \frac{4}{5} \cos 5\varphi + \frac{4}{3} \cos 3\varphi + 6 \cos \varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^5 \varphi = -\frac{1}{512} \left( \frac{1}{10} \cos 10\varphi - \frac{5}{6} \cos 6\varphi + 5 \cos 2\varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^6 \varphi = -\frac{1}{1024} \left( \frac{1}{11} \cos 11\varphi + \frac{1}{9} \cos 9\varphi - \frac{5}{7} \cos 7\varphi - \cos 5\varphi + \frac{10}{3} \cos 3\varphi + 10 \cos \varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^7 \varphi = -\frac{1}{2048} \left( \frac{1}{12} \cos 12\varphi + \frac{1}{5} \cos 10\varphi - \frac{1}{2} \cos 8\varphi - \frac{5}{8} \cos 6\varphi + \frac{5}{4} \cos 4\varphi + 10 \cos 2\varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^8 \varphi = -\frac{1}{4096} \left( \frac{1}{13} \cos 13\varphi + \frac{5}{11} \cos 11\varphi - \frac{2}{9} \cos 9\varphi - 2 \cos 7\varphi - \cos 5\varphi + \frac{25}{3} \cos 3\varphi + 20 \cos \varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^9 \varphi = -\frac{1}{8192} \left( \frac{1}{14} \cos 14\varphi + \frac{1}{5} \cos 12\varphi + \frac{1}{10} \cos 10\varphi - 2 \cos 8\varphi - \frac{19}{6} \cos 6\varphi + 5 \cos 4\varphi + \frac{45}{2} \cos 2\varphi \right)$$

$$\int d\varphi \sin^5 \varphi \cos^{10} \varphi = -\frac{1}{16384} \left( \frac{1}{15} \cos 15\varphi + \frac{5}{13} \cos 13\varphi + \frac{5}{11} \cos 11\varphi - \frac{5}{3} \cos 9\varphi - 5 \cos 7\varphi + \frac{1}{5} \cos 5\varphi + \frac{65}{3} \cos 3\varphi + 45 \cos \varphi \right)$$

Taf. IX.

$$\int \partial \varphi \sin^6 \varphi \cos^2 \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos \varphi = \frac{1}{7} \sin^7 \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos^2 \varphi = \left( \frac{1}{8} \sin^7 \varphi - \frac{1}{48} \sin^5 \varphi - \frac{5}{192} \sin^3 \varphi - \frac{5}{128} \sin \varphi \right) \cos \varphi + \frac{5}{128} \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos^3 \varphi = \left( \frac{1}{9} \cos^2 \varphi + \frac{2}{63} \right) \sin^7 \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos \varphi = -\frac{1}{64} \left( \frac{1}{7} \sin 7\varphi - \sin 5\varphi + 3 \sin 3\varphi - 5 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^2 \varphi = -\frac{1}{128} \left( \frac{1}{8} \sin 8\varphi - \frac{2}{3} \sin 6\varphi + \sin 4\varphi + 2 \sin 2\varphi - 5\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^3 \varphi = -\frac{1}{256} \left( \frac{1}{9} \sin 9\varphi - \frac{2}{7} \sin 7\varphi + \frac{8}{3} \sin 5\varphi - 6 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^4 \varphi = -\frac{1}{512} \left( \frac{1}{10} \sin 10\varphi - \frac{1}{4} \sin 8\varphi - \frac{1}{2} \sin 6\varphi + 2 \sin 4\varphi + \sin 2\varphi - 6\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^5 \varphi = -\frac{1}{1024} \left( \frac{1}{11} \sin 11\varphi - \frac{1}{9} \sin 9\varphi - \frac{5}{7} \sin 7\varphi + \sin 5\varphi + \frac{10}{3} \sin 3\varphi - 10 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^6 \varphi = -\frac{1}{2048} \left( \frac{1}{12} \sin 12\varphi - \frac{3}{4} \sin 8\varphi + \frac{15}{4} \sin 4\varphi - 10\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^7 \varphi = -\frac{1}{4096} \left( \frac{1}{13} \sin 13\varphi + \frac{1}{11} \sin 11\varphi - \frac{2}{3} \sin 9\varphi - \frac{6}{7} \sin 7\varphi + 3 \sin 5\varphi + 5 \sin 3\varphi - 20 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^8 \varphi = -\frac{1}{8192} \left( \frac{1}{14} \sin 14\varphi + \frac{1}{6} \sin 12\varphi - \frac{1}{2} \sin 10\varphi - \frac{3}{2} \sin 8\varphi + \frac{3}{2} \sin 6\varphi + \frac{15}{2} \sin 4\varphi - \frac{5}{2} \sin 2\varphi - 20\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^9 \varphi = -\frac{1}{16384} \left( \frac{1}{15} \sin 15\varphi + \frac{3}{13} \sin 13\varphi - \frac{5}{11} \sin 11\varphi - \frac{17}{9} \sin 9\varphi - \frac{3}{7} \sin 7\varphi + \frac{39}{5} \sin 5\varphi + \frac{25}{3} \sin 3\varphi - 45 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^{10} \varphi = -\frac{1}{32768} \left( \frac{1}{16} \sin 16\varphi + \frac{2}{7} \sin 14\varphi - 2 \sin 12\varphi - \frac{5}{2} \sin 8\varphi + 6 \sin 6\varphi + 16 \sin 4\varphi - 10 \sin 2\varphi - 45\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^m \varphi$$

Taf. X.

$$\int \partial \varphi \sin^7 \varphi \cos \varphi = \frac{1}{8} \sin^8 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos^2 \varphi = \frac{1}{9} \sin^8 \varphi \cos \varphi + \frac{1}{9} \int \partial \varphi \sin^7 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos^3 \varphi = \left( \frac{1}{10} \cos^2 \varphi + \frac{1}{40} \right) \sin^8 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos^4 \varphi = \left( \frac{1}{11} \cos^3 \varphi + \frac{1}{33} \cos \varphi \right) \sin^8 \varphi + \frac{1}{33} \int \partial \varphi \sin^7 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos \varphi = \frac{1}{128} \left( \frac{1}{8} \cos 8\varphi - \cos 6\varphi + \frac{7}{2} \cos 4\varphi - 7 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^2 \varphi = \frac{1}{256} \left( \frac{1}{9} \cos 9\varphi - \frac{5}{7} \cos 7\varphi + \frac{8}{5} \cos 5\varphi - 14 \cos 3\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^3 \varphi = \frac{1}{512} \left( \frac{1}{10} \cos 10\varphi - \frac{1}{2} \cos 8\varphi + \frac{1}{2} \cos 6\varphi + 2 \cos 4\varphi - 7 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^4 \varphi = \frac{1}{1024} \left( \frac{1}{11} \cos 11\varphi - \frac{1}{8} \cos 9\varphi - \frac{1}{7} \cos 7\varphi + \frac{11}{5} \cos 5\varphi - 2 \cos 3\varphi - 14 \cos \varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^5 \varphi = \frac{1}{2048} \left( \frac{1}{12} \cos 12\varphi - \frac{1}{5} \cos 10\varphi - \frac{1}{2} \cos 8\varphi + \frac{5}{5} \cos 6\varphi + \frac{5}{4} \cos 4\varphi - 10 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^6 \varphi = \frac{1}{4096} \left( \frac{1}{15} \cos 13\varphi - \frac{1}{11} \cos 11\varphi - \frac{2}{5} \cos 9\varphi + \frac{6}{7} \cos 7\varphi + 3 \cos 5\varphi - 5 \cos 3\varphi - 20 \cos \varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^7 \varphi = \frac{1}{8192} \left( \frac{1}{14} \cos 14\varphi - \frac{7}{10} \cos 10\varphi + \frac{7}{2} \cos 6\varphi - \frac{55}{2} \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^8 \varphi = \frac{1}{16384} \left( \frac{1}{15} \cos 15\varphi + \frac{1}{15} \cos 13\varphi - \frac{7}{11} \cos 11\varphi - \frac{7}{9} \cos 9\varphi + 3 \cos 7\varphi + \frac{21}{5} \cos 5\varphi - \frac{35}{5} \cos 3\varphi - 35 \cos \varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^9 \varphi = \frac{1}{32768} \left( \frac{1}{16} \cos 16\varphi + \frac{1}{7} \cos 14\varphi - \frac{1}{2} \cos 12\varphi - \frac{7}{5} \cos 10\varphi + \frac{7}{4} \cos 8\varphi + 7 \cos 6\varphi - \frac{7}{2} \cos 4\varphi - 35 \cos 2\varphi \right)$$



Taf. XI.

$$\int \partial \varphi \sin^3 \varphi \cos^* \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos \varphi = \frac{1}{9} \sin^9 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^2 \varphi = \frac{1}{10} \sin^9 \varphi \cos \varphi + \frac{1}{10} \int \partial \varphi \sin^3 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi = \left( \frac{1}{11} \cos^2 \varphi + \frac{2}{99} \right) \sin^9 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos \varphi = \frac{1}{256} \left( \frac{1}{9} \sin 9\varphi - \sin 7\varphi + 4 \sin 5\varphi - \frac{28}{5} \sin 3\varphi + 14 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^2 \varphi = \frac{1}{512} \left( \frac{1}{10} \sin 10\varphi - \frac{5}{4} \sin 8\varphi + \frac{15}{6} \sin 6\varphi - 2 \sin 4\varphi - 7 \sin 2\varphi + 14\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi = \frac{1}{1024} \left( \frac{1}{11} \sin 11\varphi - \frac{5}{9} \sin 9\varphi + \sin 7\varphi + \sin 5\varphi - \frac{22}{3} \sin 3\varphi + 14 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^4 \varphi = \frac{1}{2048} \left( \frac{1}{12} \sin 12\varphi - \frac{2}{6} \sin 10\varphi + \frac{1}{4} \sin 8\varphi + 2 \sin 6\varphi - \frac{17}{4} \sin 4\varphi - 4 \sin 2\varphi + 14\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^5 \varphi = \frac{1}{4096} \left( \frac{1}{13} \sin 13\varphi - \frac{3}{11} \sin 11\varphi - \frac{2}{9} \sin 9\varphi + 2 \sin 7\varphi - \sin 5\varphi - \frac{25}{3} \sin 3\varphi + 20 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^6 \varphi = \frac{1}{8192} \left( \frac{1}{14} \sin 14\varphi - \frac{1}{6} \sin 12\varphi - \frac{1}{2} \sin 10\varphi + \frac{3}{2} \sin 8\varphi + \frac{3}{2} \sin 6\varphi + \frac{15}{2} \sin 4\varphi - \frac{5}{2} \sin 2\varphi + 20\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^7 \varphi = \frac{1}{16384} \left( \frac{1}{15} \sin 15\varphi - \frac{1}{13} \sin 13\varphi - \frac{7}{11} \sin 11\varphi + \frac{7}{9} \sin 9\varphi + 3 \sin 7\varphi - \frac{21}{5} \sin 5\varphi - \frac{55}{5} \sin 3\varphi + 35 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^8 \varphi = \frac{1}{32768} \left( \frac{1}{16} \sin 16\varphi - \frac{2}{5} \sin 12\varphi + \frac{7}{2} \sin 8\varphi - 14 \sin 4\varphi + 35\varphi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^9 \phi$$

Taf. XII.

$$\int \partial \phi \sin^9 \phi \cos \phi = \frac{1}{10} \sin^{10} \phi$$

$$\int \partial \phi \sin^9 \phi \cos^3 \phi = \frac{1}{12} \sin^{10} \phi \cos \phi + \frac{1}{12} \int \partial \phi \sin^9 \phi$$

$$\int \partial \phi \sin^9 \phi \cos^5 \phi = \left( \frac{1}{12} \cos^2 \phi + \frac{1}{60} \right) \sin^{10} \phi$$

$$\int \partial \phi \sin^9 \phi \cos \phi = -\frac{1}{512} \left( \frac{1}{10} \cos 10\phi - \cos 8\phi + \frac{9}{2} \cos 6\phi - 12 \cos 4\phi + 21 \cos 2\phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^3 \phi = -\frac{1}{1024} \left( \frac{1}{12} \cos 12\phi - \frac{7}{9} \cos 10\phi + \frac{19}{7} \cos 8\phi - \frac{21}{5} \cos 6\phi - 2 \cos 4\phi + 42 \cos 2\phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^5 \phi = -\frac{1}{2048} \left( \frac{1}{12} \cos 12\phi - \frac{5}{5} \cos 10\phi + \frac{5}{2} \cos 8\phi - \frac{1}{3} \cos 6\phi - \frac{27}{4} \cos 4\phi + 18 \cos 2\phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^7 \phi = -\frac{1}{4096} \left( \frac{1}{12} \cos 12\phi - \frac{5}{11} \cos 10\phi + \frac{2}{3} \cos 8\phi + \frac{10}{7} \cos 6\phi - \frac{29}{5} \cos 4\phi + 3 \cos 2\phi + 36 \cos \phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^9 \phi = -\frac{1}{8192} \left( \frac{1}{14} \cos 14\phi - \frac{1}{5} \cos 12\phi + \frac{1}{10} \cos 10\phi + 2 \cos 8\phi - \frac{19}{6} \cos 6\phi - 5 \cos 4\phi + \frac{45}{2} \cos 2\phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^7 \phi = -\frac{1}{16384} \left( \frac{1}{15} \cos 15\phi - \frac{3}{13} \cos 13\phi - \frac{3}{11} \cos 11\phi + \frac{17}{9} \cos 9\phi - \frac{3}{7} \cos 7\phi - \frac{59}{5} \cos 5\phi + \frac{25}{3} \cos 3\phi + 45 \cos \phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^5 \phi = -\frac{1}{32768} \left( \frac{1}{16} \cos 16\phi - \frac{1}{7} \cos 14\phi - \frac{1}{2} \cos 12\phi + \frac{7}{5} \cos 10\phi + \frac{7}{4} \cos 8\phi - 7 \cos 6\phi - \frac{7}{2} \cos 4\phi + 35 \cos 2\phi \right)$$

$$\int \partial \phi \sin^9 \phi \cos^3 \phi = -\frac{1}{65536} \left( \frac{1}{17} \cos 17\phi - \frac{1}{15} \cos 15\phi - \frac{3}{13} \cos 13\phi + \frac{8}{11} \cos 11\phi + \frac{28}{9} \cos 9\phi - 4 \cos 7\phi - \frac{56}{5} \cos 5\phi + \frac{56}{3} \cos 3\phi + 70 \cos \phi \right)$$

Taf. XIII.

$$\int \frac{\partial \phi}{\sin^m \phi}, \quad \int \frac{\partial \phi}{\cos^m \phi}$$

$$\int \frac{\partial \phi}{\sin \phi} = \log \operatorname{Tang} \frac{\phi}{2}$$

$$\int \frac{\partial \phi}{\sin^2 \phi} = -\frac{\cos \phi}{\sin \phi} = -\operatorname{Cot} \phi$$

$$\int \frac{\partial \phi}{\sin^3 \phi} = -\frac{\cos \phi}{2 \sin^2 \phi} + \frac{1}{2} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin^4 \phi} = \left( -\frac{1}{3 \sin^3 \phi} - \frac{2}{3 \sin \phi} \right) \cos \phi = \operatorname{Cot} \phi - \frac{1}{5} \operatorname{Cot}^3 \phi$$

$$\int \frac{\partial \phi}{\sin^5 \phi} = \left( -\frac{1}{4 \sin^4 \phi} - \frac{3}{8 \sin^2 \phi} \right) \cos \phi + \frac{5}{8} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin^6 \phi} = \left( -\frac{1}{5 \sin^5 \phi} - \frac{4}{15 \sin^3 \phi} - \frac{8}{15 \sin \phi} \right) \cos \phi$$

$$\int \frac{\partial \phi}{\sin^7 \phi} = \left( -\frac{1}{6 \sin^6 \phi} - \frac{5}{24 \sin^4 \phi} - \frac{5}{16 \sin^2 \phi} \right) \cos \phi + \frac{5}{16} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin^8 \phi} = \left( -\frac{1}{7 \sin^7 \phi} - \frac{6}{35 \sin^5 \phi} - \frac{8}{35 \sin^3 \phi} - \frac{16}{35 \sin \phi} \right) \cos \phi$$

$$\int \frac{\partial \phi}{\cos \phi} = \log \operatorname{Tang} \left( 45^\circ + \frac{\phi}{2} \right)$$

$$\int \frac{\partial \phi}{\cos^2 \phi} = \frac{\sin \phi}{\cos \phi} = \operatorname{Tang} \phi$$

$$\int \frac{\partial \phi}{\cos^3 \phi} = \frac{\sin \phi}{2 \cos^2 \phi} + \frac{1}{2} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\cos^4 \phi} = \left( \frac{1}{3 \cos^3 \phi} + \frac{2}{3 \cos \phi} \right) \sin \phi = \operatorname{Tang} \phi + \frac{1}{3} \operatorname{Tang}^3 \phi$$

$$\int \frac{\partial \phi}{\cos^5 \phi} = \left( \frac{1}{4 \cos^4 \phi} + \frac{3}{8 \cos^2 \phi} \right) \sin \phi + \frac{3}{8} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\cos^6 \phi} = \left( \frac{1}{5 \cos^5 \phi} + \frac{4}{15 \cos^3 \phi} + \frac{8}{15 \cos \phi} \right) \sin \phi$$

$$\int \frac{\partial \phi}{\cos^7 \phi} = \left( \frac{1}{6 \cos^6 \phi} + \frac{5}{24 \cos^4 \phi} + \frac{5}{16 \cos^2 \phi} \right) \sin \phi + \frac{5}{16} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\cos^8 \phi} = \left( \frac{1}{7 \cos^7 \phi} + \frac{6}{35 \cos^5 \phi} + \frac{8}{35 \cos^3 \phi} + \frac{16}{35 \cos \phi} \right) \sin \phi$$

$$\int \frac{\partial \phi \sin^n \phi}{\cos \phi}, \quad \int \frac{\partial \phi \cos^n \phi}{\sin \phi}$$

Taf. XIV.

$$\int \frac{\partial \phi \sin \phi}{\cos \phi} = -\log \cos \phi = \log \sec \phi$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos \phi} = -\sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos \phi} = -\frac{\sin^2 \phi}{2} + \int \frac{\partial \phi \sin \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos \phi} = -\frac{\sin^3 \phi}{3} - \sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos \phi} = -\frac{\sin^4 \phi}{4} - \frac{\sin^2 \phi}{2} + \int \frac{\partial \phi \sin \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos \phi} = -\frac{\sin^5 \phi}{5} - \frac{\sin^3 \phi}{3} - \sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos \phi} = -\frac{\sin^6 \phi}{6} - \frac{\sin^4 \phi}{4} - \frac{\sin^2 \phi}{2} + \int \frac{\partial \phi \sin \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos \phi} = -\frac{\sin^7 \phi}{7} - \frac{\sin^5 \phi}{5} - \frac{\sin^3 \phi}{3} - \sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \cos \phi}{\sin \phi} = \log \sin \phi$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin \phi} = \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin \phi} = \frac{\cos^2 \phi}{2} + \int \frac{\partial \phi \cos \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin \phi} = \frac{\cos^3 \phi}{3} + \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin \phi} = \frac{\cos^4 \phi}{4} + \frac{\cos^2 \phi}{2} + \int \frac{\partial \phi \cos \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin \phi} = \frac{\cos^5 \phi}{5} + \frac{\cos^3 \phi}{3} + \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin \phi} = \frac{\cos^6 \phi}{6} + \frac{\cos^4 \phi}{4} + \frac{\cos^2 \phi}{2} + \int \frac{\partial \phi \cos \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin \phi} = \frac{\cos^7 \phi}{7} + \frac{\cos^5 \phi}{5} + \frac{\cos^3 \phi}{3} + \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

Taf. VII.

$$\int \partial \phi \sin^4 \phi \cos^m \phi$$


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$$\int \partial \phi \sin^4 \phi \cos \phi = \frac{1}{5} \sin^5 \phi$$

$$\int \partial \phi \sin^4 \phi \cos^2 \phi = \left( \frac{1}{6} \sin^5 \phi - \frac{1}{24} \sin^3 \phi - \frac{1}{16} \sin \phi \right) \cos \phi + \frac{1}{16} \phi$$

$$\int \partial \phi \sin^4 \phi \cos^3 \phi = \left( \frac{1}{7} \cos^2 \phi + \frac{2}{55} \right) \sin^5 \phi$$


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$$\int \partial \phi \sin^4 \phi \cos \phi = \frac{1}{16} \left( \frac{1}{5} \sin 5\phi - \sin 3\phi + 2 \sin \phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^2 \phi = \frac{1}{52} \left( \frac{1}{6} \sin 6\phi - \frac{1}{2} \sin 4\phi - \frac{1}{2} \sin 2\phi + 2\phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^3 \phi = \frac{1}{64} \left( \frac{1}{7} \sin 7\phi - \frac{1}{5} \sin 5\phi - \sin 3\phi + 3 \sin \phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^4 \phi = \frac{1}{128} \left( \frac{1}{8} \sin 8\phi - \sin 4\phi + 3\phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^5 \phi = \frac{1}{256} \left( \frac{1}{9} \sin 9\phi + \frac{1}{7} \sin 7\phi - \frac{4}{5} \sin 5\phi - \frac{4}{5} \sin 3\phi + 6 \sin \phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^6 \phi = \frac{1}{512} \left( \frac{1}{10} \sin 10\phi + \frac{1}{4} \sin 8\phi - \frac{3}{2} \sin 6\phi - 2 \sin 4\phi + \sin 2\phi + 6\phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^7 \phi = \frac{1}{1024} \left( \frac{1}{11} \sin 11\phi + \frac{1}{5} \sin 9\phi - \frac{1}{7} \sin 7\phi - \frac{11}{5} \sin 5\phi - 2 \sin 3\phi + 14 \sin \phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^8 \phi = \frac{1}{2048} \left( \frac{1}{12} \sin 12\phi + \frac{2}{5} \sin 10\phi + \frac{1}{4} \sin 8\phi - 2 \sin 6\phi - \frac{17}{4} \sin 4\phi + 4 \sin 2\phi + 14\phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^9 \phi = \frac{1}{4096} \left( \frac{1}{13} \sin 13\phi + \frac{5}{11} \sin 11\phi + \frac{2}{3} \sin 9\phi - \frac{10}{7} \sin 7\phi - \frac{29}{5} \sin 5\phi - 3 \sin 3\phi + 36 \sin \phi \right)$$

$$\int \partial \phi \sin^4 \phi \cos^{10} \phi = \frac{1}{8192} \left( \frac{1}{14} \sin 14\phi + \frac{1}{2} \sin 12\phi + \frac{11}{10} \sin 10\phi - \frac{1}{2} \sin 8\phi - \frac{15}{2} \sin 6\phi - \frac{19}{2} \sin 4\phi + \frac{27}{2} \sin 2\phi + 36\phi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^m \varphi$$

Taf. VIII.

$$\int \partial \varphi \sin^5 \varphi \cos \varphi = \frac{1}{6} \sin^6 \varphi$$

$$\int \partial \varphi \sin^5 \varphi \cos^2 \varphi = \frac{1}{7} \sin^6 \varphi \cos \varphi + \frac{1}{7} \int \partial \varphi \sin^5 \varphi$$

$$\int \partial \varphi \sin^5 \varphi \cos^3 \varphi = \left( \frac{1}{8} \cos^2 \varphi + \frac{1}{24} \right) \sin^6 \varphi$$

$$\int \partial \varphi \sin^5 \varphi \cos \varphi = -\frac{1}{32} \left( \frac{1}{6} \cos 6\varphi - \cos 4\varphi + \frac{5}{2} \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^2 \varphi = -\frac{1}{64} \left( \frac{1}{7} \cos 7\varphi - \frac{5}{5} \cos 5\varphi + \frac{1}{3} \cos 3\varphi + 5 \cos \varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^3 \varphi = -\frac{1}{128} \left( \frac{1}{8} \cos 8\varphi - \frac{1}{3} \cos 6\varphi - \frac{1}{2} \cos 4\varphi + 3 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^4 \varphi = -\frac{1}{256} \left( \frac{1}{9} \cos 9\varphi - \frac{1}{7} \cos 7\varphi - \frac{4}{5} \cos 5\varphi + \frac{4}{3} \cos 3\varphi + 6 \cos \varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^5 \varphi = -\frac{1}{512} \left( \frac{1}{10} \cos 10\varphi - \frac{5}{6} \cos 6\varphi + 5 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^6 \varphi = -\frac{1}{1024} \left( \frac{1}{11} \cos 11\varphi + \frac{1}{9} \cos 9\varphi - \frac{5}{7} \cos 7\varphi - \cos 5\varphi + \frac{10}{5} \cos 3\varphi + 10 \cos \varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^7 \varphi = -\frac{1}{2048} \left( \frac{1}{12} \cos 12\varphi + \frac{1}{5} \cos 10\varphi - \frac{1}{2} \cos 8\varphi - \frac{5}{8} \cos 6\varphi + \frac{5}{4} \cos 4\varphi + 10 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^8 \varphi = -\frac{1}{4096} \left( \frac{1}{13} \cos 13\varphi + \frac{5}{11} \cos 11\varphi - \frac{2}{9} \cos 9\varphi - 2 \cos 7\varphi - \cos 5\varphi + \frac{25}{3} \cos 3\varphi + 20 \cos \varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^9 \varphi = -\frac{1}{8192} \left( \frac{1}{14} \cos 14\varphi + \frac{1}{5} \cos 12\varphi + \frac{1}{10} \cos 10\varphi - 2 \cos 8\varphi - \frac{19}{6} \cos 6\varphi + 5 \cos 4\varphi + \frac{45}{2} \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^5 \varphi \cos^{10} \varphi = -\frac{1}{16384} \left( \frac{1}{15} \cos 15\varphi + \frac{5}{13} \cos 13\varphi + \frac{5}{11} \cos 11\varphi - \frac{5}{3} \cos 9\varphi - 5 \cos 7\varphi + \frac{1}{5} \cos 5\varphi + \frac{65}{5} \cos 3\varphi + 45 \cos \varphi \right)$$

Taf. IX.

$$\int \partial \varphi \sin^6 \varphi \cos^0 \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos \varphi = \frac{1}{7} \sin^7 \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos^2 \varphi = \left( \frac{1}{8} \sin^7 \varphi - \frac{1}{48} \sin^5 \varphi - \frac{5}{192} \sin^3 \varphi - \frac{5}{128} \sin \varphi \right) \cos \varphi + \frac{5}{128} \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos^3 \varphi = \left( \frac{1}{9} \cos^2 \varphi + \frac{2}{63} \right) \sin^7 \varphi$$

$$\int \partial \varphi \sin^6 \varphi \cos \varphi = -\frac{1}{64} \left( \frac{1}{7} \sin 7\varphi - \sin 5\varphi + 3 \sin 3\varphi - 5 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^2 \varphi = -\frac{1}{128} \left( \frac{1}{8} \sin 8\varphi - \frac{2}{3} \sin 6\varphi + \sin 4\varphi + 2 \sin 2\varphi - 5\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^3 \varphi = -\frac{1}{256} \left( \frac{1}{9} \sin 9\varphi - \frac{5}{7} \sin 7\varphi + \frac{8}{3} \sin 3\varphi - 6 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^4 \varphi = -\frac{1}{512} \left( \frac{1}{10} \sin 10\varphi - \frac{1}{4} \sin 8\varphi - \frac{1}{2} \sin 6\varphi + 2 \sin 4\varphi + \sin 2\varphi - 6\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^5 \varphi = -\frac{1}{1024} \left( \frac{1}{11} \sin 11\varphi - \frac{1}{9} \sin 9\varphi - \frac{5}{7} \sin 7\varphi + \sin 5\varphi + \frac{10}{3} \sin 3\varphi - 10 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^6 \varphi = -\frac{1}{2048} \left( \frac{1}{12} \sin 12\varphi - \frac{5}{4} \sin 8\varphi + \frac{15}{4} \sin 4\varphi - 10\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^7 \varphi = -\frac{1}{4096} \left( \frac{1}{13} \sin 13\varphi + \frac{1}{11} \sin 11\varphi - \frac{2}{5} \sin 9\varphi - \frac{6}{7} \sin 7\varphi + 3 \sin 5\varphi + 5 \sin 3\varphi - 20 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^8 \varphi = -\frac{1}{8192} \left( \frac{1}{14} \sin 14\varphi + \frac{1}{6} \sin 12\varphi - \frac{1}{2} \sin 10\varphi - \frac{5}{2} \sin 8\varphi + \frac{3}{2} \sin 6\varphi + \frac{15}{2} \sin 4\varphi - \frac{5}{2} \sin 2\varphi - 20\varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^9 \varphi = -\frac{1}{16384} \left( \frac{1}{15} \sin 15\varphi + \frac{3}{13} \sin 13\varphi - \frac{5}{11} \sin 11\varphi - \frac{17}{9} \sin 9\varphi - \frac{3}{7} \sin 7\varphi + \frac{39}{5} \sin 5\varphi + \frac{25}{3} \sin 3\varphi - 45 \sin \varphi \right)$$

$$\int \partial \varphi \sin^6 \varphi \cos^{10} \varphi = -\frac{1}{32768} \left( \frac{1}{16} \sin 16\varphi + \frac{2}{7} \sin 14\varphi - 2 \sin 12\varphi - \frac{5}{2} \sin 8\varphi + 6 \sin 6\varphi + 16 \sin 4\varphi - 10 \sin 2\varphi - 45\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^2 \varphi$$

Taf. X.

$$\int \partial \varphi \sin^7 \varphi \cos \varphi = \frac{1}{8} \sin^8 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos^2 \varphi = \frac{1}{9} \sin^8 \varphi \cos \varphi + \frac{1}{9} \int \partial \varphi \sin^7 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos^3 \varphi = \left( \frac{1}{10} \cos^2 \varphi + \frac{1}{40} \right) \sin^8 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos^4 \varphi = \left( \frac{1}{11} \cos^3 \varphi + \frac{1}{33} \cos \varphi \right) \sin^8 \varphi + \frac{1}{33} \int \partial \varphi \sin^7 \varphi$$

$$\int \partial \varphi \sin^7 \varphi \cos \varphi = \frac{1}{128} \left( \frac{1}{8} \cos 8\varphi - \cos 6\varphi + \frac{7}{2} \cos 4\varphi - 7 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^2 \varphi = \frac{1}{256} \left( \frac{1}{9} \cos 9\varphi - \frac{6}{7} \cos 7\varphi + \frac{8}{5} \cos 5\varphi - 14 \cos 3\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^3 \varphi = \frac{1}{512} \left( \frac{1}{10} \cos 10\varphi - \frac{1}{2} \cos 8\varphi + \frac{1}{2} \cos 6\varphi + 2 \cos 4\varphi - 7 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^4 \varphi = \frac{1}{1024} \left( \frac{1}{11} \cos 11\varphi - \frac{1}{5} \cos 9\varphi - \frac{1}{7} \cos 7\varphi + \frac{11}{5} \cos 5\varphi - 2 \cos 3\varphi - 14 \cos \varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^5 \varphi = \frac{1}{2048} \left( \frac{1}{12} \cos 12\varphi - \frac{1}{5} \cos 10\varphi - \frac{1}{2} \cos 8\varphi + \frac{5}{2} \cos 6\varphi + \frac{5}{4} \cos 4\varphi - 10 \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^6 \varphi = \frac{1}{4096} \left( \frac{1}{15} \cos 13\varphi - \frac{1}{11} \cos 11\varphi - \frac{2}{5} \cos 9\varphi + \frac{6}{7} \cos 7\varphi + 3 \cos 5\varphi - 5 \cos 3\varphi - 20 \cos \varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^7 \varphi = \frac{1}{8192} \left( \frac{1}{14} \cos 14\varphi - \frac{7}{10} \cos 10\varphi + \frac{7}{2} \cos 6\varphi - \frac{55}{2} \cos 2\varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^8 \varphi = \frac{1}{16384} \left( \frac{1}{15} \cos 15\varphi + \frac{1}{13} \cos 13\varphi - \frac{7}{11} \cos 11\varphi - \frac{7}{9} \cos 9\varphi + 3 \cos 7\varphi + \frac{21}{5} \cos 5\varphi - \frac{35}{3} \cos 3\varphi - 35 \cos \varphi \right)$$

$$\int \partial \varphi \sin^7 \varphi \cos^9 \varphi = \frac{1}{32768} \left( \frac{1}{16} \cos 16\varphi + \frac{1}{7} \cos 14\varphi - \frac{1}{2} \cos 12\varphi - \frac{7}{5} \cos 10\varphi + \frac{7}{4} \cos 8\varphi + 7 \cos 6\varphi - \frac{7}{2} \cos 4\varphi - 35 \cos 2\varphi \right)$$



Taf. XI.

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos \varphi = \frac{1}{9} \sin^3 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^2 \varphi = \frac{1}{10} \sin^3 \varphi \cos \varphi + \frac{1}{10} \int \partial \varphi \sin^3 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi = \left( \frac{1}{11} \cos^2 \varphi + \frac{2}{99} \right) \sin^3 \varphi$$

$$\int \partial \varphi \sin^3 \varphi \cos \varphi = \frac{1}{256} \left( \frac{1}{9} \sin 9\varphi - \sin 7\varphi + 4 \sin 5\varphi - \frac{28}{5} \sin 3\varphi + 14 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^2 \varphi = \frac{1}{512} \left( \frac{1}{10} \sin 10\varphi - \frac{5}{4} \sin 8\varphi + \frac{13}{6} \sin 6\varphi - 2 \sin 4\varphi - 7 \sin 2\varphi + 14\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^3 \varphi = \frac{1}{1024} \left( \frac{1}{11} \sin 11\varphi - \frac{5}{9} \sin 9\varphi + \sin 7\varphi + \sin 5\varphi - \frac{22}{3} \sin 3\varphi + 14 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^4 \varphi = \frac{1}{2048} \left( \frac{1}{12} \sin 12\varphi - \frac{2}{5} \sin 10\varphi + \frac{1}{4} \sin 8\varphi + 2 \sin 6\varphi - \frac{17}{4} \sin 4\varphi - 4 \sin 2\varphi + 14\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^5 \varphi = \frac{1}{4096} \left( \frac{1}{13} \sin 13\varphi - \frac{5}{11} \sin 11\varphi - \frac{2}{9} \sin 9\varphi + 2 \sin 7\varphi - \sin 5\varphi - \frac{25}{3} \sin 3\varphi + 20 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^6 \varphi = \frac{1}{8192} \left( \frac{1}{14} \sin 14\varphi - \frac{1}{6} \sin 12\varphi - \frac{1}{2} \sin 10\varphi + \frac{5}{2} \sin 8\varphi + \frac{3}{2} \sin 6\varphi + \frac{15}{2} \sin 4\varphi - \frac{5}{2} \sin 2\varphi + 20\varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^7 \varphi = \frac{1}{16384} \left( \frac{1}{15} \sin 15\varphi - \frac{1}{13} \sin 13\varphi - \frac{7}{11} \sin 11\varphi + \frac{7}{9} \sin 9\varphi + 3 \sin 7\varphi - \frac{21}{5} \sin 5\varphi - \frac{55}{3} \sin 3\varphi + 35 \sin \varphi \right)$$

$$\int \partial \varphi \sin^3 \varphi \cos^8 \varphi = \frac{1}{32768} \left( \frac{1}{16} \sin 16\varphi - \frac{2}{5} \sin 12\varphi + \frac{7}{2} \sin 8\varphi - 14 \sin 4\varphi + 35\varphi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^2 \phi$$

Taf. XII.

$$\int \partial \phi \sin^2 \phi \cos \phi = \frac{1}{10} \sin^{10} \phi$$

$$\int \partial \phi \sin^2 \phi \cos^2 \phi = \frac{1}{11} \sin^{10} \phi \cos \phi + \frac{1}{11} \int \partial \phi \sin^2 \phi$$

$$\int \partial \phi \sin^2 \phi \cos^3 \phi = \left( \frac{1}{12} \cos^2 \phi + \frac{1}{60} \right) \sin^{10} \phi$$

$$\int \partial \phi \sin^2 \phi \cos \phi = -\frac{1}{512} \left( \frac{1}{10} \cos 10\phi - \cos 8\phi + \frac{9}{2} \cos 6\phi - 12 \cos 4\phi + 21 \cos 2\phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^2 \phi = -\frac{1}{1024} \left( \frac{1}{11} \cos 11\phi - \frac{7}{9} \cos 9\phi + \frac{19}{7} \cos 7\phi - \frac{21}{5} \cos 5\phi - 2 \cos 3\phi + 42 \cos \phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^3 \phi = -\frac{1}{2048} \left( \frac{1}{12} \cos 12\phi - \frac{8}{5} \cos 10\phi + \frac{8}{3} \cos 8\phi - \frac{1}{3} \cos 6\phi - \frac{27}{4} \cos 4\phi + 18 \cos 2\phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^4 \phi = -\frac{1}{4096} \left( \frac{1}{13} \cos 13\phi - \frac{5}{11} \cos 11\phi + \frac{8}{3} \cos 9\phi + \frac{10}{7} \cos 7\phi - \frac{29}{5} \cos 5\phi + 3 \cos 3\phi + 36 \cos \phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^5 \phi = -\frac{1}{8192} \left( \frac{1}{14} \cos 14\phi - \frac{1}{3} \cos 12\phi + \frac{1}{10} \cos 10\phi + 2 \cos 8\phi - \frac{19}{6} \cos 6\phi - 5 \cos 4\phi + \frac{45}{2} \cos 2\phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^6 \phi = -\frac{1}{16384} \left( \frac{1}{15} \cos 15\phi - \frac{8}{13} \cos 13\phi - \frac{5}{11} \cos 11\phi + \frac{17}{9} \cos 9\phi - \frac{3}{7} \cos 7\phi - \frac{59}{5} \cos 5\phi + \frac{25}{3} \cos 3\phi + 45 \cos \phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^7 \phi = -\frac{1}{32768} \left( \frac{1}{16} \cos 16\phi - \frac{1}{7} \cos 14\phi - \frac{1}{2} \cos 12\phi + \frac{7}{5} \cos 10\phi + \frac{7}{4} \cos 8\phi - 7 \cos 6\phi - \frac{7}{2} \cos 4\phi + 35 \cos 2\phi \right)$$

$$\int \partial \phi \sin^2 \phi \cos^8 \phi = -\frac{1}{65536} \left( \frac{1}{17} \cos 17\phi - \frac{1}{15} \cos 15\phi - \frac{8}{13} \cos 13\phi + \frac{8}{11} \cos 11\phi + \frac{28}{9} \cos 9\phi - 4 \cos 7\phi - \frac{56}{5} \cos 5\phi + \frac{56}{3} \cos 3\phi + 70 \cos \phi \right)$$

Taf. XIII.

$$\int \frac{\partial \varphi}{\sin^m \varphi}, \quad \int \frac{\partial \varphi}{\cos^m \varphi}$$

$$\int \frac{\partial \varphi}{\sin \varphi} = \log \operatorname{Tang} \frac{\varphi}{2}$$

$$\int \frac{\partial \varphi}{\sin^2 \varphi} = -\frac{\cos \varphi}{\sin \varphi} = -\operatorname{Cot} \varphi$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi} = -\frac{\cos \varphi}{2 \sin^2 \varphi} + \frac{1}{2} \int \frac{\partial \varphi}{\sin \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi} = \left( -\frac{1}{3 \sin^3 \varphi} - \frac{2}{3 \sin \varphi} \right) \cos \varphi = \operatorname{Cot} \varphi - \frac{1}{5} \operatorname{Cot}^3 \varphi$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi} = \left( -\frac{1}{4 \sin^4 \varphi} - \frac{3}{8 \sin^2 \varphi} \right) \cos \varphi + \frac{5}{8} \int \frac{\partial \varphi}{\sin \varphi}$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi} = \left( -\frac{1}{5 \sin^5 \varphi} - \frac{4}{15 \sin^3 \varphi} - \frac{8}{15 \sin \varphi} \right) \cos \varphi$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi} = \left( -\frac{1}{6 \sin^6 \varphi} - \frac{5}{24 \sin^4 \varphi} - \frac{5}{16 \sin^2 \varphi} \right) \cos \varphi + \frac{5}{16} \int \frac{\partial \varphi}{\sin \varphi}$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi} = \left( -\frac{1}{7 \sin^7 \varphi} - \frac{6}{35 \sin^5 \varphi} - \frac{8}{35 \sin^3 \varphi} - \frac{16}{35 \sin \varphi} \right) \cos \varphi$$

$$\int \frac{\partial \varphi}{\cos \varphi} = \log \operatorname{Tang} \left( 45^\circ + \frac{\varphi}{2} \right)$$

$$\int \frac{\partial \varphi}{\cos^2 \varphi} = \frac{\sin \varphi}{\cos \varphi} = \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\cos^3 \varphi} = \frac{\sin \varphi}{2 \cos^2 \varphi} + \frac{1}{2} \int \frac{\partial \varphi}{\cos \varphi}$$

$$\int \frac{\partial \varphi}{\cos^4 \varphi} = \left( \frac{1}{3 \cos^3 \varphi} + \frac{2}{3 \cos \varphi} \right) \sin \varphi = \operatorname{Tang} \varphi + \frac{1}{3} \operatorname{Tang}^3 \varphi$$

$$\int \frac{\partial \varphi}{\cos^5 \varphi} = \left( \frac{1}{4 \cos^4 \varphi} + \frac{3}{8 \cos^2 \varphi} \right) \sin \varphi + \frac{5}{8} \int \frac{\partial \varphi}{\cos \varphi}$$

$$\int \frac{\partial \varphi}{\cos^6 \varphi} = \left( \frac{1}{5 \cos^5 \varphi} + \frac{4}{15 \cos^3 \varphi} + \frac{8}{15 \cos \varphi} \right) \sin \varphi$$

$$\int \frac{\partial \varphi}{\cos^7 \varphi} = \left( \frac{1}{6 \cos^6 \varphi} + \frac{5}{24 \cos^4 \varphi} + \frac{5}{16 \cos^2 \varphi} \right) \sin \varphi + \frac{5}{16} \int \frac{\partial \varphi}{\cos \varphi}$$

$$\int \frac{\partial \varphi}{\cos^8 \varphi} = \left( \frac{1}{7 \cos^7 \varphi} + \frac{6}{35 \cos^5 \varphi} + \frac{8}{35 \cos^3 \varphi} + \frac{16}{35 \cos \varphi} \right) \sin \varphi$$

$$\int \frac{\partial \phi \sin^n \phi}{\cos \phi}, \quad \int \frac{\partial \phi \cos^n \phi}{\sin \phi}$$

Taf. XIV.

$$\int \frac{\partial \phi \sin \phi}{\cos \phi} = -\log \cos \phi = \log \sec \phi$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos \phi} = -\sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos \phi} = -\frac{\sin^2 \phi}{2} + \int \frac{\partial \phi \sin \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos \phi} = -\frac{\sin^3 \phi}{3} - \sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos \phi} = -\frac{\sin^4 \phi}{4} - \frac{\sin^2 \phi}{2} + \int \frac{\partial \phi \sin \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos \phi} = -\frac{\sin^5 \phi}{5} - \frac{\sin^3 \phi}{3} - \sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos \phi} = -\frac{\sin^6 \phi}{6} - \frac{\sin^4 \phi}{4} - \frac{\sin^2 \phi}{2} + \int \frac{\partial \phi \sin \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos \phi} = -\frac{\sin^7 \phi}{7} - \frac{\sin^5 \phi}{5} - \frac{\sin^3 \phi}{3} - \sin \phi + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \cos \phi}{\sin \phi} = \log \sin \phi$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin \phi} = \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin \phi} = \frac{\cos^2 \phi}{2} + \int \frac{\partial \phi \cos \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin \phi} = \frac{\cos^3 \phi}{3} + \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin \phi} = \frac{\cos^4 \phi}{4} + \frac{\cos^2 \phi}{2} + \int \frac{\partial \phi \cos \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin \phi} = \frac{\cos^5 \phi}{5} + \frac{\cos^3 \phi}{3} + \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin \phi} = \frac{\cos^6 \phi}{6} + \frac{\cos^4 \phi}{4} + \frac{\cos^2 \phi}{2} + \int \frac{\partial \phi \cos \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin \phi} = \frac{\cos^7 \phi}{7} + \frac{\cos^5 \phi}{5} + \frac{\cos^3 \phi}{3} + \cos \phi + \int \frac{\partial \phi}{\sin \phi}$$

Taf. XV.

$$\int \frac{\partial \phi \sin^m \phi}{\cos^2 \phi}, \int \frac{\partial \phi \cos^m \phi}{\sin^2 \phi}$$

$$\int \frac{\partial \phi \sin \phi}{\cos^2 \phi} = \frac{1}{\cos \phi} = \sec \phi$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^2 \phi} = \frac{\sin \phi}{\cos \phi} - \phi = \text{Tang } \phi - \phi$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^2 \phi} = \left( -\sin^2 \phi + 2 \right) \frac{1}{\cos \phi} = \cos \phi + \sec \phi$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^2 \phi} = \left( -\frac{1}{2} \sin^3 \phi + \frac{5}{2} \sin \phi \right) \frac{1}{\cos \phi} - \frac{5}{2} \phi$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos^2 \phi} = \left( -\frac{1}{3} \sin^4 \phi - \frac{4}{3} \sin^2 \phi + \frac{8}{3} \right) \frac{1}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^2 \phi} = \left( -\frac{1}{4} \sin^5 \phi - \frac{5}{8} \sin^3 \phi + \frac{15}{8} \sin \phi \right) \frac{1}{\cos \phi} - \frac{15}{8} \phi$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^2 \phi} = \left( -\frac{1}{5} \sin^6 \phi - \frac{2}{5} \sin^4 \phi - \frac{8}{5} \sin^2 \phi + \frac{16}{5} \right) \frac{1}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos^2 \phi} = \left( -\frac{1}{6} \sin^7 \phi - \frac{7}{24} \sin^5 \phi - \frac{35}{48} \sin^3 \phi + \frac{35}{16} \sin \phi \right) \frac{1}{\cos \phi} - \frac{35}{16} \phi$$

$$\int \frac{\partial \phi \cos \phi}{\sin^2 \phi} = -\frac{1}{\sin \phi} = -\text{Cosec } \phi$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^2 \phi} = -\frac{\cos \phi}{\sin \phi} - \phi = -\text{Cot } \phi - \phi$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^2 \phi} = \left( \cos^2 \phi - 2 \right) \frac{1}{\sin \phi} = -\sin \phi - \text{Cosec } \phi$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^2 \phi} = \left( \frac{1}{2} \cos^3 \phi - \frac{3}{2} \cos \phi \right) \frac{1}{\sin \phi} - \frac{5}{2} \phi$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin^2 \phi} = \left( \frac{1}{3} \cos^4 \phi + \frac{4}{3} \cos^2 \phi - \frac{8}{3} \right) \frac{1}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^2 \phi} = \left( \frac{1}{4} \cos^5 \phi + \frac{5}{8} \cos^3 \phi - \frac{15}{8} \cos \phi \right) \frac{1}{\sin \phi} - \frac{15}{8} \phi$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^2 \phi} = \left( \frac{1}{5} \cos^6 \phi + \frac{2}{5} \cos^4 \phi + \frac{8}{5} \cos^2 \phi - \frac{16}{5} \right) \frac{1}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin^2 \phi} = \left( \frac{1}{6} \cos^7 \phi + \frac{7}{24} \cos^5 \phi + \frac{35}{48} \cos^3 \phi - \frac{35}{16} \cos \phi \right) \frac{1}{\sin \phi} - \frac{35}{16} \phi$$

$$\int \frac{\partial \phi \sin^m \phi}{\cos^3 \phi}, \int \frac{\partial \phi \cos^m \phi}{\sin^3 \phi}$$

Taf. XVI.

$$\int \frac{\partial \phi \sin \phi}{\cos^3 \phi} = \frac{1}{2 \cos^2 \phi}$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^3 \phi} = \frac{\sin \phi}{2 \cos^2 \phi} - \frac{1}{2} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^3 \phi} = \frac{1}{2 \cos^2 \phi} + \log \cos \phi$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^3 \phi} = \left( -\sin^2 \phi + \frac{5}{2} \sin \phi \right) \frac{1}{\cos^2 \phi} - \frac{5}{2} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos^3 \phi} = \left( -\frac{1}{2} \sin^4 \phi + 1 \right) \frac{1}{\cos^2 \phi} + 2 \log \cos \phi$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^3 \phi} = \left( -\frac{1}{2} \sin^5 \phi - \frac{5}{2} \sin^3 \phi + \frac{5}{2} \sin \phi \right) \frac{1}{\cos^2 \phi} - \frac{5}{2} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^3 \phi} = \left( -\frac{1}{4} \sin^6 \phi - \frac{5}{4} \sin^4 \phi + \frac{5}{2} \sin^2 \phi \right) \frac{1}{\cos^2 \phi} + 3 \log \cos \phi$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos^3 \phi} = \left( -\frac{1}{5} \sin^7 \phi - \frac{7}{15} \sin^5 \phi - \frac{7}{5} \sin^3 \phi + \frac{7}{2} \sin \phi \right) \frac{1}{\cos^2 \phi} - \frac{7}{2} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \cos \phi}{\sin^3 \phi} = -\frac{1}{2 \sin^2 \phi}$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^3 \phi} = -\frac{\cos \phi}{2 \sin^2 \phi} - \frac{1}{2} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^3 \phi} = -\frac{1}{2 \sin^2 \phi} - \log \sin \phi$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^3 \phi} = \left( \cos^3 \phi - \frac{3}{2} \cos \phi \right) \frac{1}{\sin^2 \phi} - \frac{5}{2} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin^3 \phi} = \left( \frac{1}{2} \cos^4 \phi - 1 \right) \frac{1}{\sin^2 \phi} - 2 \log \sin \phi$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^3 \phi} = \left( \frac{1}{3} \cos^5 \phi + \frac{5}{3} \cos^3 \phi - \frac{5}{2} \cos \phi \right) \frac{1}{\sin^2 \phi} - \frac{5}{2} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^3 \phi} = \left( \frac{1}{4} \cos^6 \phi + \frac{3}{4} \cos^4 \phi - \frac{3}{2} \cos^2 \phi \right) \frac{1}{\sin^2 \phi} - 3 \log \sin \phi$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin^3 \phi} = \left( \frac{1}{5} \cos^7 \phi + \frac{7}{15} \cos^5 \phi + \frac{7}{5} \cos^3 \phi - \frac{7}{2} \cos \phi \right) \frac{1}{\sin^2 \phi} - \frac{7}{2} \int \frac{\partial \phi}{\sin \phi}$$

Taf. XVII.

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^4 \phi}, \int \frac{\partial \phi \cos^7 \phi}{\sin^4 \phi}$$

$$\int \frac{\partial \phi \sin \phi}{\cos^4 \phi} = \frac{1}{3 \cos^3 \phi}$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^4 \phi} = \frac{\sin^3 \phi}{3 \cos^3 \phi} = \frac{1}{3} \text{Tang}^3 \phi$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^4 \phi} = \left( \sin^2 \phi - \frac{2}{3} \right) \frac{1}{\cos^3 \phi}$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^4 \phi} = \left( \frac{1}{3} \sin^3 \phi - \sin \phi \right) \frac{1}{\cos^3 \phi} + \phi = \frac{1}{3} \text{Tang}^3 \phi - \text{Tang} \phi + \phi$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos^4 \phi} = \left( -\sin^4 \phi + 4 \sin^2 \phi - \frac{8}{3} \right) \frac{1}{\cos^3 \phi}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^4 \phi} = \left( -\frac{1}{2} \sin^5 \phi + \frac{10}{3} \sin^3 \phi - \frac{5}{2} \sin \phi \right) \frac{1}{\cos^3 \phi} + \frac{5}{2} \phi$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^4 \phi} = \left( -\frac{1}{3} \sin^6 \phi - 2 \sin^4 \phi + 8 \sin^2 \phi - \frac{16}{3} \right) \frac{1}{\cos^3 \phi}$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos^4 \phi} = \left( -\frac{1}{4} \sin^7 \phi - \frac{7}{8} \sin^5 \phi + \frac{35}{6} \sin^3 \phi - \frac{35}{8} \sin \phi \right) \frac{1}{\cos^3 \phi} + \frac{35}{8} \phi$$

$$\int \frac{\partial \phi \cos \phi}{\sin^4 \phi} = -\frac{1}{3 \sin^3 \phi}$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^4 \phi} = -\frac{\cos^3 \phi}{3 \sin^3 \phi} = -\frac{1}{3} \text{Cot}^3 \phi$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^4 \phi} = \left( -\cos^2 \phi + \frac{2}{3} \right) \frac{1}{\sin^3 \phi}$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^4 \phi} = \left( -\frac{4}{3} \cos^3 \phi + \cos \phi \right) \frac{1}{\sin^3 \phi} + \phi = -\frac{1}{3} \text{Cot}^3 \phi + \text{Cot} \phi + \phi$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin^4 \phi} = \left( \cos^4 \phi - 4 \cos^2 \phi + \frac{8}{3} \right) \frac{1}{\sin^3 \phi}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^4 \phi} = \left( \frac{1}{2} \cos^5 \phi - \frac{10}{3} \cos^3 \phi + \frac{5}{2} \cos \phi \right) \frac{1}{\sin^3 \phi} + \frac{5}{2} \phi$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^4 \phi} = \left( \frac{1}{3} \cos^6 \phi + 2 \cos^4 \phi - 8 \cos^2 \phi + \frac{16}{3} \right) \frac{1}{\sin^3 \phi}$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin^4 \phi} = \left( \frac{1}{4} \cos^7 \phi + \frac{7}{8} \cos^5 \phi - \frac{35}{6} \cos^3 \phi + \frac{35}{8} \cos \phi \right) \frac{1}{\sin^3 \phi} + \frac{35}{8} \phi$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^5 \phi}, \int \frac{\partial \phi \cos^4 \phi}{\sin^5 \phi} \quad \text{Taf. XVIII.}$$

$$\int \frac{\partial \phi \sin \phi}{\cos^5 \phi} = \frac{1}{4 \cos^4 \phi}$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^5 \phi} = \left( \frac{1}{8} \sin^3 \phi + \frac{1}{8} \sin \phi \right) \frac{1}{\cos^4 \phi} - \frac{1}{8} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^5 \phi} = \frac{\sin^4 \phi}{4 \cos^4 \phi} = \frac{1}{4} \text{Tang}^4 \phi$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^5 \phi} = \left( \frac{5}{8} \sin^3 \phi - \frac{3}{8} \sin \phi \right) \frac{1}{\cos^4 \phi} + \frac{5}{8} \int \frac{\partial \phi}{\cos \phi}$$

$$\begin{aligned} \int \frac{\partial \phi \sin^5 \phi}{\cos^5 \phi} &= \left( \frac{3}{4} \sin^4 \phi - \frac{1}{2} \sin^2 \phi \right) \frac{1}{\cos^4 \phi} - \log \cos \phi \\ &= \frac{1}{4} \text{Tang}^4 \phi - \frac{1}{2} \text{Tang}^2 \phi - \log \cos \phi \end{aligned}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^5 \phi} = \left( -\sin^4 \phi + \frac{25}{8} \sin^2 \phi - \frac{15}{8} \sin \phi \right) \frac{1}{\cos^4 \phi} + \frac{15}{8} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^5 \phi} = \left( -\frac{1}{2} \sin^6 \phi + \frac{9}{4} \sin^4 \phi - \frac{5}{2} \sin^2 \phi \right) \frac{1}{\cos^4 \phi} - 3 \log \cos \phi$$

$$\int \frac{\partial \phi \cos \phi}{\sin^5 \phi} = -\frac{1}{4 \sin^4 \phi}$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^5 \phi} = \left( -\frac{1}{8} \cos^3 \phi - \frac{1}{8} \cos \phi \right) \frac{1}{\sin^4 \phi} - \frac{1}{8} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^5 \phi} = -\frac{\cos^4 \phi}{4 \sin^4 \phi} = -\frac{1}{4} \text{Cot}^4 \phi$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^5 \phi} = \left( -\frac{5}{8} \cos^3 \phi + \frac{5}{8} \cos \phi \right) \frac{1}{\sin^4 \phi} + \frac{5}{8} \int \frac{\partial \phi}{\sin \phi}$$

$$\begin{aligned} \int \frac{\partial \phi \cos^5 \phi}{\sin^5 \phi} &= \left( -\frac{5}{4} \cos^4 \phi + \frac{1}{2} \cos^2 \phi \right) \frac{1}{\sin^4 \phi} + \log \sin \phi \\ &= -\frac{1}{4} \text{Cot}^4 \phi + \frac{1}{2} \text{Cot}^2 \phi + \log \sin \phi \end{aligned}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^5 \phi} = \left( \cos^5 \phi - \frac{25}{8} \cos^3 \phi + \frac{15}{8} \cos \phi \right) \frac{1}{\sin^4 \phi} + \frac{15}{8} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^5 \phi} = \left( \frac{1}{2} \cos^6 \phi - \frac{9}{4} \cos^4 \phi + \frac{5}{2} \cos^2 \phi \right) \frac{1}{\sin^4 \phi} + 3 \log \sin \phi$$



Taf. XIX.

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^6 \phi}, \quad \int \frac{\partial \phi \cos^2 \phi}{\sin^6 \phi}$$

$$\int \frac{\partial \phi \sin \phi}{\cos^6 \phi} = \frac{1}{5 \cos^5 \phi}$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^6 \phi} = \left( -\frac{2}{15} \sin^5 \phi + \frac{1}{5} \sin^3 \phi \right) \frac{1}{\cos^5 \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^6 \phi} = \left( \frac{1}{5} \sin^2 \phi - \frac{2}{15} \right) \frac{1}{\cos^5 \phi}$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^6 \phi} = \frac{1}{5} \text{Tang}^5 \phi$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos^6 \phi} = \left( \sin^4 \phi - \frac{4}{5} \sin^2 \phi + \frac{8}{15} \right) \frac{1}{\cos^5 \phi}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^6 \phi} = \frac{1}{5} \text{Tang}^5 \phi - \frac{1}{5} \text{Tang}^3 \phi + \text{Tang} \phi - \phi$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^6 \phi} = \left( -\sin^6 \phi + 6 \sin^4 \phi - 8 \sin^2 \phi + \frac{16}{5} \right) \frac{1}{\cos^5 \phi}$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos^6 \phi} = -\frac{\sin^7 \phi}{2 \cos^5 \phi} + \frac{7}{2} \int \frac{\partial \phi \sin^6 \phi}{\cos^6 \phi}$$

$$\int \frac{\partial \phi \cos \phi}{\sin^6 \phi} = -\frac{1}{5 \sin^5 \phi}$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^6 \phi} = \left( \frac{2}{15} \cos^5 \phi - \frac{1}{5} \cos^3 \phi \right) \frac{1}{\sin^5 \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^6 \phi} = \left( -\frac{1}{5} \cos^2 \phi + \frac{2}{15} \right) \frac{1}{\sin^5 \phi}$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^6 \phi} = -\frac{1}{5} \text{Cot}^5 \phi$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin^6 \phi} = \left( -\cos^4 \phi + \frac{4}{5} \cos^2 \phi - \frac{8}{15} \right) \frac{1}{\sin^5 \phi}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^6 \phi} = -\frac{1}{5} \text{Cot}^5 \phi + \frac{1}{5} \text{Cot}^3 \phi - \text{Cot} \phi - \phi$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^6 \phi} = \left( \cos^6 \phi - 6 \cos^4 \phi + 8 \cos^2 \phi - \frac{16}{5} \right) \frac{1}{\sin^5 \phi}$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin^6 \phi} = \frac{\cos^7 \phi}{2 \sin^5 \phi} + \frac{7}{2} \int \frac{\partial \phi \cos^6 \phi}{\sin^6 \phi}$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^7 \phi}, \quad \int \frac{\partial \phi \cos^7 \phi}{\sin^7 \phi}$$

Taf. XX.

$$\int \frac{\partial \phi \sin \phi}{\cos^7 \phi} = \frac{1}{6 \cos^6 \phi}$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^7 \phi} = \left( -\frac{1}{16} \sin^5 \phi + \frac{1}{6} \sin^3 \phi + \frac{1}{16} \sin \phi \right) \frac{1}{\cos^6 \phi} - \frac{1}{16} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^7 \phi} = \left( \frac{1}{4} \sin^2 \phi - \frac{1}{12} \right) \frac{1}{\cos^6 \phi}$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^7 \phi} = \left( \frac{1}{16} \sin^5 \phi + \frac{1}{6} \sin^3 \phi - \frac{1}{16} \sin \phi \right) \frac{1}{\cos^6 \phi} + \frac{1}{16} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos^7 \phi} = \frac{1}{6} \text{Tang}^6 \phi$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^7 \phi} = \left( \frac{11}{16} \sin^5 \phi - \frac{5}{6} \sin^3 \phi + \frac{5}{16} \sin \phi \right) \frac{1}{\cos^6 \phi} - \frac{5}{16} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^7 \phi} = \frac{1}{6} \text{Tang}^6 \phi - \frac{1}{4} \text{Tang}^4 \phi + \frac{1}{2} \text{Tang}^2 \phi + \log \cos \phi$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos^7 \phi} = \left( -\sin^7 \phi + \frac{77}{16} \sin^5 \phi - \frac{35}{6} \sin^3 \phi + \frac{35}{16} \sin \phi \right) \frac{1}{\cos^6 \phi} - \frac{35}{16} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi \cos \phi}{\sin^7 \phi} = -\frac{1}{6 \sin^6 \phi}$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^7 \phi} = \left( \frac{1}{16} \cos^5 \phi - \frac{1}{6} \cos^3 \phi - \frac{1}{16} \cos \phi \right) \frac{1}{\sin^6 \phi} - \frac{1}{16} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^7 \phi} = \left( -\frac{1}{4} \cos^2 \phi + \frac{1}{12} \right) \frac{1}{\sin^6 \phi}$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^7 \phi} = \left( -\frac{1}{16} \cos^5 \phi - \frac{1}{6} \cos^3 \phi + \frac{1}{16} \cos \phi \right) \frac{1}{\sin^6 \phi} + \frac{1}{16} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin^7 \phi} = -\frac{1}{6} \text{Cot}^6 \phi$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^7 \phi} = \left( -\frac{11}{16} \cos^5 \phi + \frac{5}{6} \cos^3 \phi - \frac{5}{16} \cos \phi \right) \frac{1}{\sin^6 \phi} - \frac{5}{16} \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^7 \phi} = -\frac{1}{6} \text{Cot}^6 \phi + \frac{1}{4} \text{Cot}^4 \phi - \frac{1}{2} \text{Cot}^2 \phi - \log \sin \phi$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin^7 \phi} = \left( \cos^7 \phi - \frac{77}{16} \cos^5 \phi + \frac{35}{6} \cos^3 \phi - \frac{35}{16} \cos \phi \right) \frac{1}{\sin^6 \phi} - \frac{35}{16} \int \frac{\partial \phi}{\sin \phi}$$

Taf. XXI

$$\int \frac{\partial \phi \sin^* \phi}{\cos^3 \phi}, \quad \int \frac{\partial \phi \cos^* \phi}{\sin^3 \phi}$$

$$\int \frac{\partial \phi \sin \phi}{\cos^3 \phi} = \frac{1}{7 \cos^7 \phi}$$

$$\int \frac{\partial \phi \sin^2 \phi}{\cos^3 \phi} = \left( \frac{8}{105} \sin^7 \phi - \frac{4}{15} \sin^5 \phi + \frac{1}{5} \sin^3 \phi \right) \frac{1}{\cos^7 \phi}$$

$$\int \frac{\partial \phi \sin^3 \phi}{\cos^3 \phi} = \left( \frac{1}{5} \sin^6 \phi - \frac{2}{35} \right) \frac{1}{\cos^7 \phi}$$

$$\int \frac{\partial \phi \sin^4 \phi}{\cos^3 \phi} = \left( -\frac{2}{35} \sin^7 \phi + \frac{1}{5} \sin^5 \phi \right) \frac{1}{\cos^7 \phi}$$

$$\int \frac{\partial \phi \sin^5 \phi}{\cos^3 \phi} = \left( \frac{1}{5} \sin^4 \phi - \frac{4}{15} \sin^2 \phi + \frac{8}{105} \right) \frac{1}{\cos^7 \phi}$$

$$\int \frac{\partial \phi \sin^6 \phi}{\cos^3 \phi} = \frac{1}{7} \tan^7 \phi$$

$$\int \frac{\partial \phi \sin^7 \phi}{\cos^3 \phi} = \left( \sin^6 \phi - 2 \sin^4 \phi + \frac{8}{5} \sin^2 \phi - \frac{16}{35} \right) \frac{1}{\cos^7 \phi}$$

$$\int \frac{\partial \phi \sin^8 \phi}{\cos^3 \phi} = \frac{1}{7} \tan^7 \phi - \frac{1}{5} \tan^5 \phi + \frac{1}{5} \tan^3 \phi - \tan \phi + \phi$$

$$\int \frac{\partial \phi \cos \phi}{\sin^3 \phi} = -\frac{1}{7 \sin^7 \phi}$$

$$\int \frac{\partial \phi \cos^2 \phi}{\sin^3 \phi} = \left( -\frac{8}{105} \cos^7 \phi + \frac{4}{15} \cos^5 \phi - \frac{1}{5} \cos^3 \phi \right) \frac{1}{\sin^7 \phi}$$

$$\int \frac{\partial \phi \cos^3 \phi}{\sin^3 \phi} = \left( -\frac{1}{5} \cos^2 \phi + \frac{2}{35} \right) \frac{1}{\sin^7 \phi}$$

$$\int \frac{\partial \phi \cos^4 \phi}{\sin^3 \phi} = \left( \frac{2}{35} \cos^7 \phi - \frac{1}{5} \cos^5 \phi \right) \frac{1}{\sin^7 \phi}$$

$$\int \frac{\partial \phi \cos^5 \phi}{\sin^3 \phi} = \left( -\frac{1}{5} \cos^4 \phi + \frac{4}{15} \cos^2 \phi - \frac{8}{105} \right) \frac{1}{\sin^7 \phi}$$

$$\int \frac{\partial \phi \cos^6 \phi}{\sin^3 \phi} = -\frac{1}{7} \cot^7 \phi$$

$$\int \frac{\partial \phi \cos^7 \phi}{\sin^3 \phi} = \left( -\cos^6 \phi + 2 \cos^4 \phi - \frac{8}{5} \cos^2 \phi + \frac{16}{35} \right) \frac{1}{\sin^7 \phi}$$

$$\int \frac{\partial \phi \cos^8 \phi}{\sin^3 \phi} = -\frac{1}{7} \cot^7 \phi + \frac{1}{5} \cot^5 \phi - \frac{1}{5} \cot^3 \phi + \cot \phi + \phi$$

$$\int \frac{\partial \phi}{\sin \phi \cos^2 \phi}, \int \frac{\partial \phi}{\sin^2 \phi \cos^2 \phi} \quad \text{Taf. XXII.}$$

$$\int \frac{\partial \phi}{\sin \phi \cos \phi} = \log \text{Tang } \phi$$

$$\int \frac{\partial \phi}{\sin \phi \cos^2 \phi} = \frac{1}{\cos \phi} + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin \phi \cos^3 \phi} = \frac{1}{2 \cos^2 \phi} + \log \text{Tang } \phi$$

$$\int \frac{\partial \phi}{\sin \phi \cos^4 \phi} = \frac{1}{3 \cos^3 \phi} + \frac{1}{\cos \phi} + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin \phi \cos^5 \phi} = \frac{1}{4 \cos^4 \phi} + \frac{1}{2 \cos^2 \phi} + \log \text{Tang } \phi$$

$$\int \frac{\partial \phi}{\sin \phi \cos^6 \phi} = \frac{1}{5 \cos^5 \phi} + \frac{1}{5 \cos^3 \phi} + \frac{1}{\cos \phi} + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin \phi \cos^7 \phi} = \frac{1}{6 \cos^6 \phi} + \frac{1}{4 \cos^4 \phi} + \frac{1}{2 \cos^2 \phi} + \log \text{Tang } \phi$$

$$\int \frac{\partial \phi}{\sin \phi \cos^8 \phi} = \frac{1}{7 \cos^7 \phi} + \frac{1}{5 \cos^5 \phi} + \frac{1}{3 \cos^3 \phi} + \frac{1}{\cos \phi} + \int \frac{\partial \phi}{\sin \phi}$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos \phi} = -\frac{1}{\sin \phi} + \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^2 \phi} = -2 \text{Cot } 2\phi$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^3 \phi} = \left( \frac{1}{2 \cos^2 \phi} - \frac{3}{2} \right) \frac{1}{\sin \phi} + \frac{5}{2} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^4 \phi} = \frac{1}{3 \sin \phi \cos^3 \phi} - \frac{8}{5} \text{Cot } 2\phi$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^5 \phi} = \left( \frac{1}{4 \cos^4 \phi} + \frac{5}{8 \cos^2 \phi} - \frac{15}{8} \right) \frac{1}{\sin \phi} + \frac{15}{8} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^6 \phi} = \left( \frac{1}{5 \cos^5 \phi} + \frac{2}{5 \cos^3 \phi} \right) \frac{1}{\sin \phi} - \frac{16}{5} \text{Cot } 2\phi$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^7 \phi} = \left( \frac{1}{6 \cos^6 \phi} + \frac{7}{24 \cos^4 \phi} + \frac{35}{48 \cos^2 \phi} - \frac{35}{16} \right) \frac{1}{\sin \phi} + \frac{35}{16} \int \frac{\partial \phi}{\cos \phi}$$

$$\int \frac{\partial \phi}{\sin^2 \phi \cos^8 \phi} = \left( \frac{1}{7 \cos^7 \phi} + \frac{8}{35 \cos^5 \phi} + \frac{16}{35 \cos^3 \phi} \right) \frac{1}{\sin \phi} - \frac{128}{35} \text{Cot } 2\phi$$

Taf. XXIII.

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^2 \varphi}, \int \frac{\partial \varphi}{\sin^4 \varphi \cos^2 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos \varphi} = -\frac{1}{2 \sin^2 \varphi} + \log \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^2 \varphi} = \frac{1}{\sin^2 \varphi \cos \varphi} + 3 \int \frac{\partial \varphi}{\sin^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^3 \varphi} = -\frac{2 \cos 2 \varphi}{\sin^2 2 \varphi} + 2 \log \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^4 \varphi} = \left( \frac{1}{3 \cos^2 \varphi} + \frac{5}{3 \cos \varphi} \right) \frac{1}{\sin^2 \varphi} + 5 \int \frac{\partial \varphi}{\sin^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^5 \varphi} = \frac{1}{4 \sin^2 \varphi \cos^4 \varphi} + \frac{5}{2} \int \frac{\partial \varphi}{\sin^3 \varphi \cos^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^6 \varphi} = \left( \frac{1}{5 \cos^4 \varphi} + \frac{7}{15 \cos^3 \varphi} + \frac{7}{3 \cos \varphi} \right) \frac{1}{\sin^2 \varphi} + 7 \int \frac{\partial \varphi}{\sin^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^7 \varphi} = \left( \frac{1}{6 \cos^6 \varphi} + \frac{1}{3 \cos^4 \varphi} \right) \frac{1}{\sin^2 \varphi} + 2 \int \frac{\partial \varphi}{\sin^3 \varphi \cos^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^3 \varphi \cos^8 \varphi} = \left( \frac{1}{7 \cos^7 \varphi} + \frac{9}{35 \cos^5 \varphi} + \frac{3}{5 \cos^3 \varphi} + \frac{3}{\cos \varphi} \right) \frac{1}{\sin^2 \varphi} + 9 \int \frac{\partial \varphi}{\sin^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos \varphi} = -\frac{1}{3 \sin^3 \varphi} - \frac{1}{\sin \varphi} + \int \frac{\partial \varphi}{\cos \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^2 \varphi} = -\frac{1}{3 \cos \varphi \sin^3 \varphi} - \frac{8}{5} \cot 2 \varphi$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^3 \varphi} = \frac{1}{2 \cos^2 \varphi \sin^3 \varphi} + \frac{5}{2} \int \frac{\partial \varphi}{\sin^4 \varphi \cos \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^4 \varphi} = \left( -\frac{8}{3 \sin^3 2 \varphi} - \frac{16}{3 \sin 2 \varphi} \right) \cos 2 \varphi$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^5 \varphi} = \left( \frac{1}{4 \cos^4 \varphi} + \frac{7}{8 \cos^2 \varphi} \right) \frac{1}{\sin^3 \varphi} + \frac{55}{8} \int \frac{\partial \varphi}{\sin^4 \varphi \cos \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^6 \varphi} = \frac{1}{5 \cos^5 \varphi \sin^3 \varphi} + \frac{8}{5} \int \frac{\partial \varphi}{\sin^4 \varphi \cos^4 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^7 \varphi} = \left( \frac{1}{6 \cos^5 \varphi} + \frac{3}{8 \cos^4 \varphi} + \frac{21}{16 \cos^2 \varphi} \right) \frac{1}{\sin^3 \varphi} + \frac{105}{16} \int \frac{\partial \varphi}{\sin^4 \varphi \cos \varphi}$$

$$\int \frac{\partial \varphi}{\sin^4 \varphi \cos^8 \varphi} = \left( \frac{1}{7 \cos^7 \varphi} + \frac{2}{7 \cos^5 \varphi} \right) \frac{1}{\sin^3 \varphi} + \frac{16}{7} \int \frac{\partial \varphi}{\sin^4 \varphi \cos^4 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^2 \varphi}, \int \frac{\partial \varphi}{\sin^6 \varphi \cos^2 \varphi} \quad \text{Taf. XXIV.}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos \varphi} = -\frac{1}{4 \sin^4 \varphi} - \frac{1}{2 \sin^2 \varphi} + \log \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^2 \varphi} = \left( -\frac{1}{4 \sin^4 \varphi} - \frac{5}{8 \sin^2 \varphi} + \frac{15}{8} \right) \frac{1}{\cos \varphi} + \frac{15}{8} \int \frac{\partial \varphi}{\sin \varphi}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^3 \varphi} = -\frac{1}{4 \cos^2 \varphi \sin^4 \varphi} + \frac{3}{2} \int \frac{\partial \varphi}{\sin^3 \varphi \cos^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^4 \varphi} = \frac{1}{3 \sin^4 \varphi \cos^3 \varphi} + \frac{7}{3} \int \frac{\partial \varphi}{\sin^3 \varphi \cos^2 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^5 \varphi} = \left( -\frac{4}{\sin^4 2\varphi} - \frac{6}{\sin^2 2\varphi} \right) \cos 2\varphi + 6 \log \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^6 \varphi} = \left( \frac{1}{5 \cos^5 \varphi} + \frac{3}{5 \cos^3 \varphi} \right) \frac{1}{\sin^4 \varphi} + \frac{21}{5} \int \frac{\partial \varphi}{\sin^3 \varphi \cos^2 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^7 \varphi} = \frac{1}{6 \sin^4 \varphi \cos^6 \varphi} + \frac{5}{3} \int \frac{\partial \varphi}{\sin^3 \varphi \cos^5 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^5 \varphi \cos^8 \varphi} = \left( \frac{1}{7 \cos^7 \varphi} + \frac{11}{35 \cos^5 \varphi} + \frac{33}{35 \cos^3 \varphi} \right) \frac{1}{\sin^4 \varphi} + \frac{33}{5} \int \frac{\partial \varphi}{\sin^3 \varphi \cos^2 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos \varphi} = -\frac{1}{5 \sin^5 \varphi} - \frac{1}{3 \sin^3 \varphi} - \frac{1}{\sin \varphi} + \int \frac{\partial \varphi}{\cos \varphi}$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos^2 \varphi} = \left( -\frac{1}{5 \sin^5 \varphi} - \frac{2}{5 \sin^3 \varphi} \right) \frac{1}{\cos \varphi} - \frac{16}{5} \cot 2\varphi$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos^3 \varphi} = \left( -\frac{1}{5 \sin^5 \varphi} - \frac{7}{15 \sin^3 \varphi} - \frac{7}{3 \sin \varphi} \right) \frac{1}{\cos^2 \varphi} + 7 \int \frac{\partial \varphi}{\cos^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos^4 \varphi} = -\frac{1}{5 \sin^5 \varphi \cos^3 \varphi} + \frac{8}{5} \int \frac{\partial \varphi}{\sin^4 \varphi \cos^4 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos^5 \varphi} = \left( -\frac{1}{5 \sin^5 \varphi} - \frac{3}{5 \sin^3 \varphi} \right) \frac{1}{\cos^4 \varphi} + \frac{21}{5} \int \frac{\partial \varphi}{\sin^2 \varphi \cos^5 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos^6 \varphi} = \left( -\frac{32}{5 \sin^5 2\varphi} - \frac{128}{15 \sin^3 2\varphi} - \frac{256}{15 \sin 2\varphi} \right) \cos 2\varphi$$

$$\int \frac{\partial \varphi}{\sin^6 \varphi \cos^7 \varphi} = \frac{1}{6 \sin^5 \varphi \cos^6 \varphi} - \left( \frac{11}{30 \sin^5 \varphi} + \frac{11}{10 \sin^3 \varphi} \right) \frac{1}{\cos^4 \varphi} - \frac{77}{10} \int \frac{\partial \varphi}{\sin^2 \varphi \cos^5 \varphi}$$

Taf. XXV.

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^5 \varphi}, \int \frac{\partial \varphi}{\sin^8 \varphi \cos^5 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos \varphi} = -\frac{1}{6 \sin^6 \varphi} - \frac{1}{4 \sin^4 \varphi} - \frac{1}{2 \sin^2 \varphi} + \log \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^3 \varphi} = \left( -\frac{1}{6 \sin^6 \varphi} - \frac{7}{24 \sin^4 \varphi} - \frac{35}{48 \sin^2 \varphi} + \frac{35}{16} \right) \frac{1}{\cos \varphi} + \frac{35}{16} \int \frac{\partial \varphi}{\sin \varphi}$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^5 \varphi} = \left( -\frac{1}{6 \sin^6 \varphi} - \frac{1}{3 \sin^4 \varphi} \right) \frac{1}{\cos^3 \varphi} + 2 \int \frac{\partial \varphi}{\sin^5 \varphi \cos^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^7 \varphi} = \left( -\frac{1}{6 \sin^6 \varphi} - \frac{3}{8 \sin^4 \varphi} - \frac{21}{16 \sin^2 \varphi} \right) \frac{1}{\cos^5 \varphi} + \frac{105}{16} \int \frac{\partial \varphi}{\sin \varphi \cos^4 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^9 \varphi} = -\frac{1}{6 \cos^8 \varphi \sin^6 \varphi} + \frac{5}{5} \int \frac{\partial \varphi}{\sin^5 \varphi \cos^7 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^6 \varphi} = -\frac{1}{6 \cos^5 \varphi \sin^6 \varphi} + \left( \frac{11}{30 \cos^5 \varphi} + \frac{11}{10 \cos^3 \varphi} \right) \frac{1}{\sin^4 \varphi} + \frac{77}{10} \int \frac{\partial \varphi}{\sin^5 \varphi \cos^2 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^7 \varphi \cos^7 \varphi} = \left( -\frac{32}{3 \sin^6 2\varphi} - \frac{40}{3 \sin^4 2\varphi} - \frac{20}{\sin^2 2\varphi} \right) \cos 2\varphi + 20 \log \operatorname{Tang} \varphi$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos \varphi} = -\frac{1}{7 \sin^7 \varphi} - \frac{1}{5 \sin^5 \varphi} - \frac{1}{3 \sin^3 \varphi} - \frac{1}{\sin \varphi} + \int \frac{\partial \varphi}{\cos \varphi}$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos^3 \varphi} = \left( -\frac{1}{7 \sin^7 \varphi} - \frac{8}{35 \sin^5 \varphi} - \frac{16}{35 \sin^3 \varphi} \right) \frac{1}{\cos \varphi} - \frac{128}{35} \cot 2\varphi$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos^5 \varphi} = \left( -\frac{1}{7 \sin^7 \varphi} - \frac{9}{35 \sin^5 \varphi} - \frac{3}{5 \sin^3 \varphi} - \frac{3}{\sin \varphi} \right) \frac{1}{\cos^3 \varphi} + 9 \int \frac{\partial \varphi}{\cos^3 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos^7 \varphi} = \left( -\frac{1}{7 \sin^7 \varphi} - \frac{2}{7 \sin^5 \varphi} \right) \frac{1}{\cos^5 \varphi} + \frac{16}{7} \int \frac{\partial \varphi}{\sin^4 \varphi \cos^4 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos^9 \varphi} = \left( -\frac{1}{7 \sin^7 \varphi} - \frac{11}{35 \sin^5 \varphi} - \frac{33}{35 \sin^3 \varphi} \right) \frac{1}{\cos^7 \varphi} + \frac{33}{5} \int \frac{\partial \varphi}{\sin^2 \varphi \cos^5 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos^6 \varphi} = -\frac{1}{7 \sin^7 \varphi \cos^5 \varphi} + \frac{12}{7} \int \frac{\partial \varphi}{\sin^6 \varphi \cos^6 \varphi}$$

$$\int \frac{\partial \varphi}{\sin^8 \varphi \cos^7 \varphi} = -\frac{1}{7 \sin^7 \varphi \cos^6 \varphi} + \frac{13}{6} \int \frac{\partial \varphi}{\sin^6 \varphi \cos^7 \varphi}$$

*Bemerkungen zu den vorhergehenden Tafeln.*

1) Die Formeln S. 261 — S. 286 für das Integral  $\int \partial \varphi \sin^m \varphi \cos^n \varphi$  lassen sich auch bey dem Integral  $\int \partial \varphi \sin^m (k\varphi + l) \cos^n (k\varphi + l)$  anwenden, wenn  $k$  und  $l$  constante Größen sind. Man darf in den gegebenen Formeln nur  $k\varphi + l$  für  $\varphi$  setzen, und hierauf das Ganze mit  $\frac{1}{k}$  multipliciren, jedoch muß vorher das Integralzeichen weggeschafft, und die Formel völlig entwickelt dargestellt werden. Man findet so z. B.

$$\int \partial \varphi \cos(k\varphi + l) = \frac{1}{k} \sin(k\varphi + l)$$

$$\int \partial \varphi \sin(k\varphi + l) = -\frac{1}{k} \cos(k\varphi + l)$$

$$\int \partial \varphi \cos(k\varphi + l) \sin^n(k\varphi + l) = \frac{\sin^{n+1}(k\varphi + l)}{k(n+1)}$$

$$\int \partial \varphi \sin(k\varphi + l) \cos^n(k\varphi + l) = -\frac{\cos^{n+1}(k\varphi + l)}{k(n+1)}$$

$$\int \frac{\partial \varphi}{\sin^2(k\varphi + l) \cos^2(k\varphi + l)} = \frac{1}{k \sin^2(k\varphi + l) \cos(k\varphi + l)} - \frac{3 \cos(k\varphi + l)}{2k \sin^2(k\varphi + l)} + \frac{3}{2k} \log \operatorname{Tang} \frac{1}{2}(k\varphi + l).$$

2) Differentialformeln wie diese:  $\partial \varphi \operatorname{Tang}^m \varphi$ ,  $\partial \varphi \operatorname{Sec}^m \varphi \operatorname{Cot}^n \varphi$ ,  $\partial \varphi \operatorname{Sec}^m \varphi \operatorname{Tang}^n \varphi \operatorname{Cosec}^p \varphi$ , etc., lassen sich auf die Form  $\partial \varphi \sin^m \varphi \cos^n \varphi$  bringen, wenn anstatt  $\operatorname{Tang} \varphi$ ,  $\operatorname{Cot} \varphi$ ,  $\operatorname{Sec} \varphi$ ,  $\operatorname{Cosec} \varphi$ , ihre Werthe  $\frac{\sin \varphi}{\cos \varphi}$ ,  $\frac{\cos \varphi}{\sin \varphi}$ ,  $\frac{1}{\cos \varphi}$ ,  $\frac{1}{\sin \varphi}$  gesetzt werden.

3) Die folgenden Formeln sind, ihres häufigen Gebrauches wegen, noch zu bemerken:

$$\int \partial \varphi \sin(k\varphi + l) \cos(k'\varphi + l') = -\frac{\cos[(k+k')\varphi + l+l']}{2(k+k')} - \frac{\cos[(k-k')\varphi + l-l']}{2(k-k')}$$

$$\int \partial \varphi \sin(k\varphi + l) \sin(k'\varphi + l') = \frac{\sin[(k-k')\varphi + l-l']}{2(k-k')} - \frac{\sin[(k+k')\varphi + l+l']}{2(k+k')}$$

$$\int \partial \varphi \cos(k\varphi + l) \cos(k'\varphi + l') = \frac{\sin[(k+k')\varphi + l+l']}{2(k+k')} + \frac{\sin[(k-k')\varphi + l-l']}{2(k-k')}$$



## Taf. XXVI.

$$\int \varphi^n \partial \varphi \sin \varphi$$

## Allgemeine Formel.

$$\begin{aligned} \int \varphi^n \partial \varphi \sin \varphi = & -\varphi^n \cos \varphi + n\varphi^{n-1} \sin \varphi + n(n-1)\varphi^{n-2} \cos \varphi \\ & - n(n-1)(n-2)\varphi^{n-3} \sin \varphi \\ & - n(n-1)(n-2)(n-3)\varphi^{n-4} \cos \varphi + \dots \end{aligned}$$

## Einzelne Fälle.

$$\int \varphi \partial \varphi \sin \varphi = -\varphi \cos \varphi + \sin \varphi$$

$$\int \varphi^2 \partial \varphi \sin \varphi = -\varphi^2 \cos \varphi + 2\varphi \sin \varphi + 2 \cos \varphi$$

$$\int \varphi^3 \partial \varphi \sin \varphi = -\varphi^3 \cos \varphi + 3\varphi^2 \sin \varphi + 6\varphi \cos \varphi - 6 \sin \varphi$$

$$\begin{aligned} \int \varphi^4 \partial \varphi \sin \varphi = & -\varphi^4 \cos \varphi + 4\varphi^3 \sin \varphi + 12\varphi^2 \cos \varphi - 24\varphi \sin \varphi \\ & - 24 \cos \varphi \end{aligned}$$

$$\begin{aligned} \int \varphi^5 \partial \varphi \sin \varphi = & -\varphi^5 \cos \varphi + 5\varphi^4 \sin \varphi + 20\varphi^3 \cos \varphi - 60\varphi^2 \sin \varphi \\ & - 120\varphi \cos \varphi + 120 \sin \varphi \end{aligned}$$

$$\int \varphi^n \partial \varphi \cos \varphi$$

## Allgemeine Formel.

$$\begin{aligned} \int \varphi^n \partial \varphi \cos \varphi = & \varphi^n \sin \varphi + n\varphi^{n-1} \cos \varphi - n(n-1)\varphi^{n-2} \sin \varphi \\ & - n(n-1)(n-2)\varphi^{n-3} \cos \varphi + \dots \end{aligned}$$

## Einzelne Fälle.

$$\int \varphi \partial \varphi \cos \varphi = \varphi \sin \varphi + \cos \varphi$$

$$\int \varphi^2 \partial \varphi \cos \varphi = \varphi^2 \sin \varphi + 2\varphi \cos \varphi - 2 \sin \varphi$$

$$\int \varphi^3 \partial \varphi \cos \varphi = \varphi^3 \sin \varphi + 3\varphi^2 \cos \varphi - 6\varphi \sin \varphi - 6 \cos \varphi$$

$$\begin{aligned} \int \varphi^4 \partial \varphi \cos \varphi = & \varphi^4 \sin \varphi + 4\varphi^3 \cos \varphi - 12\varphi^2 \sin \varphi - 24\varphi \cos \varphi \\ & + 24 \sin \varphi \end{aligned}$$

$$\begin{aligned} \int \varphi^5 \partial \varphi \cos \varphi = & \varphi^5 \sin \varphi + 5\varphi^4 \cos \varphi - 20\varphi^3 \sin \varphi - 60\varphi^2 \cos \varphi \\ & + 120\varphi \sin \varphi + 120 \cos \varphi \end{aligned}$$

$$\int X \varphi dx$$

Taf. XXVII.

[X eine algebraische Function von x;  $\varphi = \text{Arc Sin } x$ ,  
Arc Cos x, Arc Tang x, etc.]

## Allgemeine Formeln.

$$\int X dx \text{ Arc Sin } x = \text{Arc Sin } x \cdot \int X dx - \int \frac{\partial x \int X dx}{V(1-x^2)}$$

$$\int X dx \text{ Arc Cos } x = \text{Arc Cos } x \cdot \int X dx + \int \frac{\partial x \int X dx}{V(1-x^2)}$$

$$\int X dx \text{ Arc Tang } x = \text{Arc Tang } x \cdot \int X dx - \int \frac{\partial x \int X dx}{1+x^2}$$

$$\int X dx \text{ Arc Cot } x = \text{Arc Cot } x \cdot \int X dx + \int \frac{\partial x \int X dx}{1+x^2}$$

$$\int X dx \text{ Arc Sec } x = \text{Arc Sec } x \cdot \int X dx - \int \frac{\partial x \int X dx}{x V(x^2-1)}$$

$$\int X dx \text{ Arc Cosec } x = \text{Arc Cosec } x \cdot \int X dx + \int \frac{\partial x \int X dx}{x V(x^2-1)}$$

$$\int X dx \text{ Arc Sin vers } x = \text{Arc Sin v. } x \cdot \int X dx - \int \frac{\partial x \int X dx}{V(2x-x^2)}$$

## Einzelne Fälle.

$$\int \partial x \text{ Arc Sin } x = x \text{ Arc Sin } x - \int \frac{x \partial x}{V(1-x^2)}$$

$$\int x^m \partial x \text{ Arc Sin } x = \frac{x^{m+1}}{m+1} \text{ Arc Sin } x - \frac{1}{m+1} \int \frac{x^{m+1} \partial x}{V(1-x^2)}$$

$$\int \frac{\partial x}{V(1-x^2)} \text{ Arc Sin } x = \frac{1}{2} (\text{Arc Sin } x)^2$$

$$\int \frac{x \partial x}{V(1-x^2)} \text{ Arc Sin } x = -\text{Arc Sin } x \cdot V(1-x^2) + x$$

$$\int \frac{x^2 \partial x}{V(1-x^2)} \text{ Arc Sin } x = \left( -\frac{1}{2} x V(1-x^2) + \frac{1}{4} \text{Arc Sin } x \right) \text{Arc Sin } x + \frac{1}{4} x^2$$

$$\int \frac{x^3 \partial x}{V(1-x^2)} \text{ Arc Sin } x = -\left( \frac{1}{3} x^2 + \frac{2}{5} \right) V(1-x^2) \cdot \text{Arc Sin } x + \frac{1}{9} x^3 + \frac{2}{3} x$$

$$\int \frac{x^4 \partial x}{V(1-x^2)} \text{ Arc Sin } x = \left[ -\left( \frac{1}{4} x^3 + \frac{5}{8} x \right) V(1-x^2) + \frac{5}{16} \text{Arc Sin } x \right]$$

$$\times \text{Arc Sin } x + \frac{1}{16} x^4 + \frac{5}{16} x^2$$

$$\int \frac{x^5 \partial x}{V(1-x^2)} \text{Arc Sin } x = -\left(\frac{1}{6}x^4 + \frac{4}{15}x^2 + \frac{8}{15}\right)VX \cdot \text{Arc Sin } x \\ + \frac{1}{25}x^5 + \frac{4}{45}x^3 + \frac{8}{15}x$$

$$\int \frac{\partial x}{(1-x^2)^{\frac{1}{2}}} \text{Arc Sin } x = \frac{x \text{Arc Sin } x}{V(1-x^2)} + \frac{1}{2} \log(1-x^2)$$

$$\int \frac{x \partial x}{(1-x^2)^{\frac{1}{2}}} \text{Arc Sin } x = \frac{\text{Arc Sin } x}{V(1-x^2)} + \frac{1}{2} \log \frac{1-x}{1+x}$$

$$\int x^m \partial x \text{Arc Cos } x = \frac{x^{m+1}}{m+1} \text{Arc Cos } x + \frac{1}{m+1} \int \frac{x^{m+1} \partial x}{V(1-x^2)}$$

$$\int x^m \partial x \text{Arc Tang } x = \frac{x^{m+1}}{m+1} \text{Arc Tang } x - \frac{1}{m+1} \int \frac{x^{m+1} \partial x}{1+x^2}$$

$$\int x^m \partial x \text{Arc Cot } x = \frac{x^{m+1}}{m+1} \text{Arc Cot } x + \frac{1}{m+1} \int \frac{x^{m+1} \partial x}{1+x^2}$$

$$\int x^m \partial x \text{Arc Sec } x = \frac{x^{m+1}}{m+1} \text{Arc Sec } x - \frac{1}{m+1} \int \frac{x^m \partial x}{V(x^2-1)}$$

$$\int x^m \partial x \text{Arc Cosec } x = \frac{x^{m+1}}{m+1} \text{Arc Cosec } x + \frac{1}{m+1} \int \frac{x^m \partial x}{V(x^2-1)}$$

$$\int x^m \partial x \text{Arc Sin vers } x = \frac{x^{m+1}}{m+1} \text{Arc Sin v. } x - \frac{1}{m+1} \int \frac{x^{m+1} \partial x}{V(2x-x^2)}$$

$$\int \frac{\partial x}{1+x^2} \text{Arc Tang } x = \frac{1}{2} (\text{Arc Tang } x)^2$$

$$\int \frac{x^2 \partial x}{1+x^2} \text{Arc Tang } x = \left(x - \frac{1}{2} \text{Arc Tang } x\right) \text{Arc Tang } x - \frac{1}{2} \log(1+x^2)$$

$$\int \frac{\partial x}{(1+x^2)^2} \text{Arc Tang } x = \left(\frac{x}{2(1+x^2)} + \frac{1}{4} \text{Arc Tang } x\right) \text{Arc Tang } x \\ + \frac{1}{4(1+x^2)}$$

$$\int \frac{\partial x}{V(1-x^2)} \text{Arc Cos } x = -\frac{1}{2} (\text{Arc Cos } x)^2$$

$$\int \frac{\partial x}{1+x^2} \text{Arc Cot } x = -\frac{1}{2} (\text{Arc Cot } x)^2$$

$$\int \frac{\partial x}{V(2x-x^2)} \text{Arc Sin vers } x = \frac{1}{2} (\text{Arc Sin v. } x)^2$$

$$\int X dx \log Z$$

Taf. XXVIII.

(X, Z, algebraische Functionen von x)

Allgemeine Formel.

$$\int X dx \log Z = \log Z \cdot \int X dx - \int \frac{\partial Z / X dx}{Z}$$

Einzelne Fälle.

$$\int X dx \log x = \log x \cdot \int X dx - \int \frac{\partial x / X dx}{x}$$

$$\int x^m dx \log x = \frac{x^{m+1}}{m+1} \left( \log x - \frac{1}{m+1} \right)$$

$$\int (a+bx)^m dx \log x = \frac{(a+bx)^{m+1}}{(m+1)b} \log x - \frac{1}{(m+1)b} \int \frac{\partial x (a+bx)^{m+1}}{x}$$

$$\int x^{-1} dx \log x = \int \frac{\partial x}{x} \log x = \frac{1}{2} \log^2 x$$

$$\int \frac{\partial x}{a+bx} \log x = \frac{1}{b} \log x \cdot \log(a+bx) - \frac{1}{b} \int \frac{\partial x}{x} \log(a+bx)$$

Hieraus erhält man entweder \*)

$$\int \frac{\partial x}{a+bx} \log x = \frac{1}{b} \log x \cdot \log \frac{a+bx}{a} - \frac{x}{a} + \frac{bx^2}{2a^2} - \frac{b^2 x^3}{3^2 a^3} + \text{etc.}$$

oder

$$\begin{aligned} \int \frac{\partial x}{a+bx} \log x &= \frac{1}{b} \log x \cdot \log(a+bx) - \frac{1}{2b} (\log bx)^2 + \frac{a}{b^2 x} - \frac{a^2}{2^2 b^3 x^2} \\ &+ \frac{a^3}{3^2 b^4 x^3} - \frac{a^4}{4^2 b^5 x^4} + \text{etc.} \end{aligned}$$

$$\int x^m dx \log(a+bx) = \frac{x^{m+1}}{m+1} \log(a+bx) - \frac{b}{m+1} \int \frac{x^{m+1} \partial x}{a+bx}$$

$$\int \frac{\partial x}{x} \log(a+bx) = \log a \cdot \log x + \frac{bx}{a} - \frac{b^2 x^2}{2^2 a^2} + \frac{b^3 x^3}{3^2 a^3} - \text{etc.}$$

$$\int \frac{\partial x}{x} \log(a+bx) = \frac{1}{2} (\log bx)^2 - \frac{a}{bx} + \frac{a^2}{2^2 b^2 x^2} - \frac{a^3}{3^2 b^3 x^3} + \text{etc.}$$

\*) Man s. die beiden letzten Formeln auf dieser Seite. Es wird nämlich  $\log(a+bx)$  in eine nach den Potenzen von  $x$  steigende oder fallende Reihe verwandelt, mit  $\frac{\partial x}{x}$  multiplicirt und hierauf integrirt.

Taf. XXIX.

$$\int X dx \log^n x$$

Allgemeine Formel.

$$\int X dx \log^n x = X' \log^n x - n X'' \log^{n-1} x + n(n-1) X''' \log^{n-2} x + n(n-1)(n-2) X'''' \log^{n-3} x + \text{etc.}$$

$$X = \int X dx, X'' = \int \frac{X' dx}{x}, X''' = \int \frac{X'' dx}{x}, \text{etc.}$$

Einzelne Fälle.

$$\int x^m dx \log^n x = \frac{x^{m+1}}{m+1} \left( \log^n x - \frac{n}{m+1} \log^{n-1} x + \frac{n(n-1)}{(m+1)^2} \log^{n-2} x - \frac{n(n-1)(n-2)}{(m+1)^3} \log^{n-3} x + \text{etc.}^*) \right)$$

$$\int x^{-1} dx \log^n x = \int \frac{dx}{x} \log^n x = \frac{1}{n+1} \log^{n+1} x$$

$$\int x^m dx \log x = \frac{x^{m+1}}{m+1} \left( \log x - \frac{1}{m+1} \right)$$

$$\int x^m dx \log^2 x = \frac{x^{m+1}}{m+1} \left( \log^2 x - \frac{2}{m+1} \log x + \frac{2 \cdot 1}{(m+1)^2} \right)$$

$$\int x^m dx \log^3 x = \frac{x^{m+1}}{m+1} \left( \log^3 x - \frac{3}{m+1} \log^2 x + \frac{3 \cdot 2}{(m+1)^2} \log x - \frac{3 \cdot 2 \cdot 1}{(m+1)^3} \right)$$

$$\int \frac{x^m dx}{\sqrt{\log x}} = \frac{x^{m+1}}{(m+1) \sqrt{\log x}} \left( 1 + \frac{1}{(2m+2) \log x} + \frac{1 \cdot 3}{[(2m+2) \log x]^2} + \frac{1 \cdot 3 \cdot 5}{[(2m+2) \log x]^3} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{[(2m+2) \log x]^4} + \text{in infinit.} \right)$$

$$\int \frac{x^m dx}{\sqrt{\log \frac{1}{x}}} = \frac{x^{m+1}}{(m+1) \sqrt{\log \frac{1}{x}}} \left( 1 + \frac{1}{(2m+2) \log x} + \frac{1 \cdot 3}{[(2m+2) \log x]^2} + \frac{1 \cdot 3 \cdot 5}{[(2m+2) \log x]^3} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{[(2m+2) \log x]^4} + \text{in infinit.} \right)$$

$\left\{ \begin{array}{l} \text{Die erste von den beiden Integralformeln } \int \frac{x^m dx}{\sqrt{\log x}}, \int \frac{x^m dx}{\sqrt{\log \frac{1}{x}}} \\ \text{wird imaginär, wenn } x \text{ zwischen } 0 \text{ und } 1 \text{ fällt, die zweite wird} \\ \text{es, wenn } x > 1. \end{array} \right\}$

\*) Die Reihe bricht ab, wenn  $n$  eine ganze positive Zahl ist. Wenn  $n$  eine ganze negative Zahl ist, läßt sich ebenfalls eine endliche Reihe finden. M. a. die folg. Seite.

$$\int \frac{X dx}{\log^n x}$$

Taf. XXX.

Allgemeine Formel.

$$\int \frac{X dx}{\log^n x} = -\frac{Xx}{(n-1)\log^{n-1}x} - \frac{X'x}{(n-1)(n-2)\log^{n-2}x} - \frac{X''x}{(n-1)(n-2)(n-3)\log^{n-3}x} - \text{etc.}$$

$$X' = \frac{\partial(Xx)}{\partial x}, \quad X'' = \frac{\partial(X'x)}{\partial x}, \quad X''' = \frac{\partial(X''x)}{\partial x}, \quad \text{etc.}$$

Einzelne Fälle.

$$\begin{aligned} \int \frac{x^m dx}{\log^n x} &= -\frac{x^{m+1}}{(n-1)\log^{n-1}x} - \frac{(m+1)x^{m+1}}{(n-1)(n-2)\log^{n-2}x} \\ &\quad - \frac{(m+1)^2 x^{m+1}}{(n-1)(n-2)(n-3)\log^{n-3}x} - \dots \\ &\quad - \frac{(m+1)^{n-2} x^{m+1}}{(n-1)(n-2)(n-3)\dots 2 \cdot 1 \log x} + \frac{(m+1)^{n-1}}{(n-1)(n-2)\dots 2 \cdot 1} \int \frac{x^m dx}{\log x} \end{aligned}$$

$$\int \frac{dx}{x \log^n x} = \frac{1}{n-1} \log^{n-1} x$$

$$\begin{aligned} \int \frac{dx}{\log x} &= \log \log x + \frac{\log x}{1} + \frac{1}{2} \cdot \frac{\log^2 x}{1 \cdot 2} + \frac{1}{3} \cdot \frac{\log^3 x}{1 \cdot 2 \cdot 3} + \frac{1}{4} \cdot \frac{\log^4 x}{1 \cdot 2 \cdot 3 \cdot 4} \\ &\quad + \frac{1}{5} \cdot \frac{\log^5 x}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \text{in infinit.} \end{aligned}$$

$$\int \frac{x^m dx}{\log x} = \int \frac{dy}{\log y} \quad \text{für } y = x^{m+1}$$

$$\int \frac{x^m dx}{\log^2 x} = -\frac{x^{m+1}}{\log x} + \frac{m+1}{1} \int \frac{x^m dx}{\log x}$$

$$\int \frac{x^m dx}{\log^3 x} = -\frac{x^{m+1}}{2 \log^2 x} - \frac{(m+1)x^{m+1}}{2 \cdot 1 \log x} + \frac{(m+1)^2}{2 \cdot 1} \int \frac{x^m dx}{\log x}$$

$$\begin{aligned} \int \frac{dx}{\log^3 x} &= \log \log x - \frac{\log x}{1} + \frac{1}{2} \cdot \frac{\log^2 x}{1 \cdot 2} - \frac{1}{3} \cdot \frac{\log^3 x}{1 \cdot 2 \cdot 3} + \frac{1}{4} \cdot \frac{\log^4 x}{1 \cdot 2 \cdot 3 \cdot 4} \\ &\quad - \frac{1}{5} \cdot \frac{\log^5 x}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \dots + \text{in infinit.} \end{aligned}$$

$$\int' \frac{dx}{V \log \frac{1}{x}} = V\pi \quad \left[ \begin{array}{l} \text{Das Integral von } x=0 \text{ bis } x=1 \text{ genommen.} \\ \text{(Euler Comment. Acad. Petrop. Tom. XVI. p. 111.)} \end{array} \right]$$

$$\int' dx \left( \log \frac{1}{x} \right)^{\frac{2n+1}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n+1)}{2^{n+1}} V\pi \quad (\text{Ebend.})$$

Taf. XXXI.

$$\int a^x X dx$$

Allgemeine Formeln.

$$\int a^x X dx = \frac{a^x X}{\log a} - \frac{a^x X'}{\log^2 a} + \frac{a^x X''}{\log^3 a} - \frac{a^x X'''}{\log^4 a} + \dots$$

$$X' = \frac{\partial X}{\partial x}, X'' = \frac{\partial^2 X}{\partial x^2}, X''' = \frac{\partial^3 X}{\partial x^3}, \text{ etc.}$$

$$\int a^x X dx = a^x X_1 - a^x X_2 \log a + a^x X_3 \log^2 a - a^x X_4 \log^3 a + a^x X_5 \log^4 a - \dots$$

$$X_1 = \int X dx, X_2 = \int X_1 dx, X_3 = \int X_2 dx, \text{ etc.}$$

Einzelne Fälle.

$$\int a^x x^n dx = \frac{a^x x^n}{\log a} - \frac{n a^x x^{n-1}}{\log^2 a} + \frac{n(n-1) a^x x^{n-2}}{\log^3 a} - \frac{n(n-1)(n-2) a^x x^{n-3}}{\log^4 a} + \dots + \frac{n(n-1)(n-2) \dots 2 \cdot 1 a^x}{\log^{n+1} a}$$

$$\int \frac{a^x dx}{x^n} = -\frac{a^x}{(n-1)x^{n-1}} - \frac{a^x \log a}{(n-1)(n-2)x^{n-2}} - \frac{a^x \log^2 a}{(n-1)(n-2)(n-3)x^{n-3}} - \dots - \frac{a^x \log^{n-2} a}{(n-1)(n-2) \dots 2 \cdot 1 x} + \frac{\log^{n-1} a}{(n-1)(n-2) \dots 2 \cdot 1} \int \frac{a^x dx}{x}$$

$$\int a^x dx = \frac{a^x}{\log a}, \int a^{mx} dx = \frac{a^{mx}}{m \log a}, \int e^{mx} dx = \frac{e^{mx}}{m}$$

$$\int a^x x dx = \frac{a^x x}{\log a} - \frac{a^x}{\log^2 a}$$

$$\int a^x x^2 dx = \frac{a^x x^2}{\log a} - \frac{2 a^x x}{\log^2 a} + \frac{2 \cdot 1 a^x}{\log^3 a}$$

$$\int a^x x^3 dx = \frac{a^x x^3}{\log a} - \frac{3 a^x x^2}{\log^2 a} + \frac{3 \cdot 2 a^x x}{\log^3 a} - \frac{3 \cdot 2 \cdot 1 a^x}{\log^4 a}$$

$$\int \frac{a^x dx}{x} = \log x + \frac{x \log a}{1} + \frac{x^2 \log^2 a}{1 \cdot 2 \cdot 2} + \frac{x^3 \log^3 a}{1 \cdot 2 \cdot 3 \cdot 3} + \frac{x^4 \log^4 a}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 4} + \frac{x^5 \log^5 a}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 5} + \text{in infinit.}$$

$$\int \frac{a^x dx}{x^2} = -\frac{a^x}{x} + \log a \int \frac{a^x dx}{x}$$

$$\int \frac{a^x \partial x}{x^3} = -\frac{a^x}{2x^2} - \frac{a^x \log a}{2 \cdot 1 x} + \frac{\log^2 a}{2 \cdot 1} \int \frac{a^x \partial x}{x}$$

$$\int \frac{a^x \partial x}{x^4} = -\frac{a^x}{3x^3} - \frac{a^x \log a}{3 \cdot 2 x^2} - \frac{a^x \log^2 a}{3 \cdot 2 \cdot 1 x} + \frac{\log^3 a}{3 \cdot 2 \cdot 1} \int \frac{a^x \partial x}{x}$$

$$\int \frac{a^x \partial x}{Vx} = \frac{a^x}{Vx} \left( \frac{1}{\log a} + \frac{1}{2x \log^2 a} + \frac{1 \cdot 3}{2^2 x^2 \log^3 a} + \frac{1 \cdot 3 \cdot 5}{2^3 x^3 \log^4 a} + \text{etc.} \right)$$

$$\int \frac{a^x \partial x}{Vx} = \frac{a^x}{Vx} \left( \frac{2x}{1} - \frac{2^2 x^2 \log a}{1 \cdot 3} + \frac{2^3 x^3 \log^2 a}{1 \cdot 3 \cdot 5} - \frac{2^4 x^4 \log^3 a}{1 \cdot 3 \cdot 5 \cdot 7} + \text{etc.} \right)$$

$$\int \frac{a^x \partial x}{1-x} = a^x \left[ \frac{1}{(1-x) \log a} - \frac{1}{(1-x)^2 \log^2 a} + \frac{1 \cdot 2}{(1-x)^3 \log^3 a} - \frac{1 \cdot 2 \cdot 3}{(1-x)^4 \log^4 a} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{(1-x)^5 \log^5 a} - \text{etc.} \right]$$

$$\int a^{mx} x^n \partial x = \frac{1}{m^{n+1}} \int a^y y^n \partial y \text{ für } y = mx$$

$$\int x^{m+1} x^n \partial x = \int \left( 1 + \frac{nx \log x}{1} + \frac{n^2 x^2 \log^2 x}{1 \cdot 2} + \frac{n^3 x^3 \log^3 x}{1 \cdot 2 \cdot 3} + \text{etc.} \right) x^m \partial x$$

$$\begin{aligned} &= *) \quad x^{m+1} \left( \frac{1}{m+1} - \frac{nx}{(m+2)^2} + \frac{n^2 x^2}{(m+3)^3} - \frac{n^3 x^3}{(m+4)^4} + \text{etc.} \right) \\ &+ \frac{nx^{m+2} \log x}{1} \left( \frac{1}{m+2} - \frac{nx}{(m+3)^2} + \frac{n^2 x^2}{(m+4)^3} - \frac{n^3 x^3}{(m+5)^4} + \text{etc.} \right) \\ &+ \frac{n^2 x^{m+3} \log^2 x}{1 \cdot 2} \left( \frac{1}{m+3} - \frac{nx}{(m+4)^2} + \frac{n^2 x^2}{(m+5)^3} - \frac{n^3 x^3}{(m+6)^4} + \text{etc.} \right) \\ &+ \frac{n^3 x^{m+4} \log^3 x}{1 \cdot 2 \cdot 3} \left( \frac{1}{m+4} - \frac{nx}{(m+5)^2} + \frac{n^2 x^2}{(m+6)^3} - \frac{n^3 x^3}{(m+7)^4} + \text{etc.} \right) \\ &\quad \text{etc.} \quad \text{etc.} \quad \text{etc.} \end{aligned}$$

$$\int e^{-x} \partial x = Vx \quad \left[ \text{Das Integral von } x = -\infty \text{ bis } x = +\infty. \right]$$

[Laplace, Mécan. cel. livre X. No. 5.]

\*) Durch Integrirung der Differentiale  $x^m \partial x$ ,  $x^{m+1} \partial x \log x$ ,  $x^{m+2} \partial x \log^2 x$ ,  $x^{m+3} \partial x \log^3 x$ , etc. Der Werth des Integrals  $\int x^{m+1} x^n \partial x$  zwischen den Grenzen 0 und 1 genommen, jedoch unter der Voraussetzung, daß  $m+1$  eine positive Zahl sey, ist

$$= \frac{1}{m+1} - \frac{n}{(m+2)^2} + \frac{n^2}{(m+3)^3} - \frac{n^3}{(m+4)^4} + \frac{n^4}{(m+5)^5} - \text{etc.}$$

*Handwritten note:*  $\int \sin q x^2 + \int \cos q x^2$  ...  
 Bernol. Prog. An 30, 1776.



Taf. XXXII.  $\int e^{ax} dx \sin^m x, \int e^{ax} dx \cos^n x$

Reductionsformeln.

$$\int e^{ax} dx \sin^n x = \frac{e^{ax} \sin^{n-1} x (a \sin x - n \cos x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2} \int e^{ax} dx \sin^{n-2} x$$

$$\int e^{ax} dx \cos^n x = \frac{e^{ax} \cos^{n-1} x (a \cos x + n \sin x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2} \int e^{ax} dx \cos^{n-2} x$$

Einzelne Formeln.

$$\int e^{ax} dx \sin x = \frac{e^{ax} (a \sin x - \cos x)}{a^2 + 1}$$

$$\int e^{ax} dx \sin^2 x = \frac{e^{ax} \sin x (a \sin x - 2 \cos x)}{a^2 + 4} + \frac{1 \cdot 2}{a(a^2 + 4)} e^{ax}$$

$$\int e^{ax} dx \sin^3 x = \frac{e^{ax} \sin^2 x (a \sin x - 3 \cos x)}{a^2 + 9} + \frac{2 \cdot 3 e^{ax} (a \sin x - \cos x)}{(a^2 + 1)(a^2 + 9)}$$

$$\int e^{ax} dx \cos x = \frac{e^{ax} (a \cos x + \sin x)}{a^2 + 1}$$

$$\int e^{ax} dx \cos^2 x = \frac{e^{ax} \cos x (a \cos x + 2 \sin x)}{a^2 + 4} + \frac{1 \cdot 2}{a(a^2 + 4)} e^{ax}$$

$$\int e^{ax} dx \cos^3 x = \frac{e^{ax} \cos^2 x (a \cos x + 3 \sin x)}{a^2 + 9} + \frac{2 \cdot 3 e^{ax} (a \cos x + \sin x)}{(a^2 + 1)(a^2 + 9)}$$

$$\int e^{ax} dx \sin kx = \frac{e^{ax} (a \sin kx - k \cos kx)}{a^2 + k^2}$$

$$\int e^{ax} dx \cos kx = \frac{e^{ax} (a \cos kx + k \sin kx)}{a^2 + k^2}$$

Mit Hülfe der beiden letzten Formeln kann auch das Integral  $\int e^{ax} dx \sin^m x \cos^n x$  gefunden werden, wenn nämlich  $\sin^m x \cos^n x$  nach Sinus und Cosinus der vielfachen Winkel entwickelt wird, wodurch man lauter Monomen von der Form  $e^{ax} dx \sin kx$ ,  $e^{ax} dx \cos kx$ ,  $e^{ax} dx$ , erhält.

$$\int \frac{(f + g \cos \varphi) d\varphi}{(a + b \cos \varphi)^n}$$

Taf. XXXIII.

Reductionsformel.

$$\begin{aligned} \int \frac{(f + g \cos \varphi) d\varphi}{(a + b \cos \varphi)^n} &= \frac{(ag - bf) \sin \varphi}{(n-1)(a^2 - b^2)(a + b \cos \varphi)^{n-1}} \\ &+ \frac{1}{(n-1)(a^2 - b^2)} \int \frac{[(n-1)(af - bg) + (n-2)(ag - bf) \cos \varphi] d\varphi}{(a + b \cos \varphi)^{n-1}} \end{aligned}$$

Einzelne Fälle.

$$\begin{aligned} \int \frac{d\varphi}{a + b \cos \varphi} &= \frac{2}{V(a^2 - b^2)} \text{Arc Tang} \frac{(a-b) \text{Tang} \frac{1}{2} \varphi}{V(a^2 - b^2)} \\ &= \frac{1}{V(a^2 - b^2)} \text{Arc Tang} \frac{\sin \varphi V(a^2 - b^2)}{b + a \cos \varphi} \\ &= \frac{1}{V(a^2 - b^2)} \text{Arc Sin} \frac{\sin \varphi V(a^2 - b^2)}{a + b \cos \varphi} \\ &= \frac{1}{V(a^2 - b^2)} \text{Arc Cos} \frac{b + a \cos \varphi}{a + b \cos \varphi} \end{aligned}$$

$$\int \frac{d\varphi}{a + b \cos \varphi} = \frac{1}{V(b^2 - a^2)} \log \frac{b + a \cos \varphi + \sin \varphi V(b^2 - a^2)}{a + b \cos \varphi}$$

Der erste von diesen beiden Werthen für  $b < a$ ,  
der zweite, für  $b > a$ ; für  $b = a$  ist

$$\int \frac{d\varphi}{a + a \cos \varphi} = \frac{1}{a} \int \frac{d\varphi}{1 + \cos \varphi} = \frac{1}{a} \text{Tang} \frac{1}{2} \varphi$$

$$\int \frac{d\varphi \sin \varphi}{a + b \cos \varphi} = -\frac{1}{b} \log(a + b \cos \varphi)$$

$$\int \frac{d\varphi \cos \varphi}{a + b \cos \varphi} = \frac{\varphi}{b} - \frac{a}{b} \int \frac{d\varphi}{a + b \cos \varphi}$$

$$\int \frac{d\varphi}{(a + b \cos \varphi)^2} = \frac{1}{a^2 - b^2} \left( \frac{-b \sin \varphi}{a + b \cos \varphi} + a \int \frac{d\varphi}{a + b \cos \varphi} \right)$$

$$\int \frac{d\varphi \cos \varphi}{(a + b \cos \varphi)^2} = \frac{1}{a^2 - b^2} \left( \frac{a \sin \varphi}{a + b \cos \varphi} - b \int \frac{d\varphi}{a + b \cos \varphi} \right)$$

## Taf. XXXIV.

## Entwicklung des Integrals

$$\int \partial \varphi (1 + n \cos \varphi)^p$$

nach vielfachen Winkeln. \*)

I.  $p$  positiv.  $p = +m$ 

Allgemeine Formel.

$$\int \partial \varphi (1 + n \cos \varphi)^m = A\varphi + B \sin \varphi + \frac{1}{2} C \sin 2\varphi + \frac{1}{3} D \sin 3\varphi \\ + \frac{1}{4} E \sin 4\varphi + \frac{1}{5} F \sin 5\varphi + \text{etc.}$$

$$A = 1 + \frac{1}{2} {}^m\mathfrak{B}n^2 + \frac{1 \cdot 3}{2 \cdot 4} {}^m\mathfrak{D}n^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} {}^m\mathfrak{F}n^6 \\ + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} {}^m\mathfrak{H}n^8 + \text{etc.}$$

$$B = 2n \left( \frac{1}{2} {}^m\mathfrak{A} + \frac{1 \cdot 3}{2 \cdot 4} {}^m\mathfrak{C}n^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} {}^m\mathfrak{E}n^4 \right. \\ \left. + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} {}^m\mathfrak{G}n^6 + \text{etc.} \right)$$

$$C = \frac{2mnA - 2B}{(m+2)n}, \quad D = \frac{(m-1)nB - 4C}{(m+3)n} \\ E = \frac{(m-2)nC - 6D}{(m+4)n}, \quad F = \frac{(m-3)nD - 8E}{(m+5)n} \\ G = \frac{(m-4)nE - 10F}{(m+6)n}, \quad H = \frac{(m-5)nF - 12G}{(m+7)n}$$

etc.

[Die Reihen für  $A$  und  $B$  brechen ab, wenn]  
 $m$  eine ganze Zahl ist.]

\*) Das Integral  $\int \partial \varphi (a + b \cos \varphi)^p$  lässt sich auf dieses zurückführen, wenn  $\frac{b}{a} = n$  gesetzt wird: denn es ist

$$\int \partial \varphi (a + b \cos \varphi)^p = a^p \int \partial \varphi (1 + n \cos \varphi)^p$$

## Einzelne Fälle.

 Für  $m = 1$  ist

$$A = 1, B = n, (C, D, E, \text{ etc.} = 0).$$

 Für  $m = 2$  ist

$$A = 1 + \frac{1}{2}n^2, B = 2n, C = \frac{1}{2}n^2, (D, E, \text{ etc.} = 0).$$

 Für  $m = 3$  ist

$$A = 1 + \frac{3}{2}n^2, B = 3n + \frac{5}{4}n^3, C = \frac{3}{2}n^2,$$

$$D = \frac{1}{4}n^3, (E, F, \text{ etc.} = 0).$$

 Für  $m = 4$  ist

$$A = 1 + 3n^2 + \frac{5}{8}n^4, B = 4n + 3n^3, C = 3n^2 + \frac{1}{8}n^4,$$

$$D = n^3, E = \frac{1}{8}n^4, (F, G, \text{ etc.} = 0).$$

 Für  $m = \frac{1}{2}$  ist

$$A = 1 - \frac{1 \cdot 1}{4 \cdot 4}n^2 - \frac{1 \cdot 1 \cdot 3 \cdot 5}{4 \cdot 4 \cdot 8 \cdot 8}n^4 - \frac{1 \cdot 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{4 \cdot 4 \cdot 8 \cdot 8 \cdot 12 \cdot 12}n^6 - \text{etc.}$$

$$B = \frac{1}{2}n + \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 8}n^3 + \frac{1 \cdot 1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 8 \cdot 8 \cdot 12}n^5 + \text{etc.}$$

$$C = \frac{2nA - 4B}{5n}, D = \frac{-nB - 8C}{7n}, \text{ etc.}$$

 II.  $p$  negativ.  $p = -m$ 

Reductionsformel.

Es werde gesetzt

$$\int \partial \varphi (1 + n \cos \varphi)^{-m} = A\varphi + B \sin \varphi + \frac{1}{2}C \sin 2\varphi + \frac{1}{3}D \sin 3\varphi \\ + \frac{1}{4}E \sin 4\varphi + \frac{1}{5}F \sin 5\varphi + \text{etc.}$$

$$\int \partial \varphi (1 + n \cos \varphi)^{-m-1} = A'\varphi + B' \sin \varphi + \frac{1}{2}C' \sin 2\varphi + \frac{1}{3}D' \sin 3\varphi \\ + \frac{1}{4}E' \sin 4\varphi + \frac{1}{5}F' \sin 5\varphi + \text{etc.}$$

so ist

$$A' = \frac{2m A - (m-1)nB}{2m(1-n^2)} = A + \frac{n\partial A}{m\partial n}$$

$$B' = \frac{2(A - A')}{n} = B + \frac{n\partial B}{m\partial n}$$

$$C' = \frac{2(B - B') - 2nA'}{n} = C + \frac{n\partial C}{m\partial n}$$

$$D' = \frac{2(C - C') - nB'}{n} = D + \frac{n\partial D}{m\partial n}$$

$$E' = \frac{2(D - D') - nC'}{n} = E + \frac{n\partial E}{m\partial n}$$

etc.

etc.

Mit Hülfe dieser doppelten Reductionsformeln, welche einander wechselseitig zur Prüfung dienen können, lassen sich die Coefficienten  $A, B, C, D$ , etc., für die Werthe  $p = -2, -3, -4$ , etc., aus den Werthen derselben für  $p = -1$  bestimmen. Für  $p = -1$  ist aber

$$A = \frac{1}{V(1-n^2)}$$

$$B = \frac{2-2V(1-n^2)}{nV(1-n^2)}$$

$$C = \frac{4-2n^2-4V(1-n^2)}{n^2V(1-n^2)}$$

$$D = \frac{8-6n^2-2(4-n^2)V(1-n^2)}{n^3V(1-n^2)}$$

$$E = \frac{16-16n^2+2n^4-2(8-4n^2)V(1-n^2)}{n^4V(1-n^2)}$$

.....

$$A = \frac{2}{V(1-n^2)} \left( \frac{1-V(1-n^2)}{n} \right)^{\mu}$$

Die Werthe von  $A, B, C, D$ , etc., für  $p = -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}$ , etc., lassen sich durch die nämlichen Reductionsformeln aus den Werthen derselben für  $p = \frac{1}{2}$  (S. 299) herleiten; sie lassen sich aber nicht wohl anders als durch Reihen ausdrücken.

Mit der Auflösung der Winkelfunction  $(1 + n \cos \varphi)^p$  in eine Reihe von der Form  $A + B \cos \varphi + C \cos 2\varphi + D \cos 3\varphi + \text{etc.}$  worauf es bei der Integration des Differentials  $d\varphi(1 + n \cos \varphi)^p$  einzig und allein ankommt, haben sich die Analysten vielfältig beschäftigt. Sie hat vorzüglich in der Astronomie ihren Nutzen, wo sie in der Form  $(r^2 + r'^2 - rr' \cos \varphi)^p$ , oder in dieser etwas einfacheren  $(1 + a^2 - a \cos \varphi)^p$  vorkommt. Die ausführlichste Belehrung darüber findet man in Eulers *Instit. calc. integr.* und in dem *Traité du calc. diff. et integr.* von Lacroix. Laplace giebt im zweiten Buche seiner *Mécanique céleste* die folgenden Reductionsformeln.

Es sey nach seiner Bezeichnung

$$(1 + a^2 - a \cos \varphi)^{-s} = \frac{1}{2} b_{,s}^{(0)} + b_{,s}^{(1)} \cos \varphi + b_{,s}^{(2)} \cos 2\varphi + \text{etc.}$$

$$(1 + a^2 - a \cos \varphi)^{-s-1} = \frac{1}{2} b_{,s+1}^{(0)} + b_{,s+1}^{(1)} \cos \varphi + b_{,s+1}^{(2)} \cos 2\varphi + \text{etc.}$$

so ist

$$b_{,s}^{(i)} = \frac{(i-1)(1+a^2)b_{,s}^{(i-1)} - (i+s-2)a b_{,s}^{(i-2)}}{(i-s)a}$$

$$b_{,s+1}^{(i)} = \frac{(s+i)(1+a^2)b_{,s}^{(i)} - 2(i-s+1)a b_{,s}^{(i+1)}}{s(1-a^2)^2}$$

$$b_{,s+1}^{(i)} = \frac{(s-i)(1+a^2)b_{,s}^{(i)} + 2(i+s-1)a b_{,s}^{(i-1)}}{s(1-a^2)^2}$$

Für die Werthe von  $b_{,s}^{(0)}$ ,  $b_{,s}^{(1)}$ , giebt er folgende Reihen:

$$b_{,s}^{(0)} = 2 \left[ 1 + s^2 \cdot a^2 + \left( \frac{s(s+1)}{1 \cdot 2} \right)^2 \cdot a^4 + \left( \frac{s(s+1)(s+2)}{1 \cdot 2 \cdot 3} \right)^2 \cdot a^6 + \text{etc.} \right]$$

$$b_{,s}^{(1)} = 2a \left[ s + s \cdot \frac{s(s+1)}{1 \cdot 2} \cdot a^2 + \frac{s(s+1)}{1 \cdot 2} \cdot \frac{s(s+1)(s+2)}{1 \cdot 2 \cdot 3} \cdot a^4 + \text{etc.} \right]$$

Hieraus erhält man, wenn  $s = -\frac{1}{2}$  gesetzt wird, mithin für  $p = \frac{1}{2}$  folgende Reihen:

$$\frac{1}{2} b_{-\frac{1}{2}}^{(0)} = 1 + \left(\frac{1}{2}\right)^2 \alpha^2 + \left(\frac{1 \cdot 1}{2 \cdot 4}\right)^2 \alpha^4 + \left(\frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6}\right)^2 \alpha^6 + \text{etc.}$$

$$b_{-\frac{1}{2}}^{(1)} = -\alpha \left( 1 - \frac{1 \cdot 1}{2 \cdot 4} \alpha^2 - \frac{1}{4} \cdot \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6} \alpha^4 \right. \\ \left. - \frac{1 \cdot 3}{4 \cdot 6} \cdot \frac{1 \cdot 1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \alpha^6 - \text{etc.} \right)$$

Diese Reihen convergiren sehr.schnell, wenn  $\alpha$  ein etwas kleiner Bruch ist. Mit Hülfe derselben und der angegebenen Reductionsformeln lassen sich nun die Werthe von  $b_{-\frac{1}{2}}^{(2)}$ ,  $b_{-\frac{1}{2}}^{(5)}$ , etc.,  $b_{-\frac{1}{2}}^{(2)}$ ,  $b_{-\frac{1}{2}}^{(3)}$ , etc., wie auch ihre Differentiale in Beziehung auf  $\alpha$ , wenn es, wie in dem angeführten Werke erfordert wird, sehr leicht finden.